



Groundwater Modelling

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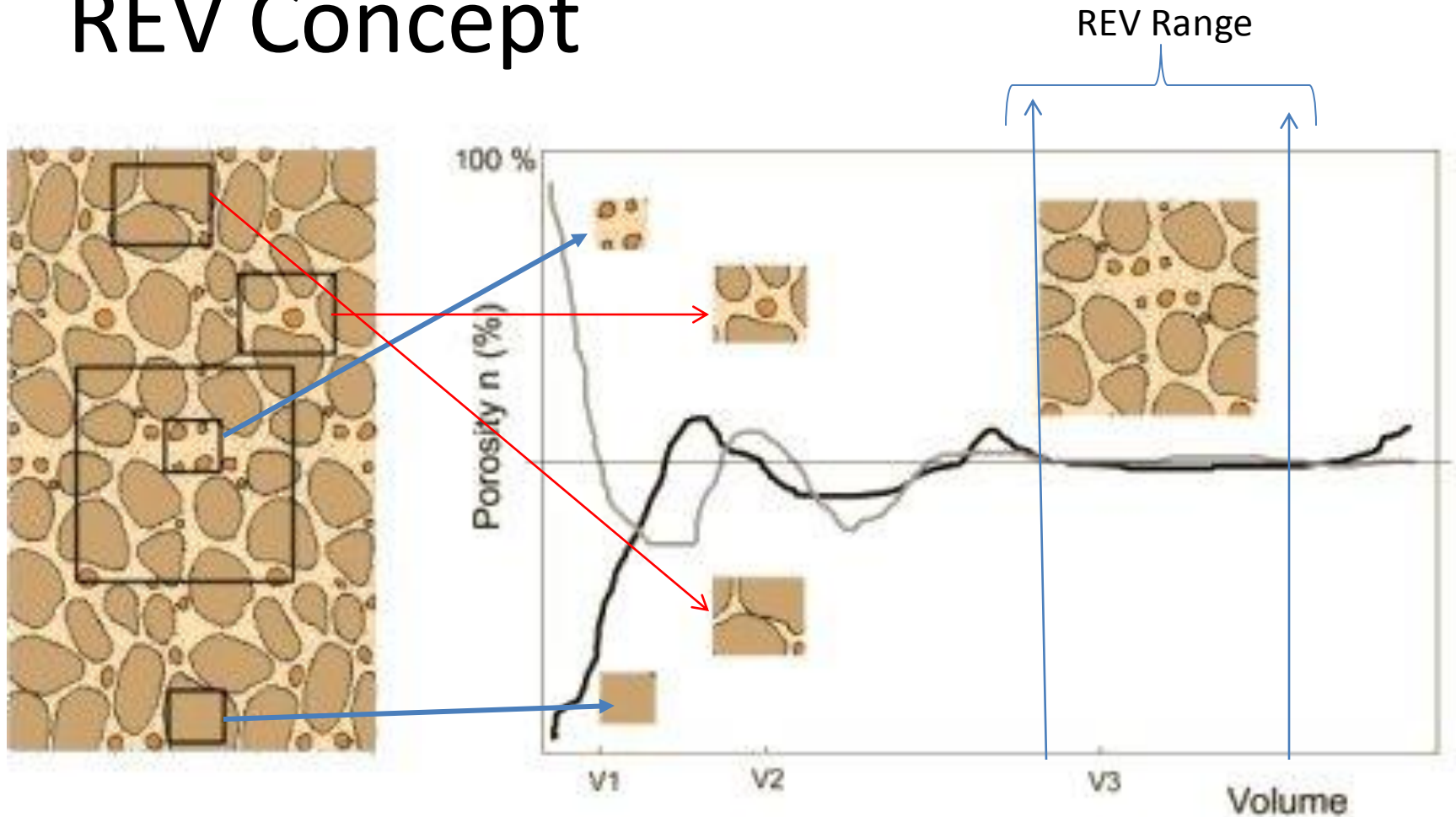
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Recap

