



Groundwater Modelling

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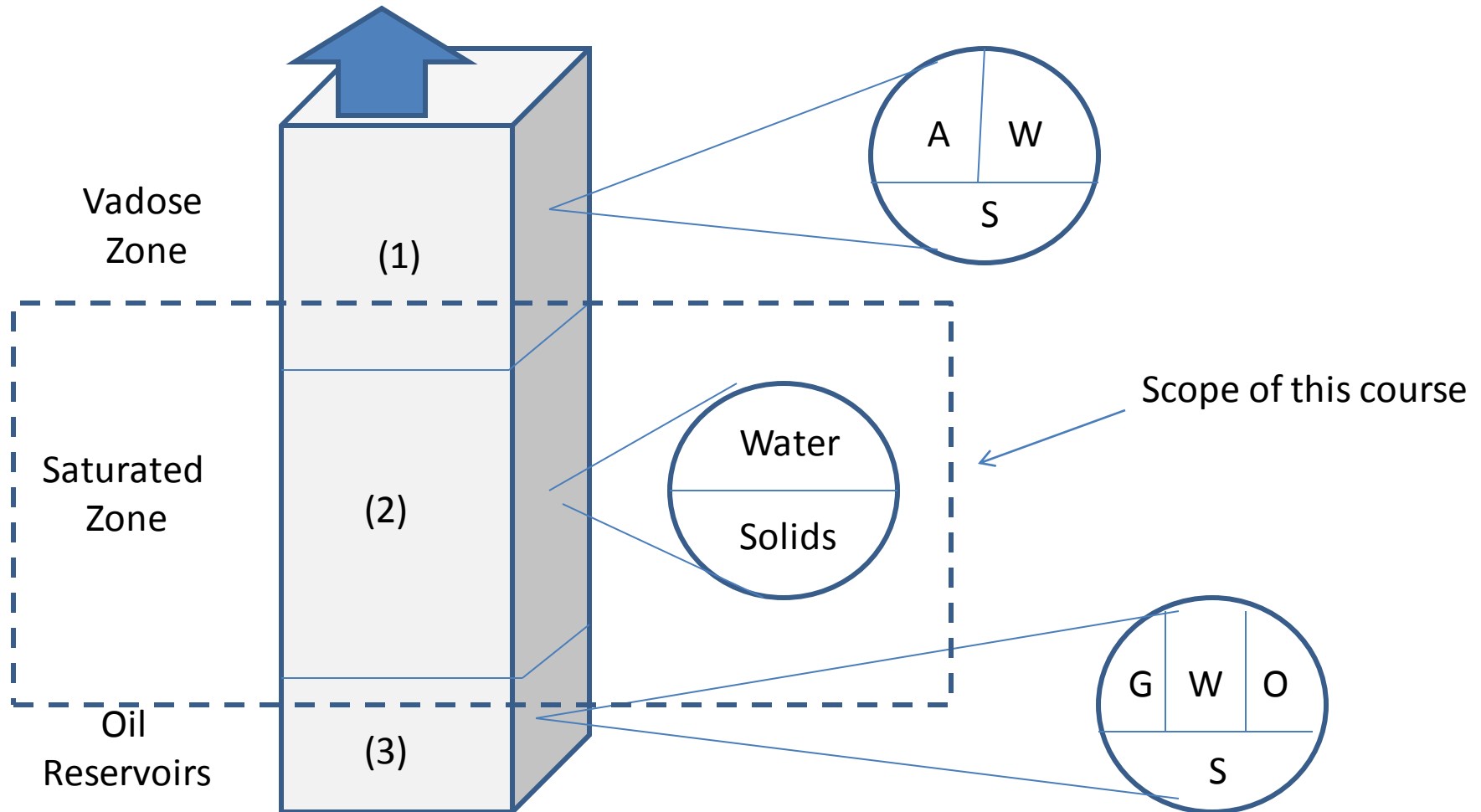
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Recap

