



Groundwater Modelling

Gurmu Deressa(Msc.)

Lecturer of Water Resource and Irrigation Engineering
Wollo University, Kombolcha Institute of Technology

Office Hours: TH 2:00 – 3:00 PM or by Appt; You can usually stop by if the door is open

Ph: +25117131813

E-mail: gurmud@gmail.com

Recap

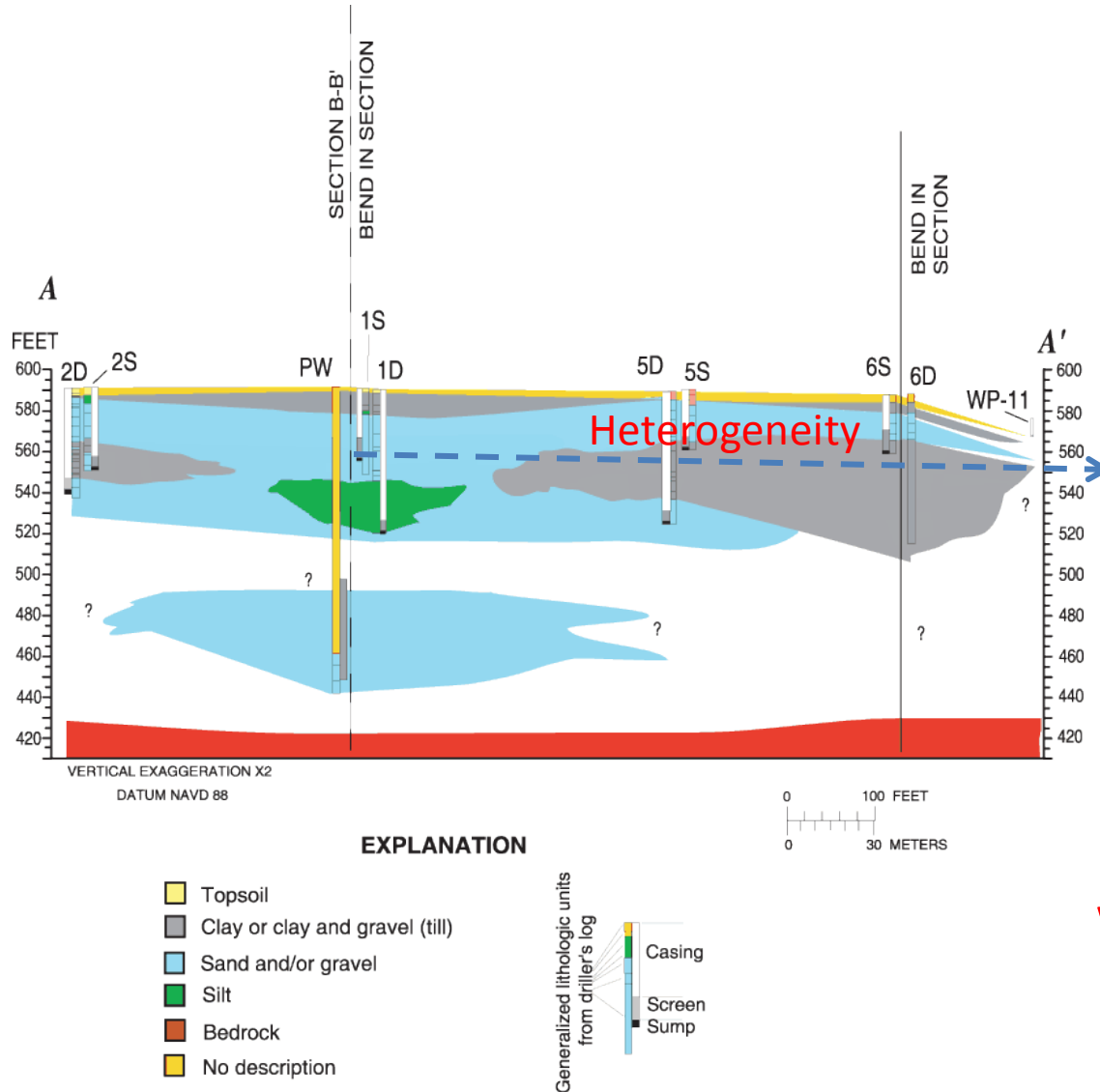
- Aquifer Heterogeneity
- How to estimate gradient:
 - Graphical method
 - Fitting an equation of a plane
- Measuring Hydraulic Head
- Making Water Level Measurements

$$v = \frac{Q}{A} = -K \frac{dh}{dx}$$

Aquifer Heterogeneity

- Refers to variation of aquifer properties along a given direction
 - How does hydraulic conductivity (K) vary along X-axis
- Heterogeneity is particularly evident along streambeds
 - Bigger rocks (pores) are at the headwaters
 - Finer particles (pores) are at the tailwaters
 - Easier to move particles that are small