- Darcy's Law
- Three Velocities in Porous Media
- Ways to obtain estimates of hydraulic conductivity(K):
 1. Empirical equations
 2. Field Methods:
 - Auger-hole testing
 - Field Tracer tests
 - Field Pumping Tests
 3. Laboratory tests:
 - Constant head and
 - falling head tests

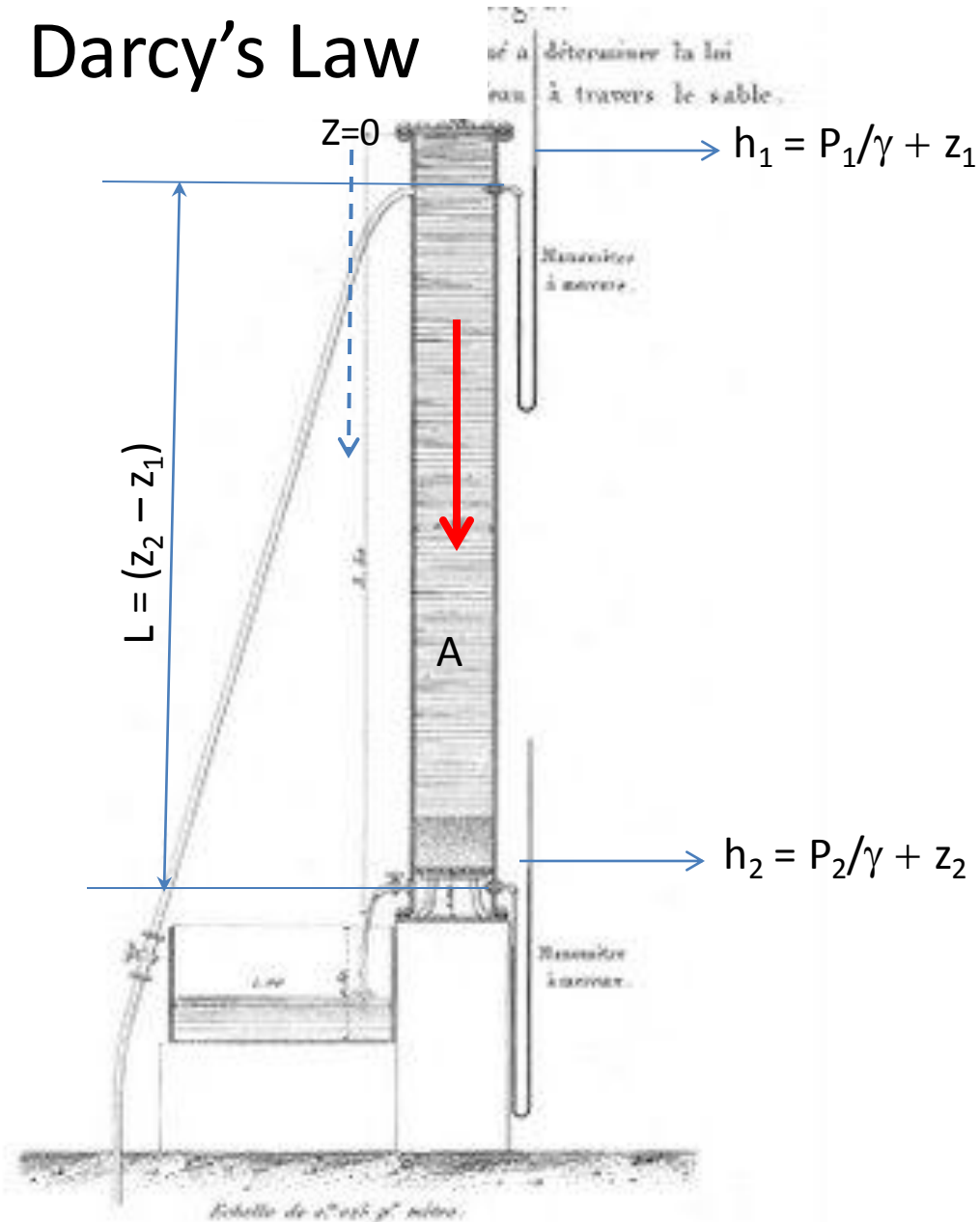
REV Concept



Porosity = 1 when completely inside a pore and Porosity = 0 when completely on the solid