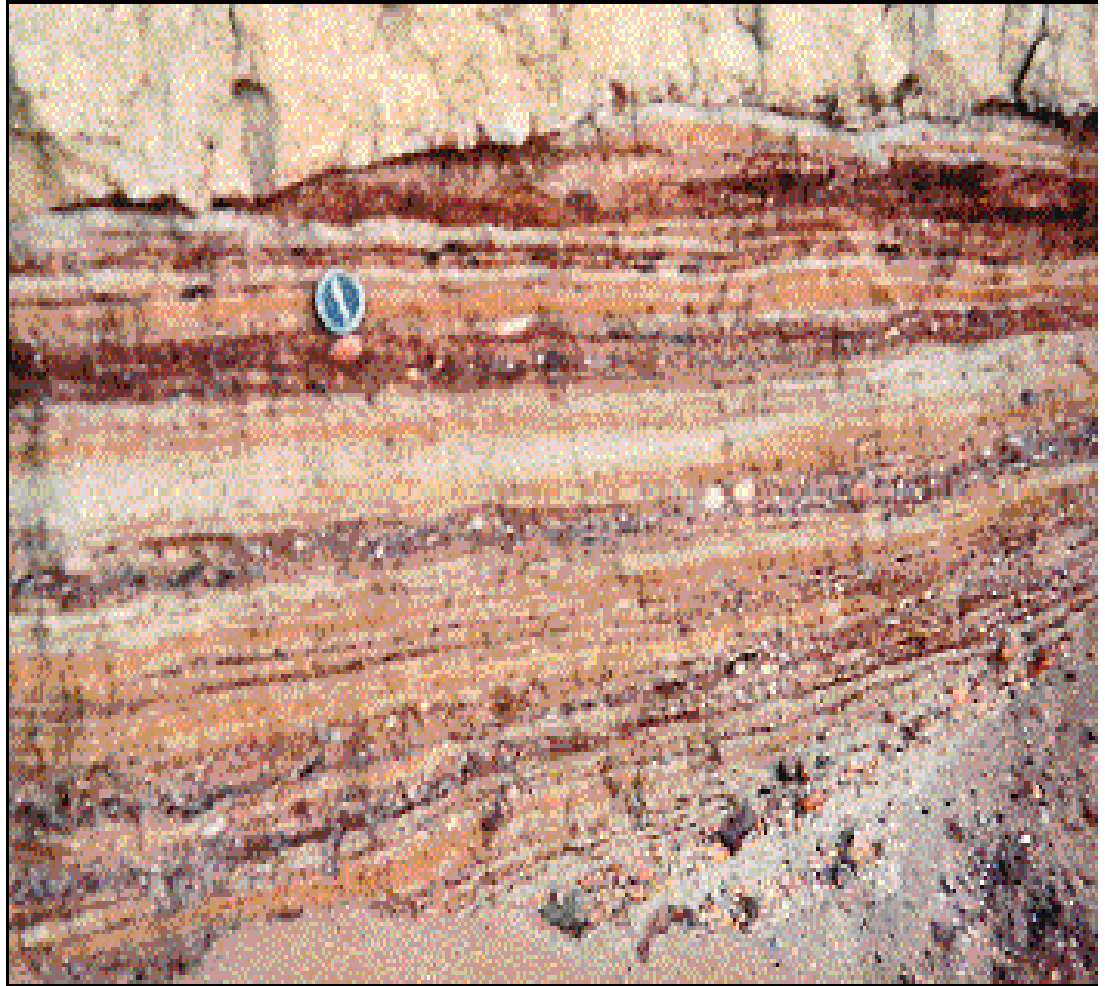
- Aquifer types based on geology
 - Sand and Gravel; Fractured bedrock, sandstone, limestone (Karst)
- Aquifer types based on fluid mechanics
 - Confined, unconfined and leaky
- Porosity
- Porosity-density relationship
- Specific Yield and Specific retention

Recap the Subsurface Environment



Aquifer Types

- Sand and Gravel Formations
 - Typically unconsolidated or partially consolidated
 - About 90% of the aquifers are of this type
 - Alluvial deposits
 - May contain silt, clays

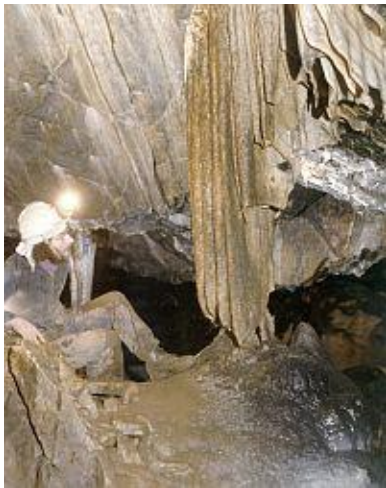


Aquifer Types

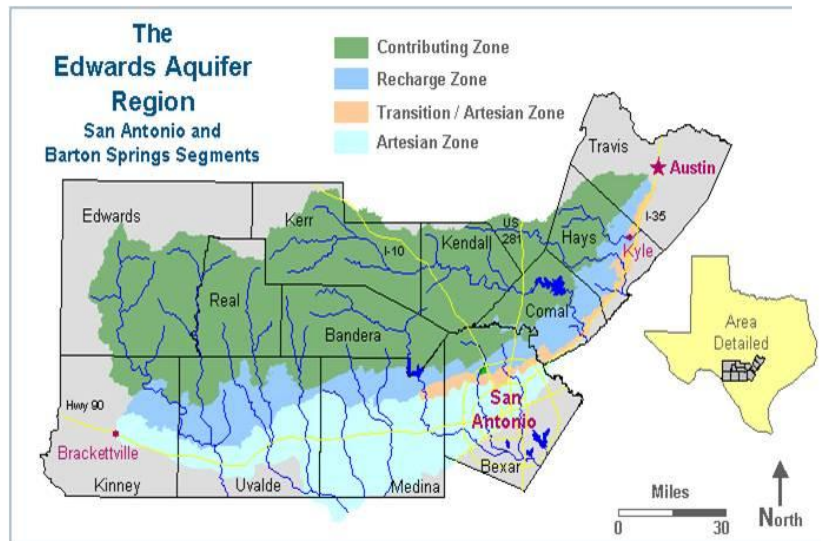
- Limestone (Karst) Formation
 - Dissolution of calcium carbonate
 - Primary and secondary porosity
 - Pore sizes can vary dramatically