Aquifer Heterogeneity

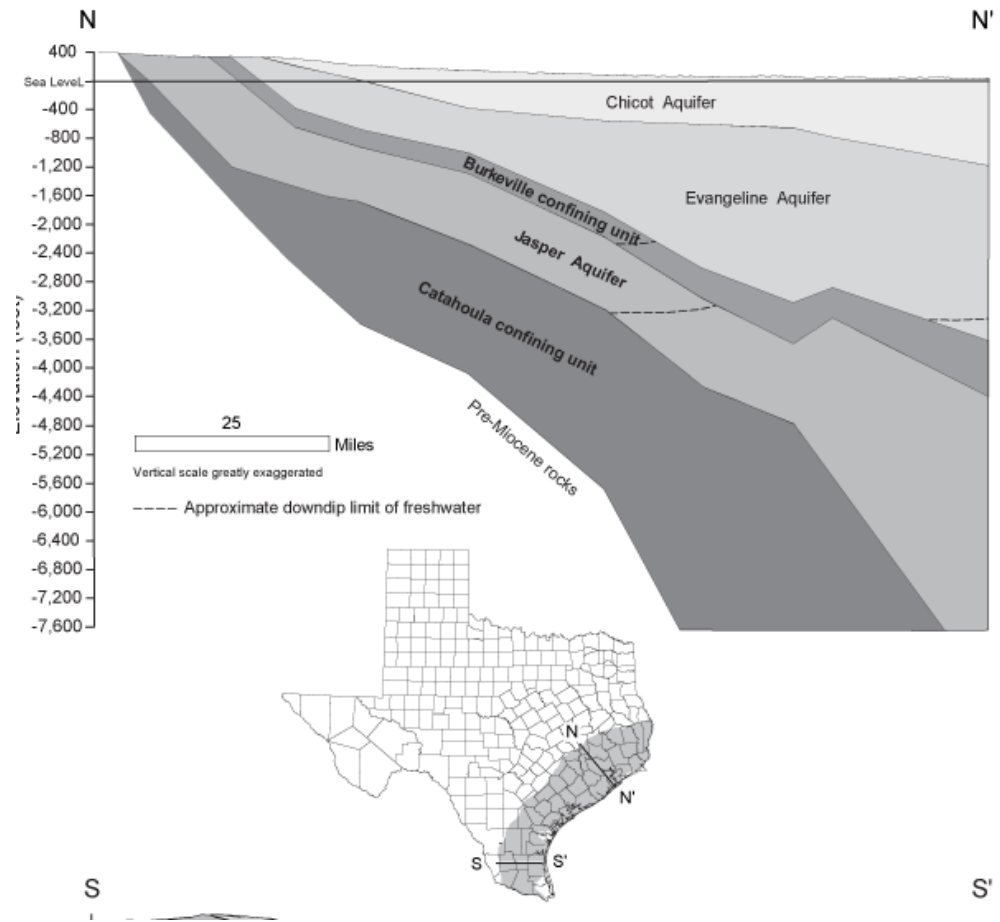
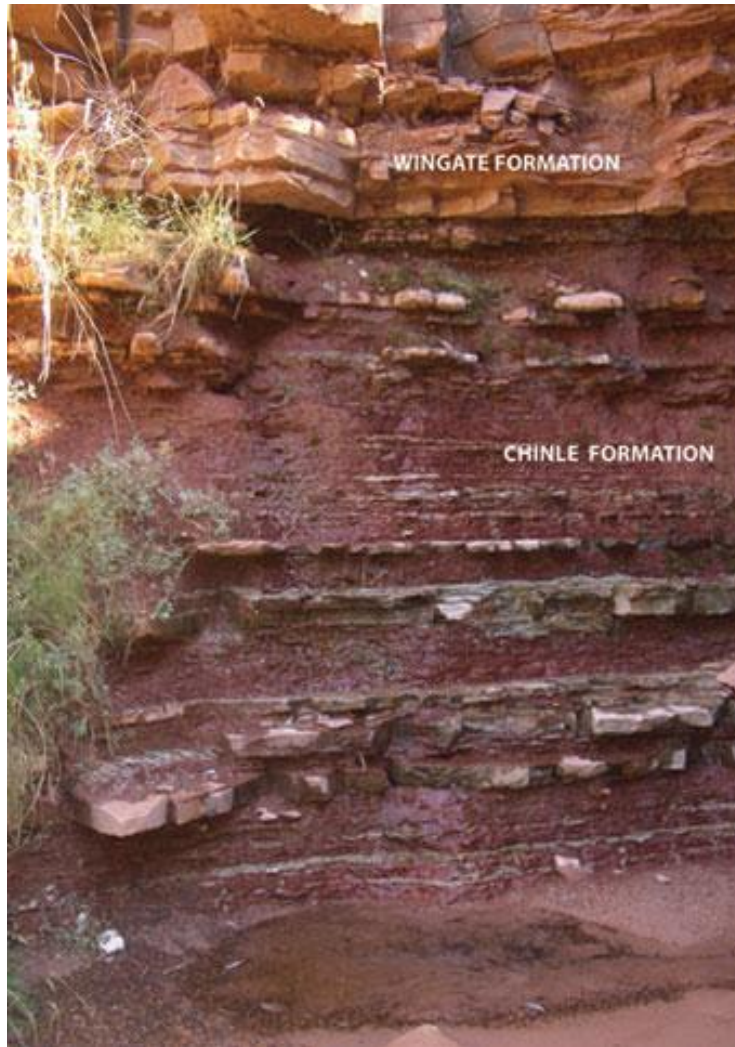


Heterogeneity deals with changes that are greater than REV

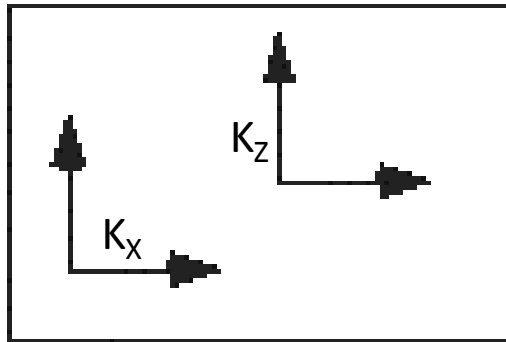
Anisotropy

- Properties of the aquifer change with direction
 - Hydraulic conductivity along X-axis is not the same along Z-axis
- Anisotropy is very common in aquifers
 - Particularly in the vertical dimension
 - Deposition of the aquifer occurs over large time periods
 - Lower layers are more compacted than the upper layers

