



Groundwater Modelling

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Recap

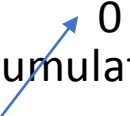
- Groundwater Flow – Steady-state Conditions
- Water Balance
 - Accumulation = In – Out $\pm$ Source/Sinks
- Groundwater-surface water interactions
 - Recharge = Discharge
 - Recharge = Pumping
 - Safe Yield
- Groundwater management doctrines
 - Rule of capture
 - Stakeholder driven management

Application of Darcy's law

- We can apply Darcy's law when we know both K and hydraulic conductivity
 - We need to know the cross-sectional area as well
- Cross-sectional area depends upon aquifer thickness in confined aquifers
- Cross-sectional area depends upon hydraulic head in unconfined aquifer
 - Complicates mathematics

Groundwater Flow – Steady-state Conditions

- Steady-state conditions implies there is no net accumulation within the aquifer
 - Net accumulation in the aquifer is zero
 - The sum of all inflows and sources equals the sum of all outflows and sinks
- Steady-state conditions set in after a long-period of time $t \rightarrow \infty$
 - The system reaches equilibrium in terms of inflows and outflows
- Steady-state conditions seldom set in real world systems
 - However, we assume steady-state to sometime simplify our design calculations


$$\text{Accumulation} = \text{Inflows} - \text{Outflows} \pm \text{sources/sinks}$$

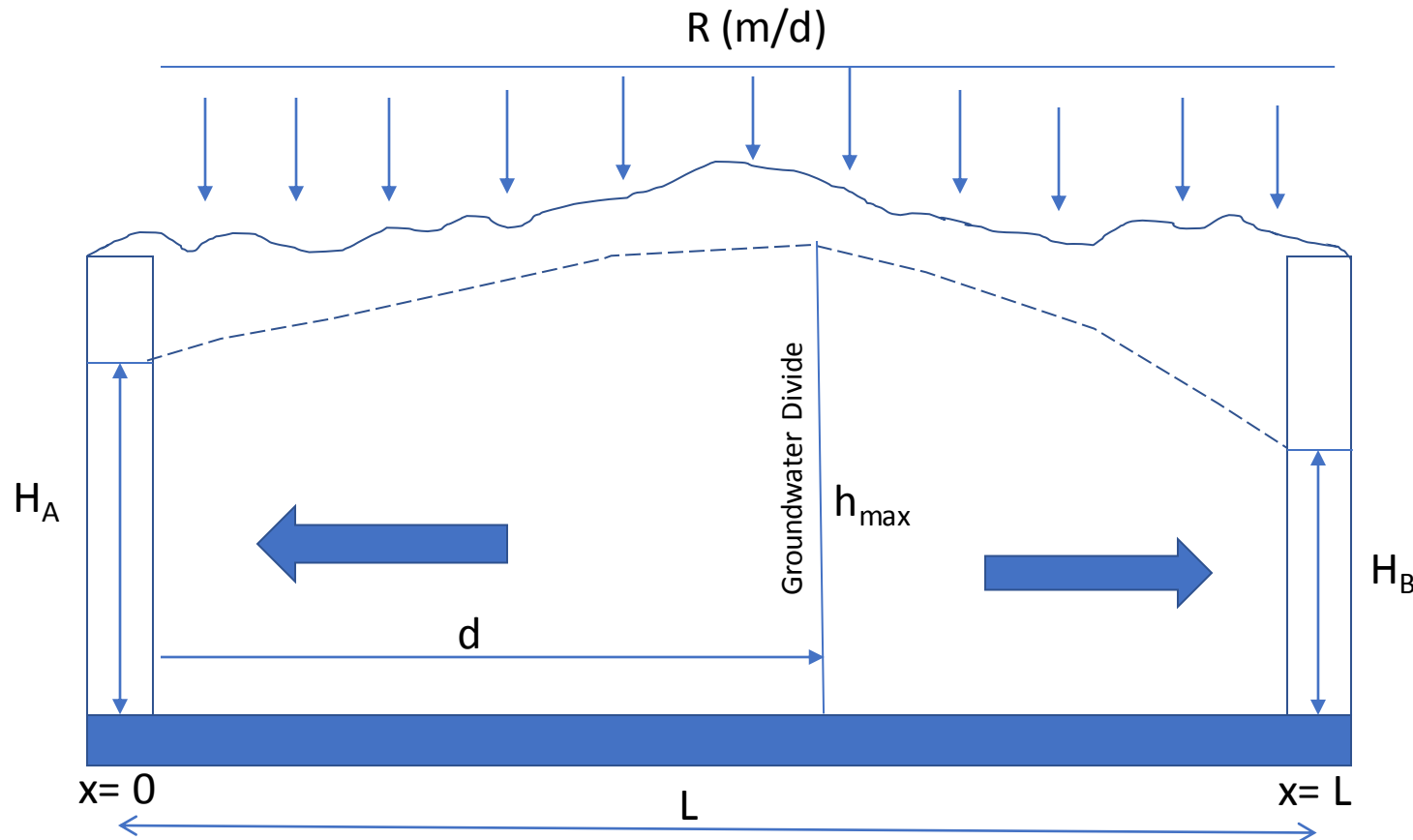
Steady-state conditions

- Steady-state conditions are useful to analyze certain practical problems:
 - Estimation of groundwater flow to streams (baseflows)
 - Long-term pumping tests in aquifers
 - Confined aquifers
 - Unconfined aquifers