(Floridian Aquifer Core)

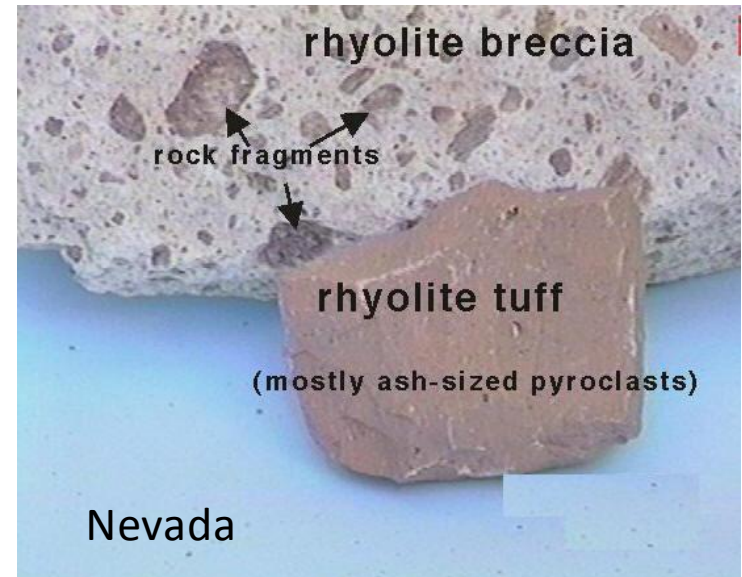


(Sinkhole in Ireland)