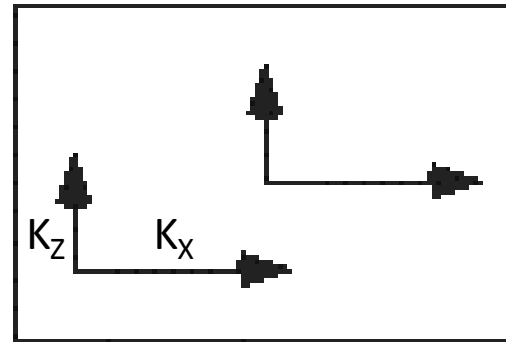
Anisotropy



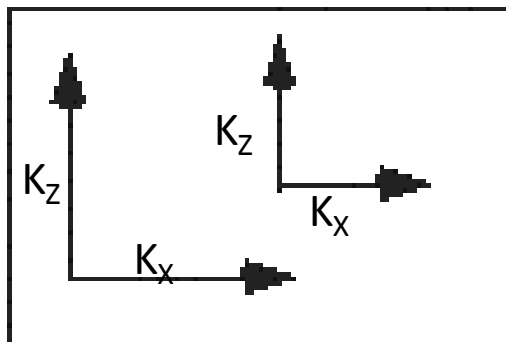
Isotropy and Heterogeneity



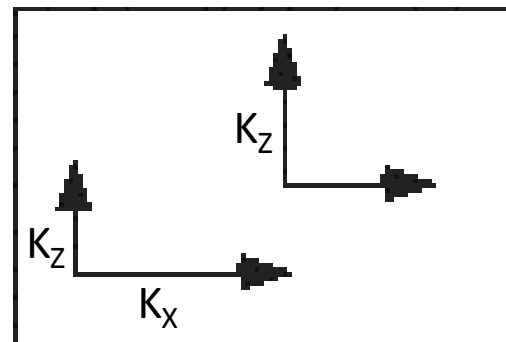
Isotropic, Homogeneous



Anisotropic, Homogeneous



Isotropic, Heterogeneous



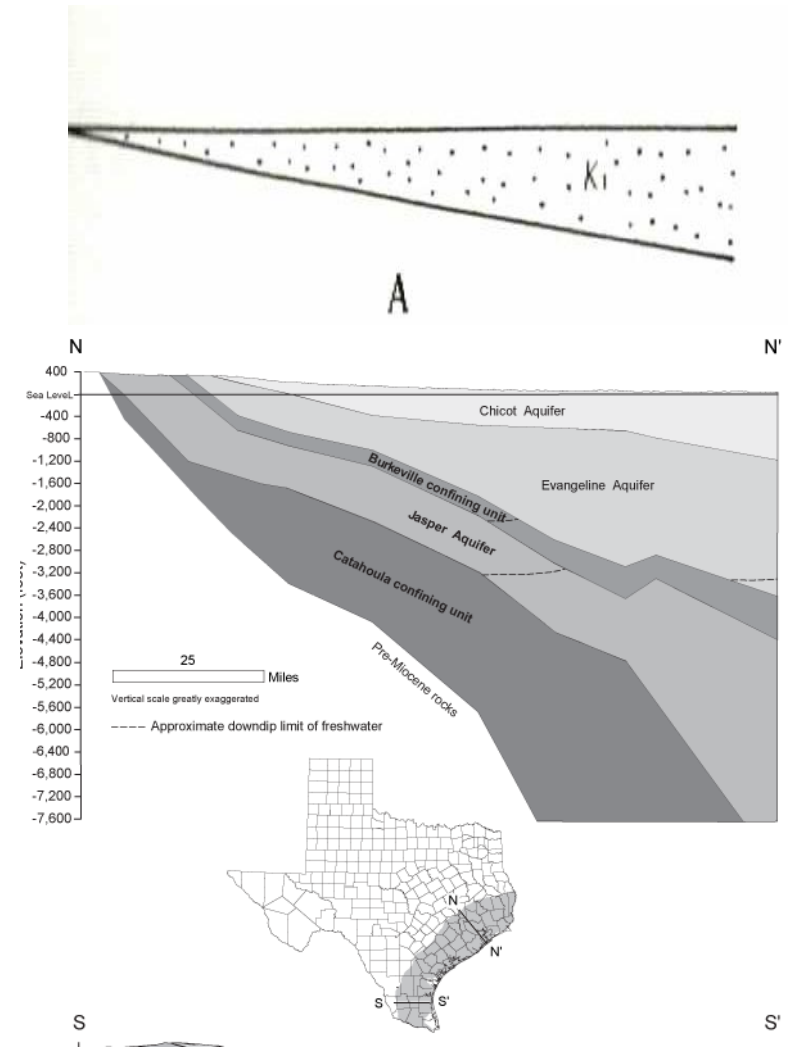
Anisotropic, Heterogeneous

Darcy's Law: Heterogeneity & Anisotropy

- Darcy's law is based on the assumptions of homogeneity and isotropy
- One approach to deal with heterogeneity and isotropy is to develop “Equivalent Hydraulic Conductivity”
 - A representative value that provides a reasonable representation of the system
 - Upholds conservation principles
 - Continuity equation

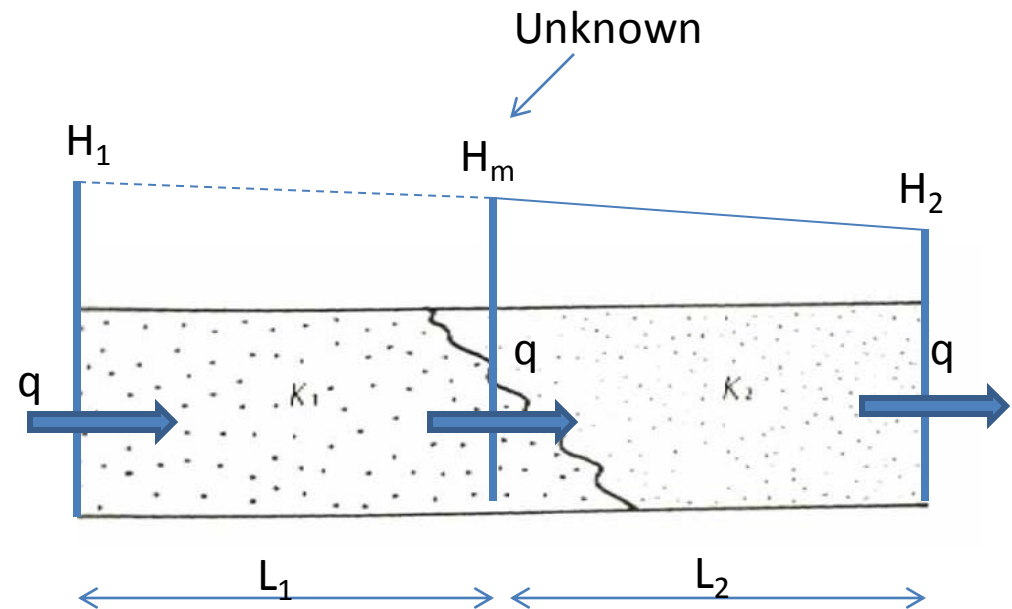
Heterogeneity in Aquifers

- The aquifer may pinch out
 - Assume K is constant?
 - Not heterogeneous
 - The thickness is variable
 - Must account for changes in cross-sectional area when applying Darcy's law



Heterogeneity in Aquifers

- Horizontal changes in depositional history
- To determine Equivalent K apply Darcy's law separately
 - Continuity equation holds



$$q = -K_1 \left(\frac{h_m - h_1}{L_1} \right)$$

$$q = -K_2 \left(\frac{h_2 - h_m}{L_2} \right)$$

$$q = -K_{eff} \left(\frac{h_2 - h_1}{L_1 + L_2} \right)$$

Three Unknowns: h_m , q , K_{eff}

Must Know: K_1 , K_2 , L_1 , L_2

Heterogeneity

Darcy's Law

$$q = -K_1 \left(\frac{h_m - h_1}{L_1} \right) = -K_1 \left(\frac{dh_1}{L_1} \right) \Rightarrow dh_1 = -q \left(\frac{L_1}{K_1} \right)$$

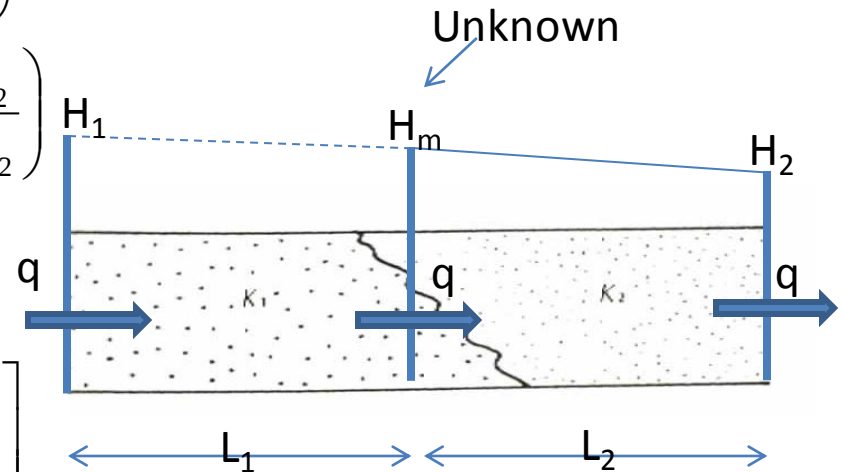
$$q = -K_2 \left(\frac{h_2 - h_m}{L_2} \right) = -K_2 \left(\frac{dh_2}{L_2} \right) \Rightarrow dh_2 = -q \left(\frac{L_2}{K_2} \right)$$