A Porous Media REV contains both solids and pore spaces (that is the continuum of the system)

Darcy's Law



$$Q \propto A$$

$$Q \propto \frac{dh}{dl} = \frac{h_2 - h_1}{L}$$

$$Q \propto A \left(\frac{dh}{dl} \right)$$

$$Q = -KA \left(\frac{dh}{dl} \right)$$

Velocity heads are neglected because the flow is extremely slow

Darcy's law

- Fundamental equation for the flow through porous media
 - Flowrate is proportional to the cross-sectional area
 - Flowrate is proportional to the hydraulic gradient
- The constant of proportionality is referred to as the
- Flowrate per unit area is referred to as the specific discharge or the Darcy velocity

$$v = \frac{Q}{A} = -K \frac{dh}{dx}$$

Three Velocities in Porous Media

- We can envision 3 types of velocities in the porous media

- **Darcy velocity:** This is the average rate at which the water front is moving through the entire cross-section

$$q = \frac{Q}{A} = -K \frac{dh}{dx}$$

- Flux term

- **Average interstitial velocity:** The average rate at which the water front is moving in the pores

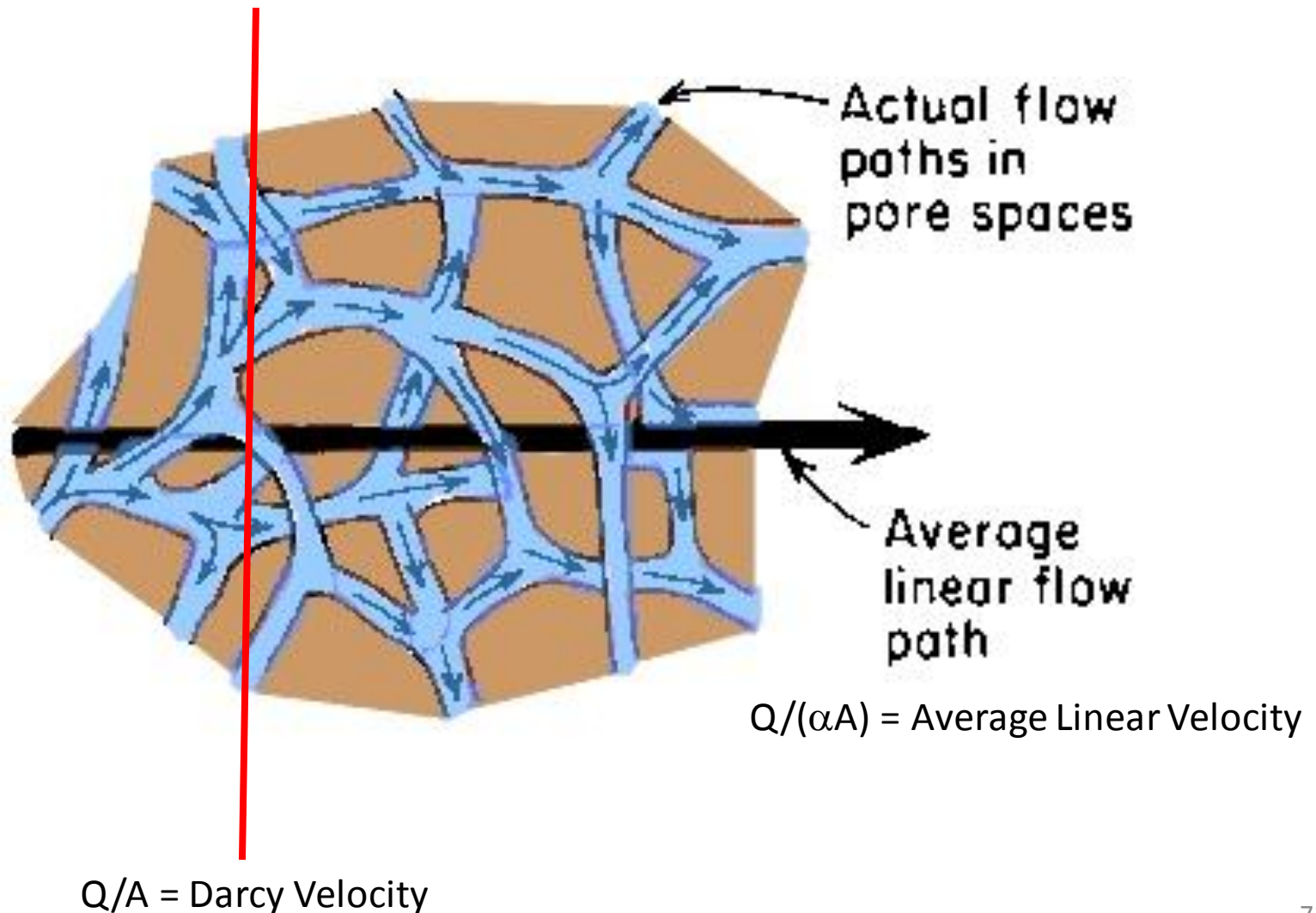
$$v_a = \frac{Q}{\alpha A} = -\frac{K}{\alpha} \frac{dh}{dx}$$