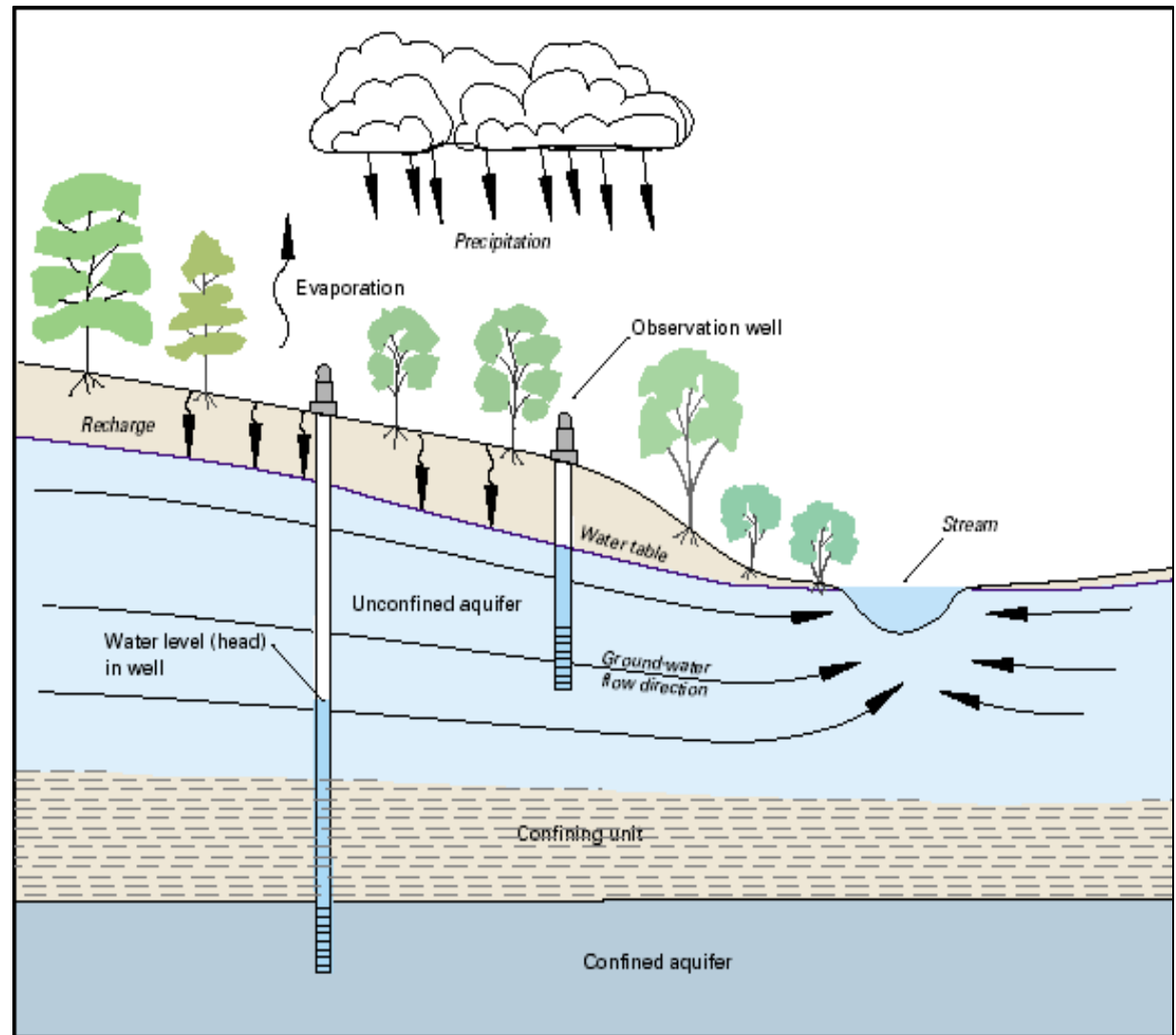
Aquifer Types

- Volcanic Tuff
 - Rhyolites
- Fractured bedrock
 - Basalt
- Sandstone formations
 - Best flow at joints

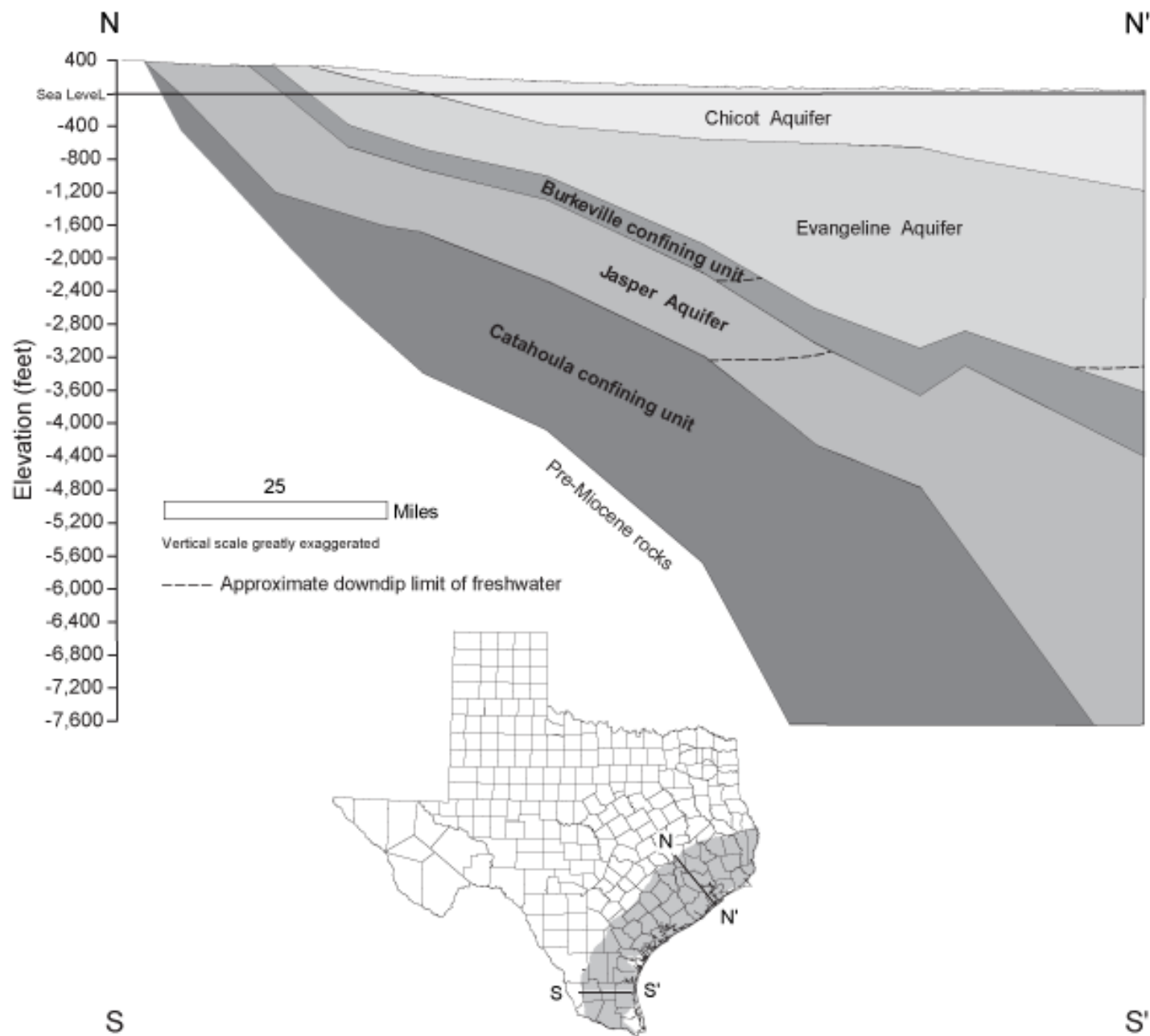


Aquifer Types

- Unconfined
 - Connected to the atmosphere
 - Typically shallower
 - Water table mimics topography
- Confined
 - Water under pressure
 - Between two confining layers
 - Typically deeper
- Leaky
 - Semi-confined