Baseflow to streams

- Consider an aquifer between two streams
- The streams fully penetrate the aquifer
- The stream stage (hydraulic head) is assumed constant
 - The width of the stream is wide that any water additions will cause negligible changes in head
- There is a constant recharge applied to the aquifer, R (m/d)
 - The recharge does not change with time

Conceptual Schematic



Where does the groundwater divide occur?

What is the baseflow to each stream ?

The heads in rivers stay roughly constant as the rivers are wide and cause water to spread out
Water level in the aquifer rises due to recharge and results in a mound formation
Creates a groundwater divide and causes water to flow in both directions

Steady-state solution

- The head at any point in the aquifer (h); the location of the groundwater divide from the origin (d) and the maximum height of the water table ($h_{\max}$) at the groundwater divide are given by the following equations

$$h^2 = h_a^2 - \frac{(h_a^2 - h_b^2)}{L}x + \frac{R}{K}(L - x)x \quad \left. \vphantom{h^2 = h_a^2 - \frac{(h_a^2 - h_b^2)}{L}x + \frac{R}{K}(L - x)x} \right\} \text{Head at any point that is a distance } x \text{ from the origin}$$

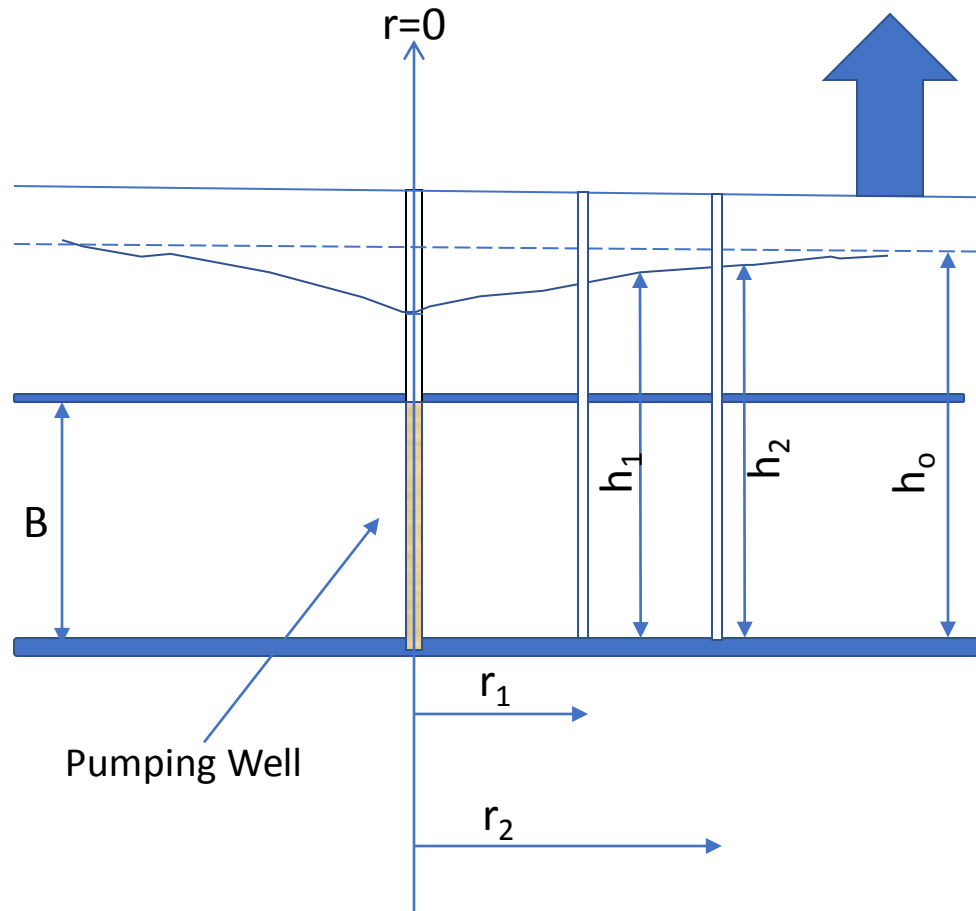
$$d = \frac{L}{2} - \frac{K}{R} \frac{(h_a^2 - h_b^2)}{2L} \quad \left. \vphantom{d = \frac{L}{2} - \frac{K}{R} \frac{(h_a^2 - h_b^2)}{2L}} \right\} \text{Distance from the origin where the groundwater divide occurs}$$

$$h_{\max}^2 = h_a^2 - \frac{(h_a^2 - h_b^2)}{L}d + \frac{R}{K}(L - d)d \quad \left. \vphantom{h_{\max}^2 = h_a^2 - \frac{(h_a^2 - h_b^2)}{L}d + \frac{R}{K}(L - d)d} \right\} \text{Height of the water table at groundwater divide}$$