- Groundwater movement only occurs in the pores

- **Actual pore velocity:** Highly heterogeneous and varies from pore to pore

- Can only be characterized in some statistical sense

Average Linear and Actual Velocity



Validity of Darcy's Law

- Darcy's law is applicable when the flow in the porous media is laminar
 - v is the Darcy velocity (flux)
- Reynolds number stated using effective grain size (d_{10}) should be between less than 1 – 10
 - Only a range is specified because turbulence first starts in larger pores and spreads to smaller pores
- In most natural systems the Reynolds number is < 1
 - Darcy's law is therefore valid in most aquifers
- Deviations from Darcy's law do occur in nature
 - In the vicinity of the pumping wells
 - In Karst terrains with large conduits
 - In dense clays with very small diameters (nano-pores)
 - Knudsen flow due to electrical charges

$$R = \frac{\rho v D_{10}}{\mu}$$

Darcy's law

- Reasonable approximation of the flow in most aquifers
- May not be suitable when the pores are extremely small
 - Micro to nanopores
- May not be suitable when gradients are very steep
 - Near a well
- May not be suitable when the flows are high
 - Karst Conduits

Often used as a first-cut engineering approximation even in cases it is not fully valid



$$v = \frac{Q}{A} = -K \frac{dh}{dx}$$

To apply Darcy's Law

- Need to know Hydraulic Conductivity
- Need to know the gradient
- (Need to know the cross-sectional area)

Hydraulic Conductivity

- Hydraulic conductivity is a function of both aquifer and fluid (water) properties

$$K = \frac{k\rho g}{\mu}$$

Intrinsic Permeability (darcy)