$$q = -K_{eff} \left(\frac{h_2 - h_1}{L_1 + L_2} \right) = -K_{eff} \left(\frac{h_2 - h_m + h_m - h_1}{L} \right)$$

$$q = - \left[\frac{K_{eff}}{L} \right] (dh_1 + dh_2) \Rightarrow (dh_1 + dh_2) = -q \left[\frac{L}{K_{eff}} \right]$$

$$-q \left(\frac{L_1}{K_1} + \frac{L_2}{K_2} \right) = -q \left[\frac{L}{K_{eff}} \right]$$

$$\frac{L_1 + L_2}{K_{eff}} = \frac{L_1}{K_1} + \frac{L_2}{K_2} \quad \left. \vphantom{\frac{L_1 + L_2}{K_{eff}}} \right\} \text{Harmonic Mean}$$

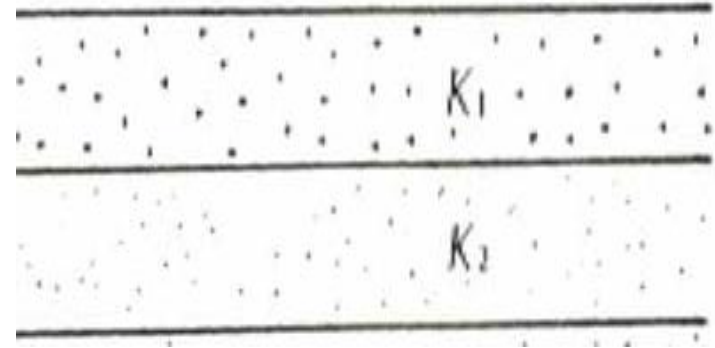


$$q = q_1 = q_m = q_2$$

Continuity Equation

Vertically Stratified Aquifers

- Vertical Stratification is very common in aquifers
 - The aquifer is comprised of several layers
 - Unconsolidated sand and gravel formations
 - Each layer can be considered “effectively homogeneous”



Equivalent K_H

$$q_1 = -K_1 i z_1$$

$$q_2 = -K_2 i z_2$$

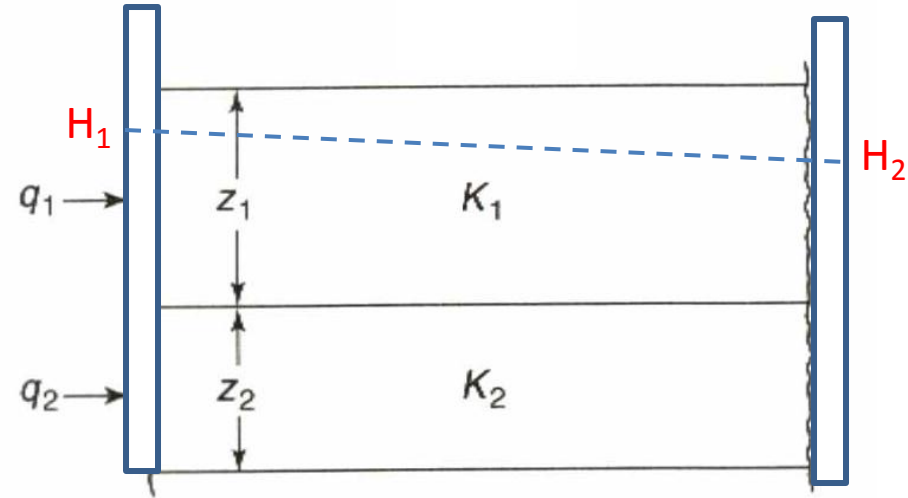
$$q = -K_e i (z_1 + z_2)$$

$$q = q_1 + q_2 \quad \leftarrow \text{Continuity Equation}$$

$$-K_e i (z_1 + z_2) = -K_1 i z_1 - K_2 i z_2$$

$$K_e = \frac{K_1 z_1 + K_2 z_2}{z_1 + z_2}$$

Darcy's law



$$i = \frac{dh}{dl} = \left[\frac{H_2 - H_1}{L} \right]$$

Assumptions

1. Each layer is isotropic
2. The layer thickness is constant
3. The wells fully penetrate the layers
4. Steady-state conditions

Weighted Arithmetic Mean

Final Thoughts on K

- K is the hydraulic conductivity
- Function of fluid and aquifer matrix
 - Intrinsic permeability
- K can be estimated from empirical formulas or by field and laboratory testing
 - Field testing is preferred
- Aquifers are seldom homogeneous, isotropic
 - Exhibit vertical and horizontal variations in K due to depositional characteristics
- We can estimate an equivalent (or effective) K
 - Representative K value for horizontal and/or vertical layering

Hydraulic Gradient

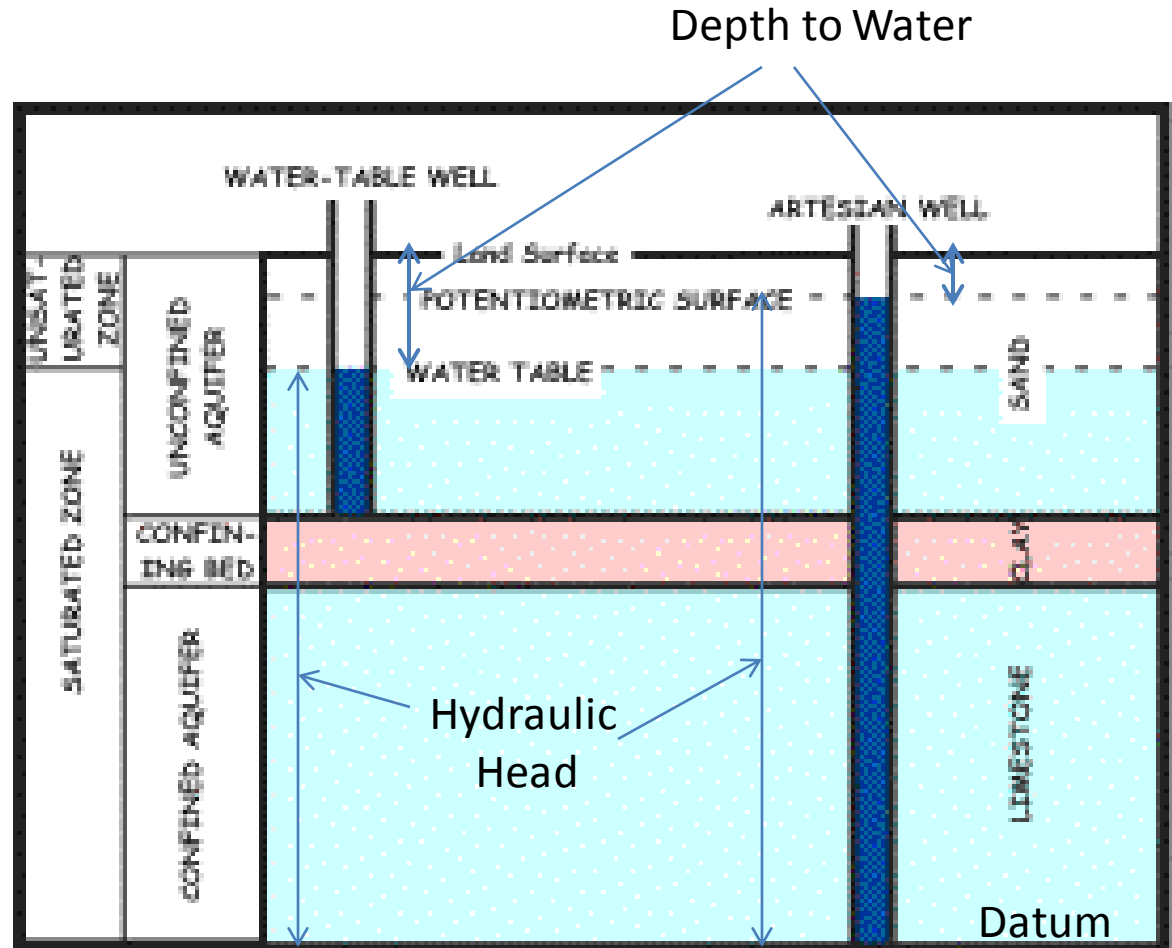
This is the second piece of information needed to apply Darcy's law