Flow in Confined Aquifer

- Assume a fully penetrating well tapping a confined aquifer
 - The aquifer is homogeneous and isotropic
 - Hydraulic conductivity is constant
 - Flow to the well is radial
 - Water flows from all directions
 - Steady-state conditions
 - Hydraulic head varies only in space and not in time
 - Inflow to the well from the aquifer = outflow (pumping) from the well
 - Initial hydraulic head is uniform around the well

Conceptualization and Equation



$$Q = -K(2\pi rB) \frac{dh}{dr}$$

$$\frac{Q}{2\pi B} \int_{r_1}^{r_2} \frac{dr}{r} = -K \int_{h_1}^{h_2} dh$$

$$\frac{Q}{2\pi B} \ln\left(\frac{r_2}{r_1}\right) = -K(h_2 - h_1)$$

$$|K| = \frac{Q}{2\pi B} \frac{1}{(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right)$$

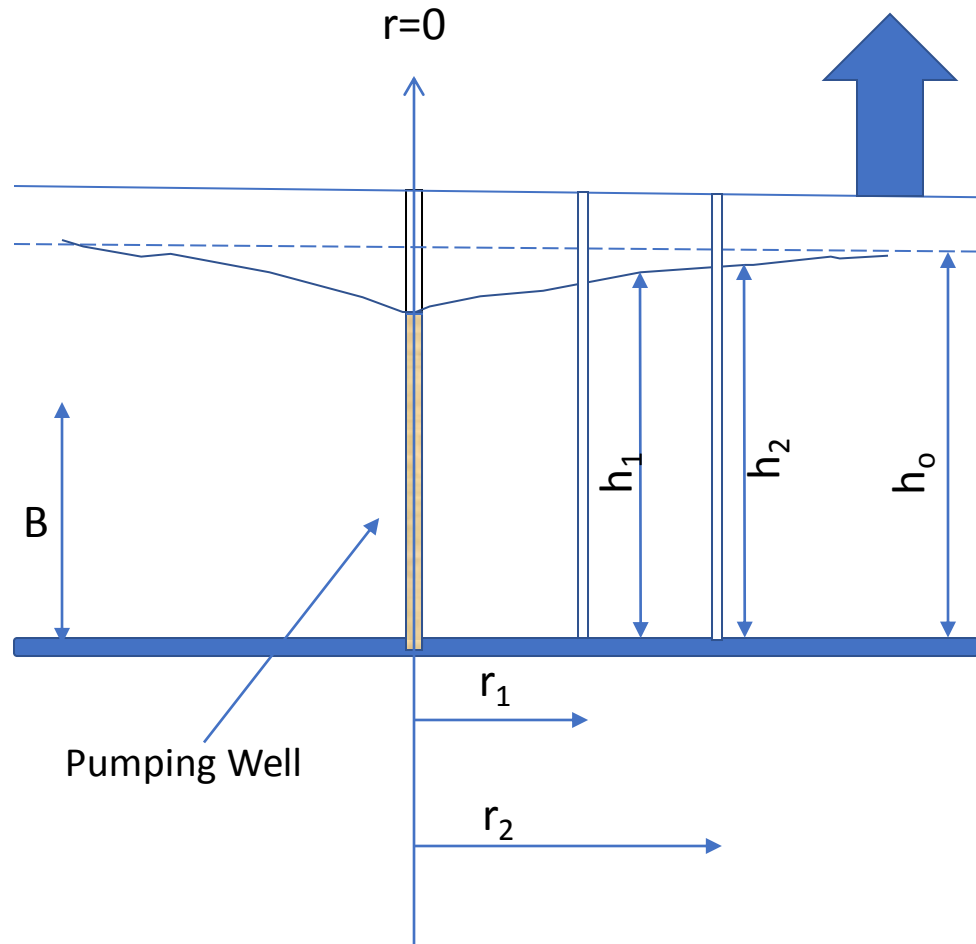
(Negative sign simply indicates flow is towards the well)

$s = h_o - h$ Where, s , is the drawdown

Flow in Unconfined Aquifer

- Well fully penetrates the aquifer
- The aquifer is homogeneous and isotropic
- Flow to the well is radial
- Pumping is steady
- Steady-state conditions
 - Inflow to well from the aquifer = outflow from the well due to pumping
- Unconfined conditions
 - Water drains under gravity
- Initial aquifer head is uniform around the well

Flow in Unconfined Aquifer



$$Q = -K(2\pi h) \frac{dh}{dr}$$

$$\frac{Q}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = -K \int_{h_1}^{h_2} h \, dh$$

$$\frac{Q}{\pi} \ln\left(\frac{r_2}{r_1}\right) = -K(h_2^2 - h_1^2)$$

$$|K| = \frac{Q}{\pi} \frac{1}{(h_2^2 - h_1^2)} \ln\left(\frac{r_2}{r_1}\right)$$

(Negative sign simply indicates flow is towards the well)