Acceleration due to gravity

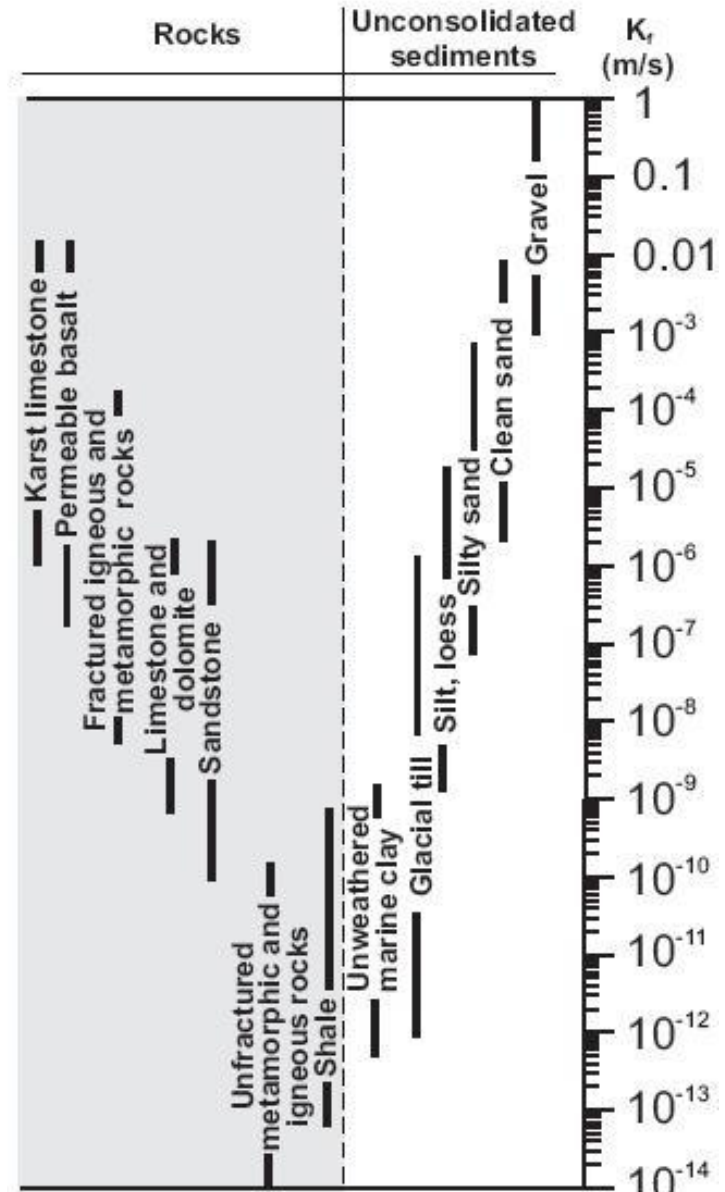
Dynamic viscosity

Fluid density

$$1 \text{ Darcy} = 0.987 \mu\text{m}^2$$

Typical Hydraulic Conductivity Values

- Hydraulic Conductivity varies over several orders of magnitude.
 - Karst and Sand & Gravel most permeable
 - Shales and unfractured rocks are least permeable



Determination of Hydraulic Conductivity

- There are several methods described in the literature
 - Empirical equations for permeability
 - Kozney-Carman formula $k = cd^2$
 - Based on grain-size distribution
 - The coefficient, c , is a function of porosity and grain shape factor
 - Extremely crude; should be avoided when possible
 - When data from alternative methods are available

Field Methods

- Tracer Tests

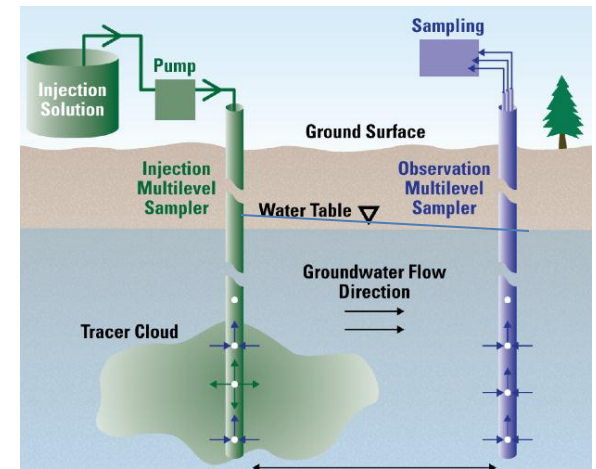
- Difficult to perform

- Need to know porosity
 - Wells need to be close
 - Tracer movement affected by other processes
 - Flow direction must be known