Measuring Hydraulic Head

- We use wells to measure hydraulic heads
 - Measure the height of the water in the well at any given time from the land surface
 - Subtract from a known datum to obtain hydraulic heads
- The water level in the well must be stable
 - If the water level is falling or rising, we are accumulating water in the system
 - Not at equilibrium as such non-steady state
 - Stable water level measurements are referred to as **static water levels**

Hydraulic Head

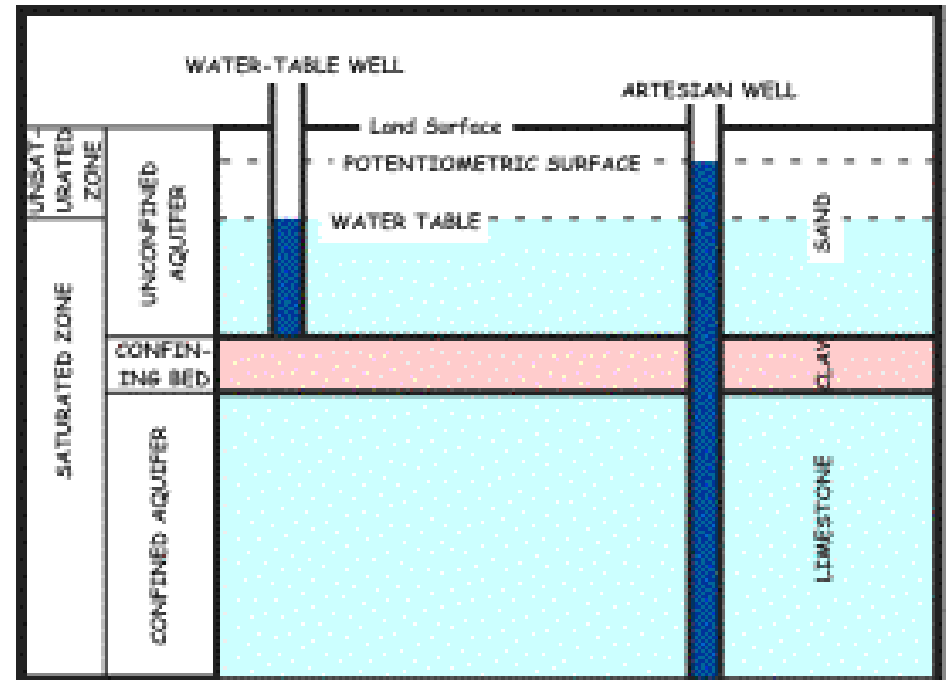
- Hydraulic head can be measured using a well in both confined and unconfined formations



Important to specify the Reference Datum when Reporting Hydraulic Heads

Water Table & Potentiometric Surface

- The phrase “Water Table” refers to the water surface in an unconfined aquifer
 - This is where the water is in the aquifer
- The phrase Potentiometric level (Pot surface) is used to describe water level in a well tapping a confined aquifer



The water level in the well need not coincide with the water level in the aquifer formation in which the well is drilled

Water Table → Water level in the well and the aquifer are the same (unconfined aquifer)

Pot. Surface → Water level in the well not at the same location as the aquifer water level (Confined)

Making Water Level Measurements

- There are three basic approaches
 - Use of an e-line
 - Vented pressure transducer
 - Non-vented pressure transducer