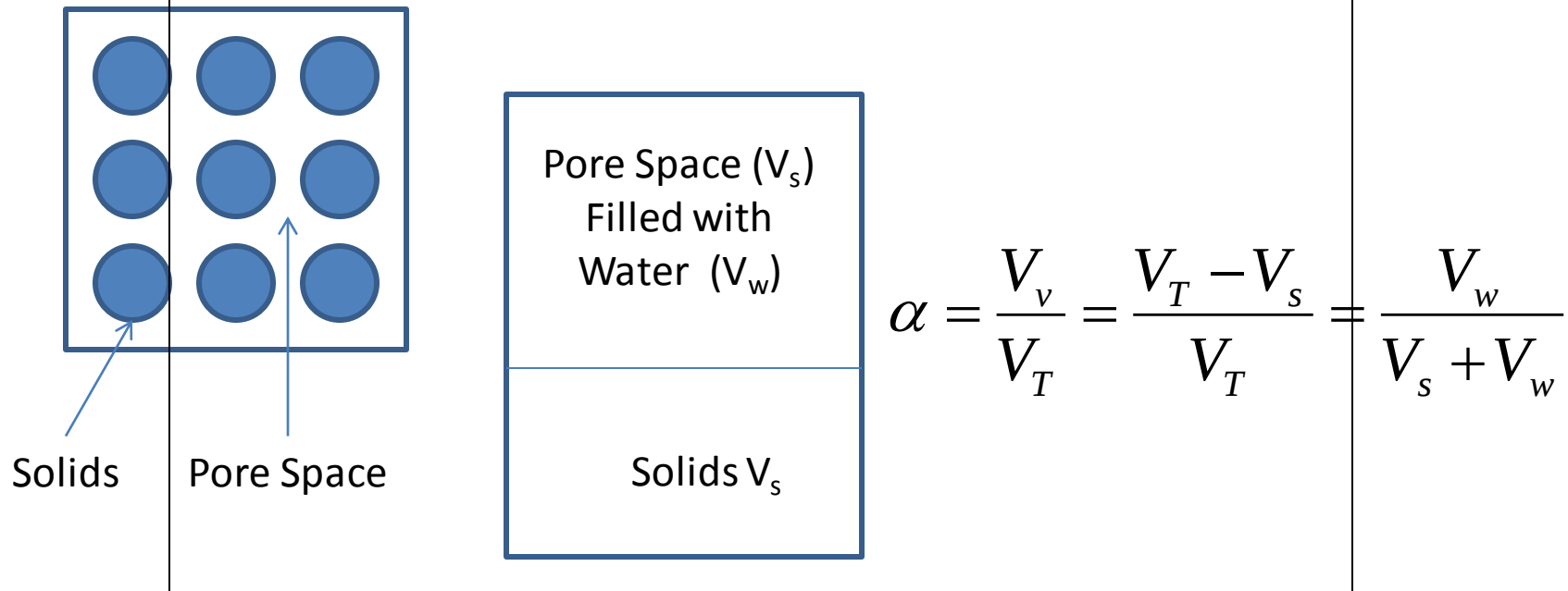


An aquifer can be both confined and unconfined at different locations



Rock Properties Affecting Groundwater

- A porous matrix comprises of pore space and solids
 - Porosity $\rightarrow$ volume of voids/total volume



In aquifers the pore-space is completely filled with water

Knowledge of Porosity is necessary to calculate the volume of water stored in the aquifer

Porosity

- The pore-space in aquifers is not uniform
 - The grain size is a surrogate for the pore-size
- Pore-sizes can be classified into three types
 - Sub-capillary pores
 - Very tiny the forces of adhesion hold water
 - Capillary pores
 - Water held by surface tension (typically there is trapped air)
 - Super-capillary pores
 - Larger pores water is held in compression

Total and Effective Porosity

- Total Porosity → considers the volume of all voids
 - Some of the voids are not inter-connected
 - Also referred to as vugs
- Effective Porosity → considers on the volume of the interconnected voids
 - Flow only takes place through the interconnected voids
- For unconsolidated sediments the total and effective porosities are close

Porosity – Density Relationship

$$\alpha = \frac{V_w}{V_T} = \frac{V_T - V_s}{V_T} = 1 - \frac{V_s}{V_T}$$

M_s = Mass of the Solids

$M_T = M_s + M_w$ = Total Mass

$M_T \approx M_s$ as $M_w \ll M_s$

We can re-write the porosity expression as:

$$\alpha = 1 - \frac{\left(\frac{V_s}{M_s} \right)}{\left(\frac{V_T}{M_s} \right)}$$

$$\begin{array}{cc} \text{Particle} & \text{Bulk} \\ \text{Density} & \text{Density} \\ \hline \rho_m = \frac{M_s}{V_s} & \text{and} \quad \rho_d = \frac{M_s}{V_T} \end{array}$$