$$v_a = \frac{K}{\alpha} \frac{h}{L}$$

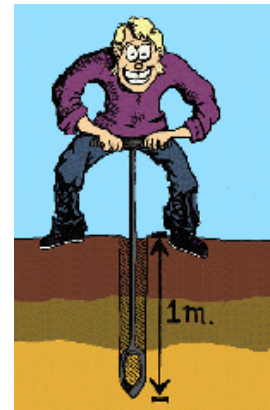
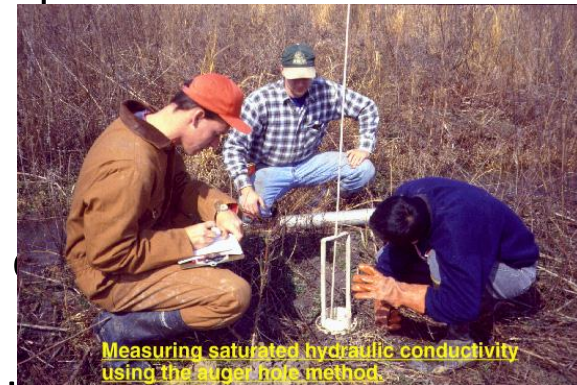
$$v_a = \frac{L}{t}$$

$$K = \frac{\alpha L^2}{ht}$$



- Auger-hole tests

- Simple test
 - Water of known volume is bailed
 - Recovery is used to estimate K
 - Well suited for shallow water-tables
 - Value of K is a local estimate
 - Aquifer is disturbed



$$K(m/d) = \frac{C}{864} \left(\frac{dy}{dt} \right)$$

- Pumping tests

- The best method for K
 - Will spend a considerable amount of time later in the class



Laboratory Methods

- Hydraulic Conductivity determined using a permeameter

- Constant Head Permeameter
- Falling Head Permeameter

$$v = \frac{Q}{A} = -K \frac{dh}{dx}$$

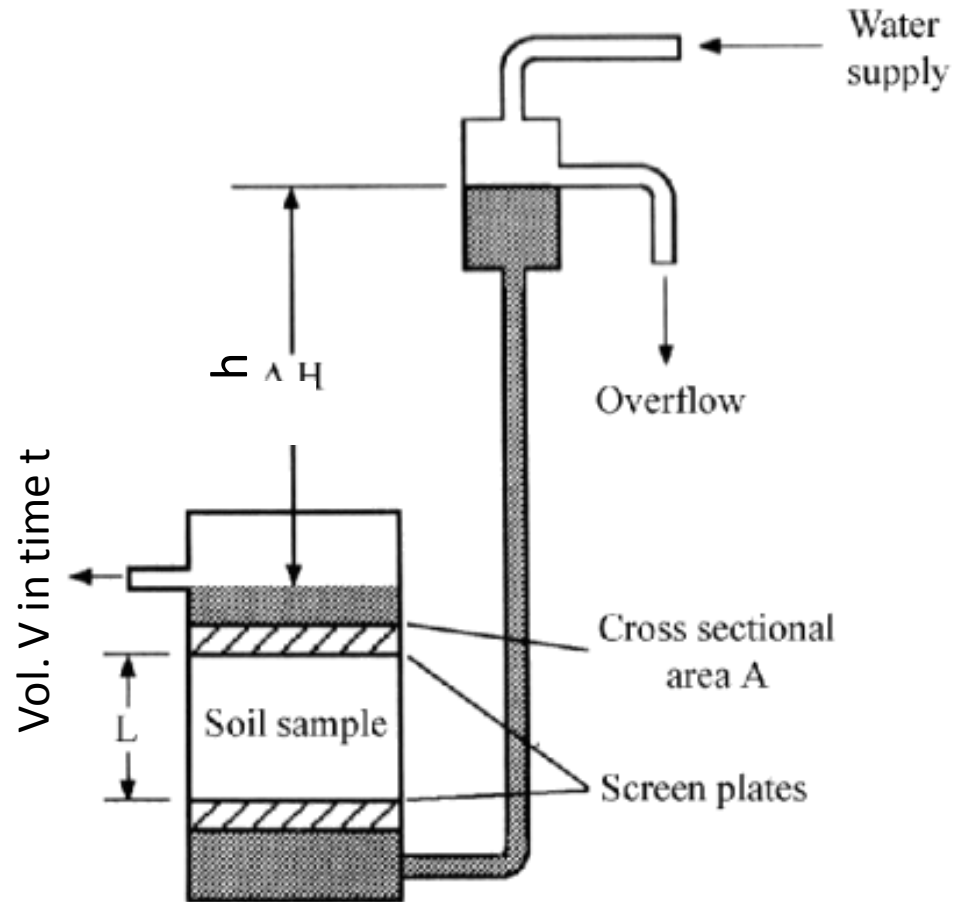
- Basic principle behind permeameter
 - Maintain flow through a small sample
 - The cross-sectional area is known
 - Measure the flowrate and head loss
 - Estimate the unknown K (hydraulic conductivity)