<https://www.youtube.com/watch?v=Exb965b9NRM>



Vented Pressure
Transducer



Unvented
Pressure
Transducer

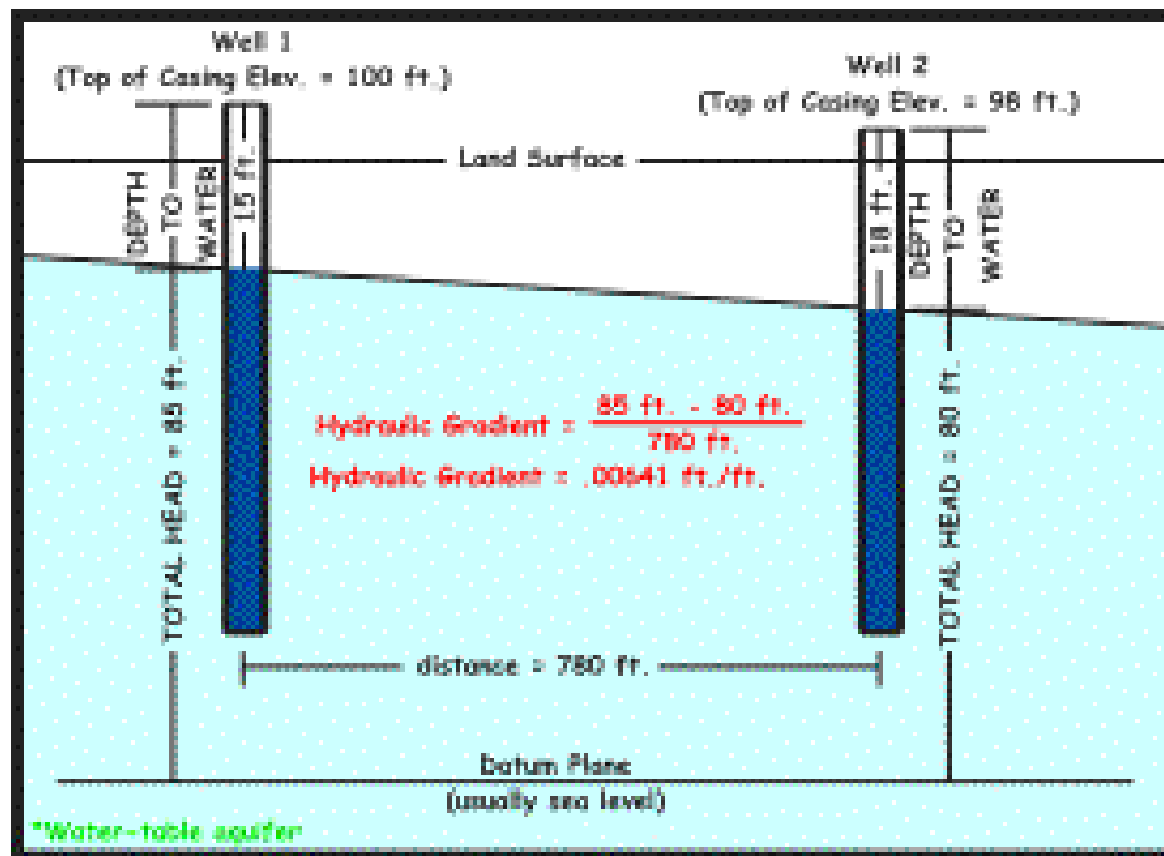
Hydraulic Head

- Summation of pressure and potential heads
 - Velocity heads are small and neglected
- Measured with respect to a chosen datum
 - Mean Sea Level (MSL) or bottom of the aquifer is typically chosen but need not be so
- All heads within an aquifer (in the area of interest) must be referenced with respect to the same datum
 - This is important if we want to apply Darcy's law

The term Hydraulic Head can be used in both Confined and Unconfined Aquifer Formations

Hydraulic Gradient

- Ratio of difference in Hydraulic Heads between two wells to the distance between them along a given axis (direction)
 - Rate of change of the hydraulic head w.r.t. to distance



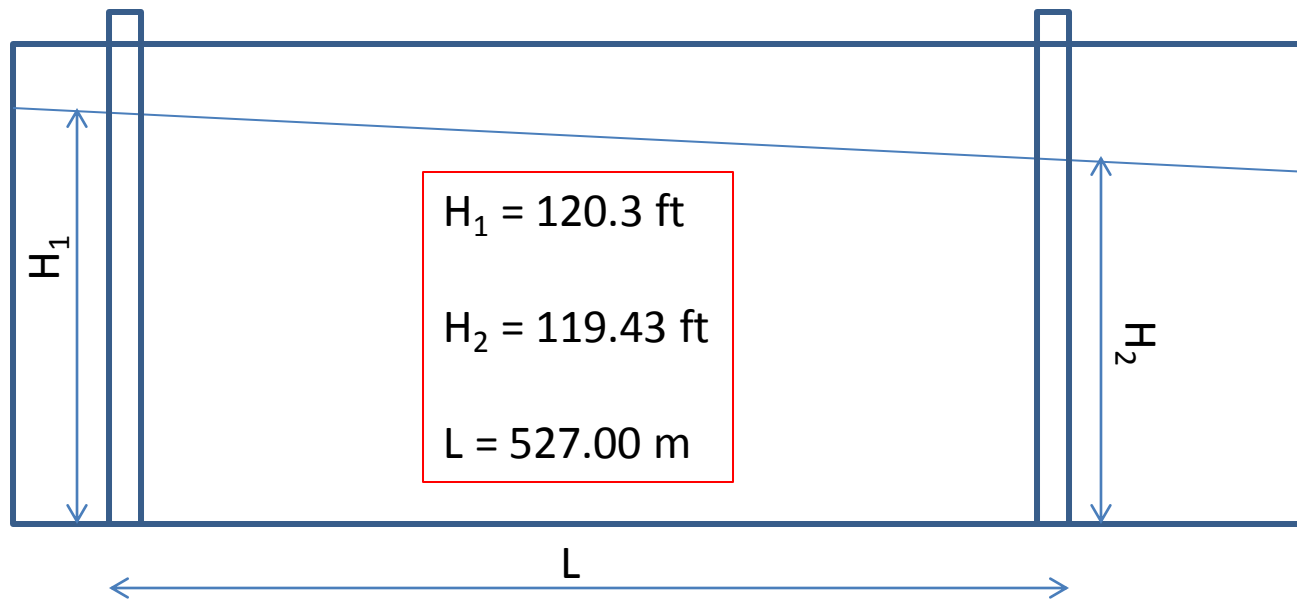
Hydraulic Gradients

- Hydraulic Gradients must be calculated from Hydraulic Heads
 - Do not use water levels as the land surface is usually not flat (i.e., datum is not constant)
- Estimating the hydraulic gradients between two wells is straightforward
 - However, in the field these two wells must aligned along the groundwater flow direction
 - Which means we must know the direction of the flow before we make gradient measurements
 - Typically not possible

We will need 3 or more randomly located wells to calculate the hydraulic gradient

Calculating Gradients

- Gradient Calculations are relatively simple when the direction of groundwater flow is known



Vertical Gradients

- Gradients can be calculated in the vertical plane as well

Determine H_B and the vertical velocity

Using (1) as the datum

$$i_{AB} = \frac{H_B - 27}{27}$$

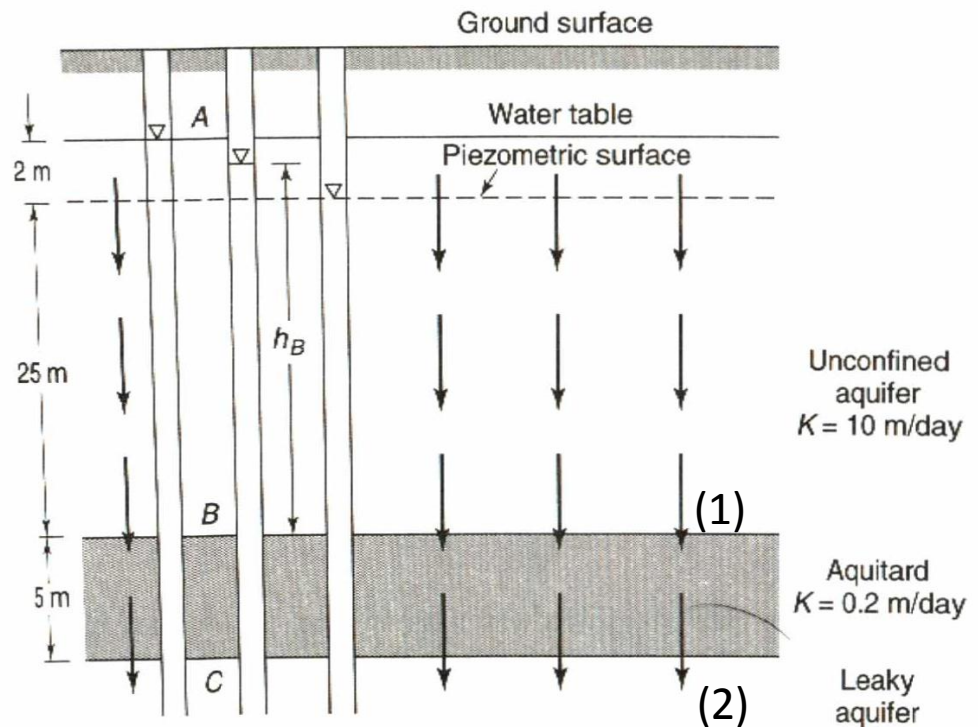
$$q = -Ki_{AB} = 10 \left(\frac{H_B - 27}{27} \right)$$