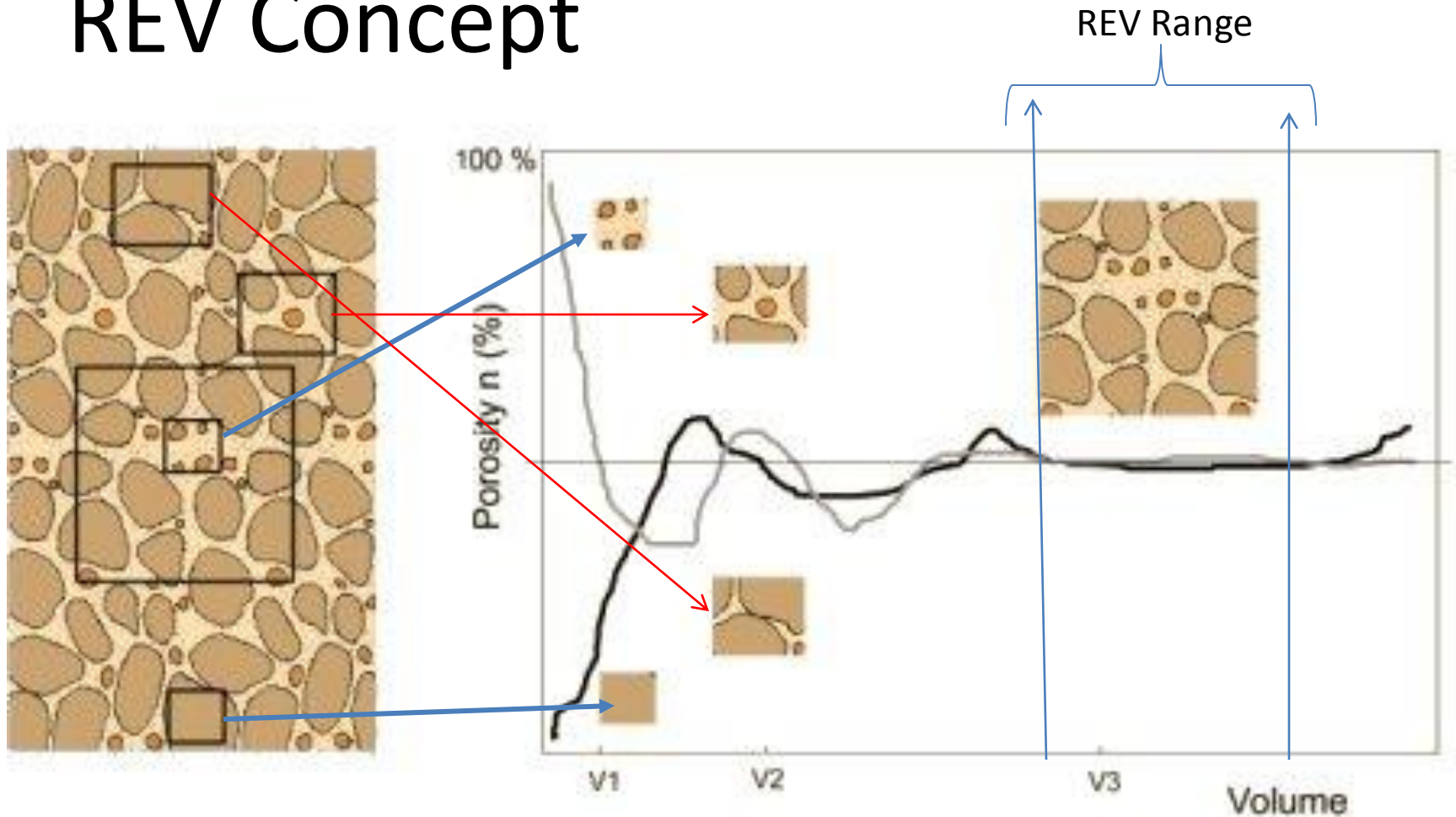
$$\alpha = 1 - \frac{\left(\frac{1}{\rho_s} \right)}{\left(\frac{1}{\rho_d} \right)}$$

$$\boxed{\alpha = 1 - \frac{\rho_d}{\rho_m}}$$

Representative Elementary Volume (REV)

- We can in theory write equations to describe groundwater flow at every point (pore) in the porous media
 - Microscopic approach
- Not very practical to do so because we cannot describe the porous media with such detail
- We describe the flow over an average volume of the porous media
 - Continuum approach
 - Common practice in Physics and Engineering

REV Concept



Porosity = 1 when completely inside a pore and Porosity = 0 when completely on the solid

A Porous Media REV contains both solids and pore spaces (that is the continuum of the system)

Specific Yield and Specific Retention

- Gravity drainage plays a major role in movement of unconfined aquifers
 - They are not subjected to significant gage pressures
 - Gravity cannot drain the aquifer completely
 - Some water will be held back due to capillary action
 - Some water trapped in disconnected pores
- Specific Retention (S_r): Volume fraction of water that will be retained against the force of gravity
- Specific Yield (S_y): Volume fraction of water that will drain under the influence of gravity