Constant Head Test

- Water enters from the bottom
- Passes through the sample
- Overflow is collected over time

$$Q = \frac{V}{t} = -KA \left(-\frac{h}{L} \right)$$

$$K = \frac{VL}{Aht}$$



- The sample should be as undisturbed as possible
- The sample should be fully saturated (with no entrapped air)
- Run the experiment with several heads and average

Falling Head Test

$$Q = AV = (\pi r^2) \frac{dh}{dt}$$

Flowrate in the tube

$$Q = -K(\pi r_s^2) \left[\frac{h}{L} \right]$$

Darcy's law

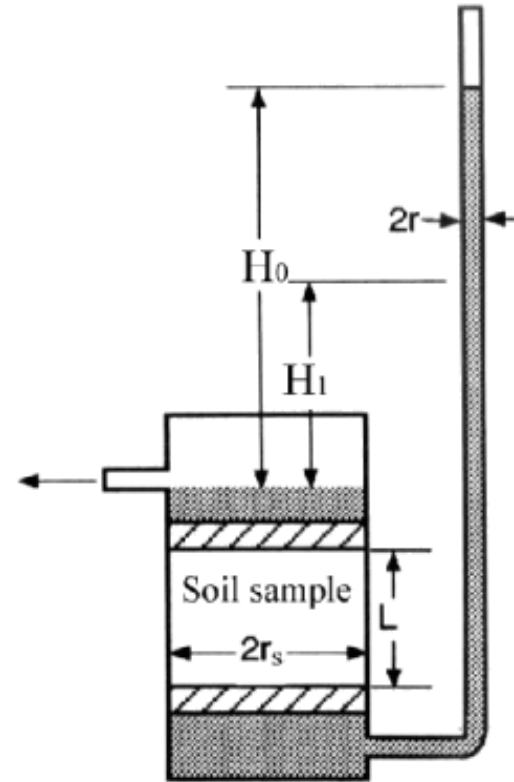
(Note the hydraulic head, h , is unknown)

$$Q = AV = (\pi r^2) \frac{dh}{dt} = -K(\pi r_c^2) \left[\frac{h}{L} \right]$$

$$\frac{dh}{dt} = \left(\frac{K}{L} \right) \left(\frac{r_s^2}{r^2} \right) h$$

Separate the variables & integrate

$$\int_{H_o}^{H_1} \frac{dh}{h} = \int_0^t \left(-\frac{K}{L} \right) \left(\frac{r_s^2}{r^2} \right) dt$$



$$\ln \left(\frac{H_1}{H_o} \right) = -K \left(\frac{r_s^2 t}{r^2 L} \right)$$

$$K = \left\{ -\ln \left(\frac{H_1}{H_o} \right) \left(\frac{r^2 L}{r_s^2 t} \right) \right\}$$

Laboratory Tests

- Laboratory tests of K are more reliable than empirical equations
 - Cheaper than field tests
- The values from laboratory tests are usually a lot different than what is measured in the field
 - Small sample size → Not all heterogeneities are sampled
- Obtaining relatively undisturbed cores is usually problematic
 - Unconsolidated sediments
 - Repacking usually is not accurate

Used more often by Geotechnical Engineers than Groundwater Hydrologists