Using (2) as datum

Gradient between B & c

$$i_{BC} = \frac{30 - (H_B + 5)}{5}$$

$$q_{BC} = -Ki_{BC} = -0.2 \left(\frac{30 - (H_B + 5)}{5} \right)$$



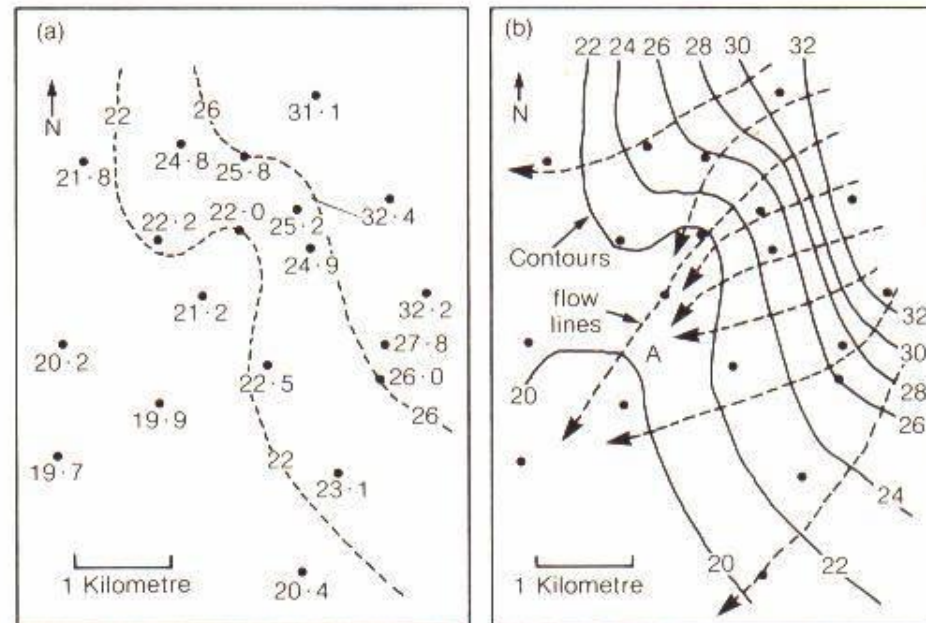
From Continuity: $q_{AB} = q_{BC}$

Two Equations two Unknowns

Solve using GOAL SEEK
in EXCEL

Gradient using Contour Plots

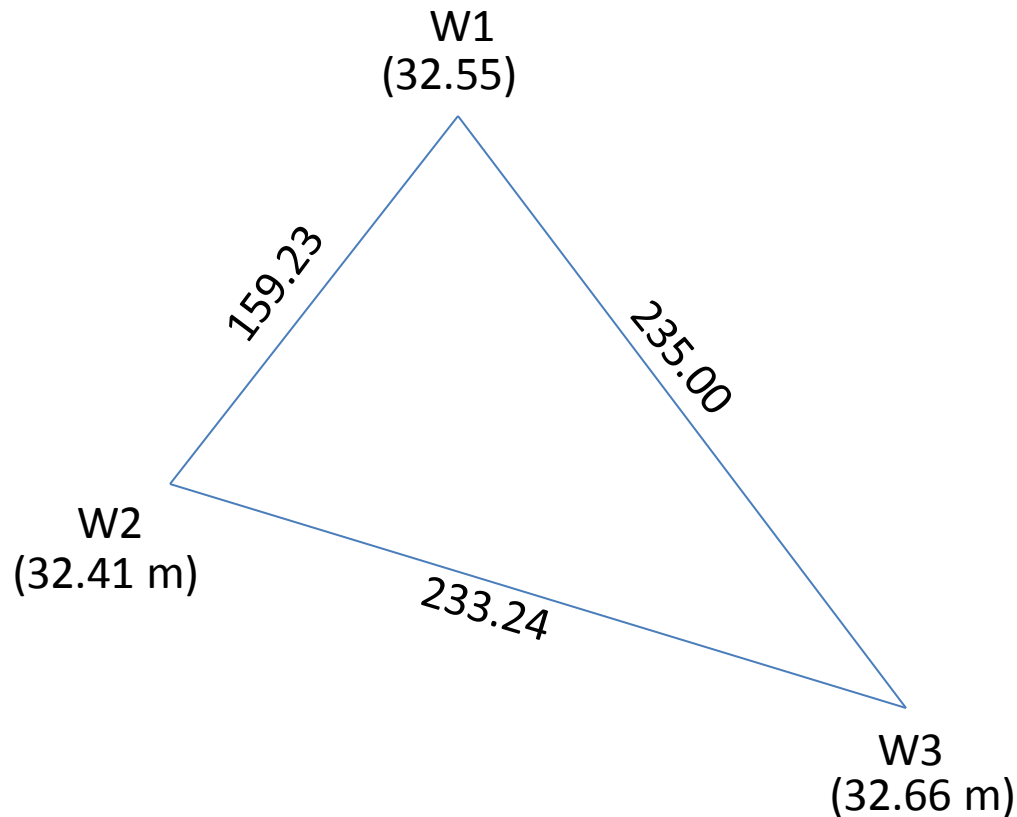
- The direction of groundwater flow and the magnitude of the gradient can be obtained via contour plots
 - Contours: Lines connecting areas having equal hydraulic heads within a region
 - Also referred to as equipotential lines
- The flow takes place perpendicular to the equipotential lines
 - Referred to as flowlines or flowpaths



Gradient from 3 Wells

- The groundwater flow and hydraulic gradient can be estimated from three wells
 - The three wells should form a triangle
 - The wells should be screened over the same interval
 - If there is groundwater flow heads in at least two of these wells will be different
 - Typically, all three wells will have different heads