Specific Yield and Specific Retention

- Specific Retention:

$$S_r = \frac{w_r}{V_T}$$

- Specific Yield

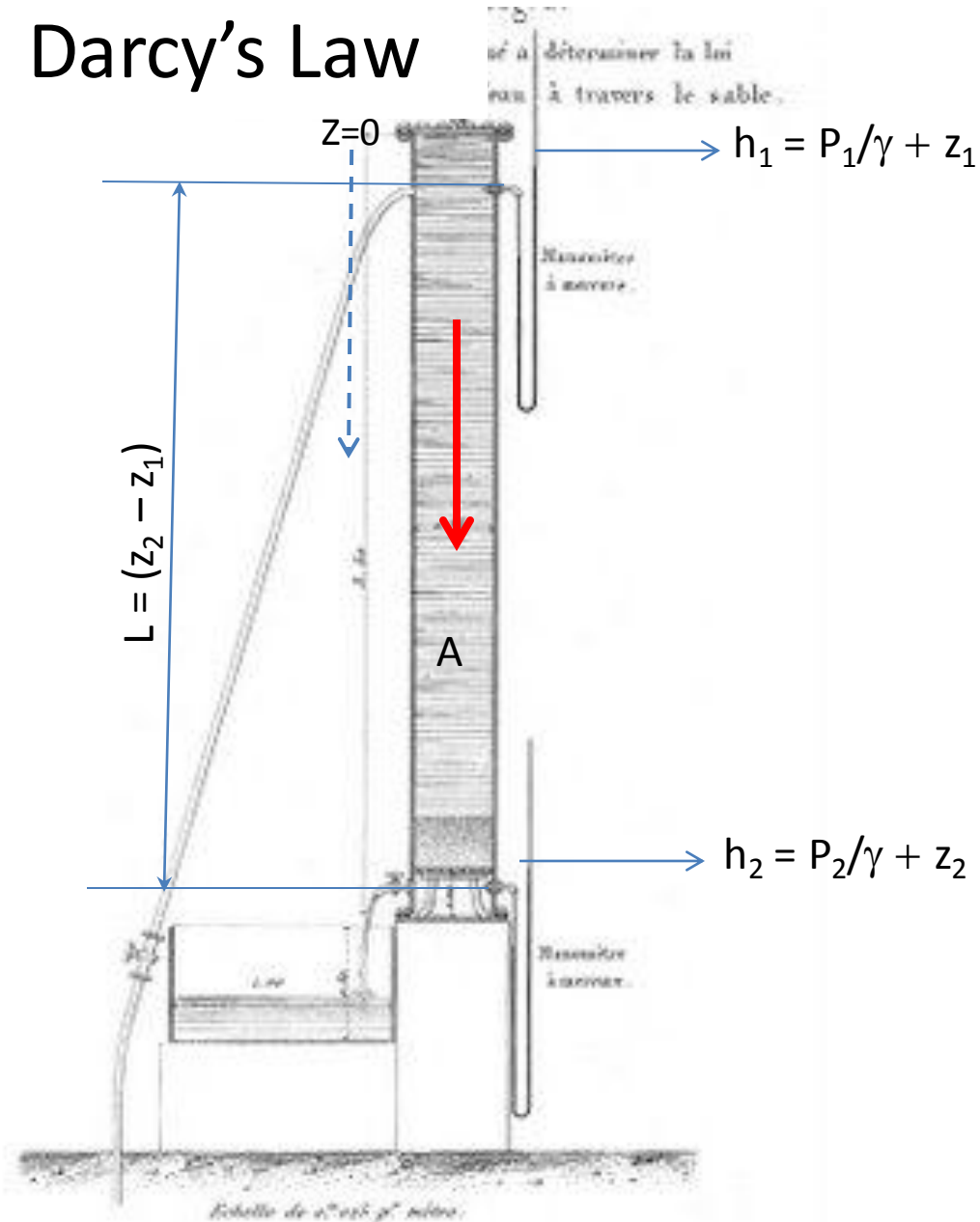
$$S_y = \frac{w_y}{V_T}$$

$$\alpha = \frac{V_w}{V_T} = \frac{w_r + w_y}{V_T}$$

$$\alpha = \frac{w_r}{V_T} + \frac{w_y}{V_T} = S_r + S_y$$

Water in the pore is either retained or drained (yielded)

Darcy's Law



$$Q \propto A$$

$$Q \propto \frac{dh}{dl} = \frac{h_2 - h_1}{L}$$

$$Q \propto A \left(\frac{dh}{dl} \right)$$

$$Q = -KA \left(\frac{dh}{dl} \right)$$

Velocity heads are neglected because the flow is extremely slow

Darcy's law

- Fundamental equation for the flow through porous media
 - Flowrate is proportional to the cross-sectional area
 - Flowrate is proportional to the hydraulic gradient
- The constant of proportionality is referred to as the hydraulic conductivity
- Flowrate per unit area is referred to as the specific discharge or the Darcy velocity

$$v = \frac{Q}{A} = -K \frac{dh}{dx}$$

Hydraulic Conductivity

- Hydraulic conductivity is a function of both aquifer and fluid (water) properties

$$K = \frac{k\rho g}{\mu}$$

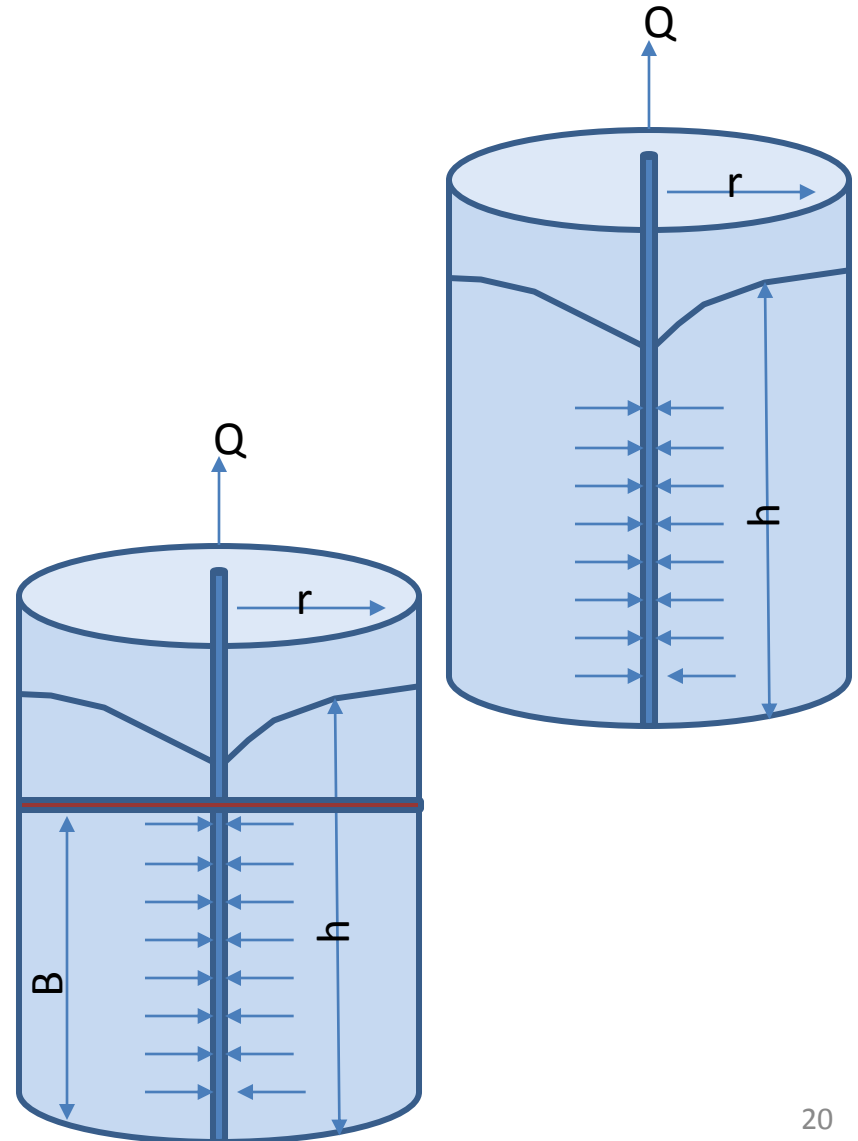
Diagram illustrating the components of the hydraulic conductivity equation:

- k : Intrinsic Permeability (darcy)
- ρ : Fluid density
- g : Acceleration due to gravity
- μ : Dynamic viscosity

$$1 \text{ Darcy} = 0.987 \mu\text{m}^2$$

Darcy's Law in Radial Coordinates

- Darcy's law in radial coordinates is given as:
 - $Q = -KA \frac{dh}{dr}$
 - A in confined aquifer
 - $A = 2\pi rB$
 - B is the aquifer thickness
 - A in unconfined aquifer
 - $A = 2\pi rh$
 - Area varies with hydraulic head



Validity of Darcy's Law

- Darcy's law is applicable when the flow in the porous media is laminar
 - v is the Darcy velocity (flux)
- Reynolds number stated using effective grain size (d_{10}) should be between less than 1 – 10
 - Only a range is specified because turbulence first starts in larger pores and spreads to smaller pores
- In most natural systems the Reynolds number is < 1
 - Darcy's law is therefore valid in most aquifers
- Deviations from Darcy's law do occur in nature
 - In the vicinity of the pumping wells
 - In Karst terrains with large conduits
 - In dense clays with very small diameters (nano-pores)
 - Knudsen flow due to electrical charges

$$R = \frac{\rho v D_{10}}{\mu}$$

Three Velocities in Porous Media

- We can envision 3 types of velocities in the porous media

- **Darcy velocity:** This is the average rate at which the water front is moving through the entire cross-section

$$q = \frac{Q}{A} = -K \frac{dh}{dx}$$

- Flux term

- **Average interstitial velocity:** The average rate at which the water front is moving in the pores

$$v_a = \frac{Q}{\alpha A} = -\frac{K}{\alpha} \frac{dh}{dx}$$

- Groundwater movement only occurs in the pores

- **Actual pore velocity:** Highly heterogeneous and varies from pore to pore

- Can only be characterized in some statistical sense