Three Well Graphical Scheme

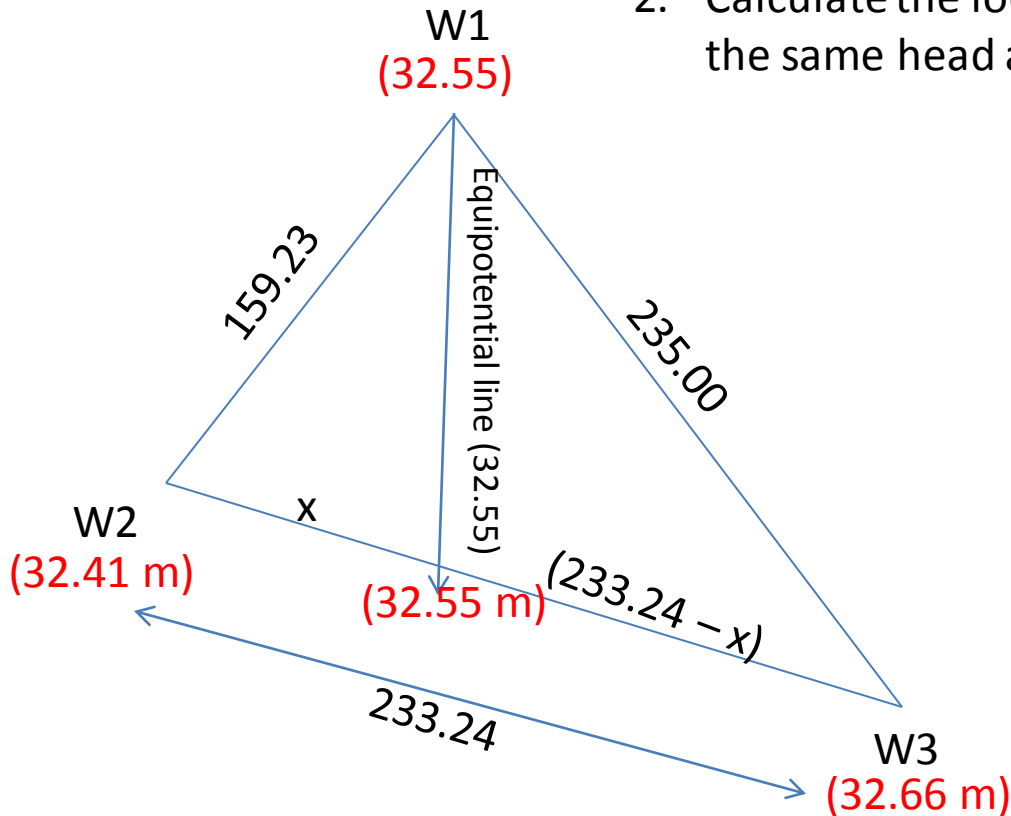


All dimensions
in meters

Step 1: Join the wells to form a triangle (need not be right-angled)
Step 2: Calculate the distances between the wells

Three Well Graphical Scheme

1. W2 has the lowest head; W3 has the highest head (W1 has the intermediate head)
2. Calculate the location on the line W2-W3 that has the same head as W1 (linear interpolation)



$$\frac{x}{(32.55 - 32.41)} = \frac{(233.24 - x)}{(32.66 - 32.55)}$$

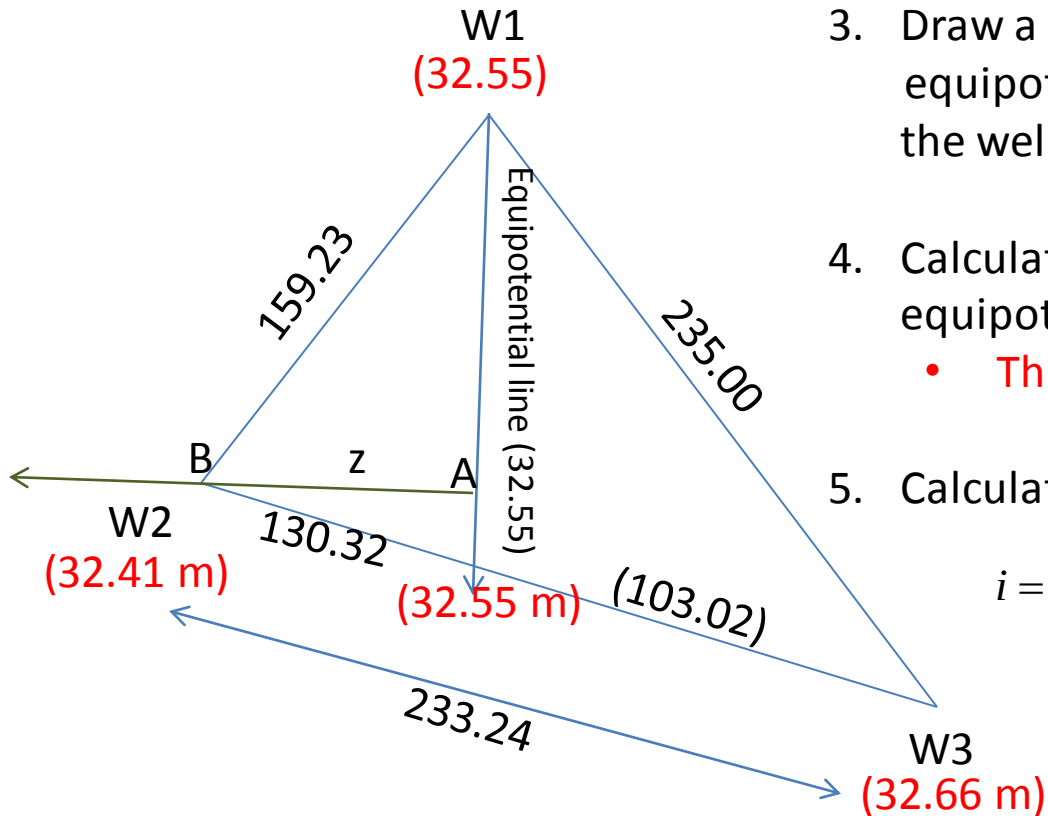
$$\frac{(233.24 - x)}{x} = \frac{0.11}{0.14} = 0.79$$

$$\frac{233.24}{x} - 1 = 0.79$$

$$\frac{233.24}{x} = 1.79$$

$$x = \frac{233.24}{1.79} = 130.32$$

Three Well Graphical Procedure



3. Draw a line Perpendicular to the equipotential line and passing through the well with the lowest head
4. Calculate the distance AB (line joining equipotential line to the lowest well)
 - This is where a graph is handy
5. Calculate the gradient as follows:

$$i = \frac{H_2 - H_1}{z} \sim \frac{32.41 - 32.55}{115.5} \sim -0.0011$$

Hydraulic Gradient – Planar Method

- The points connecting the three wells form a planar surface
 - Use the equation of the plane to compute the gradient
- The equation of the plane in 2D is given as follows:

$$H = aX + bY + z$$

- Three unknowns (a, b, Z)
- From three wells we get 3 values of (H, X, Y)
- Solve for the three unknown

Hydraulic Gradient – Matrix Method

- We can express the equations in the matrix form

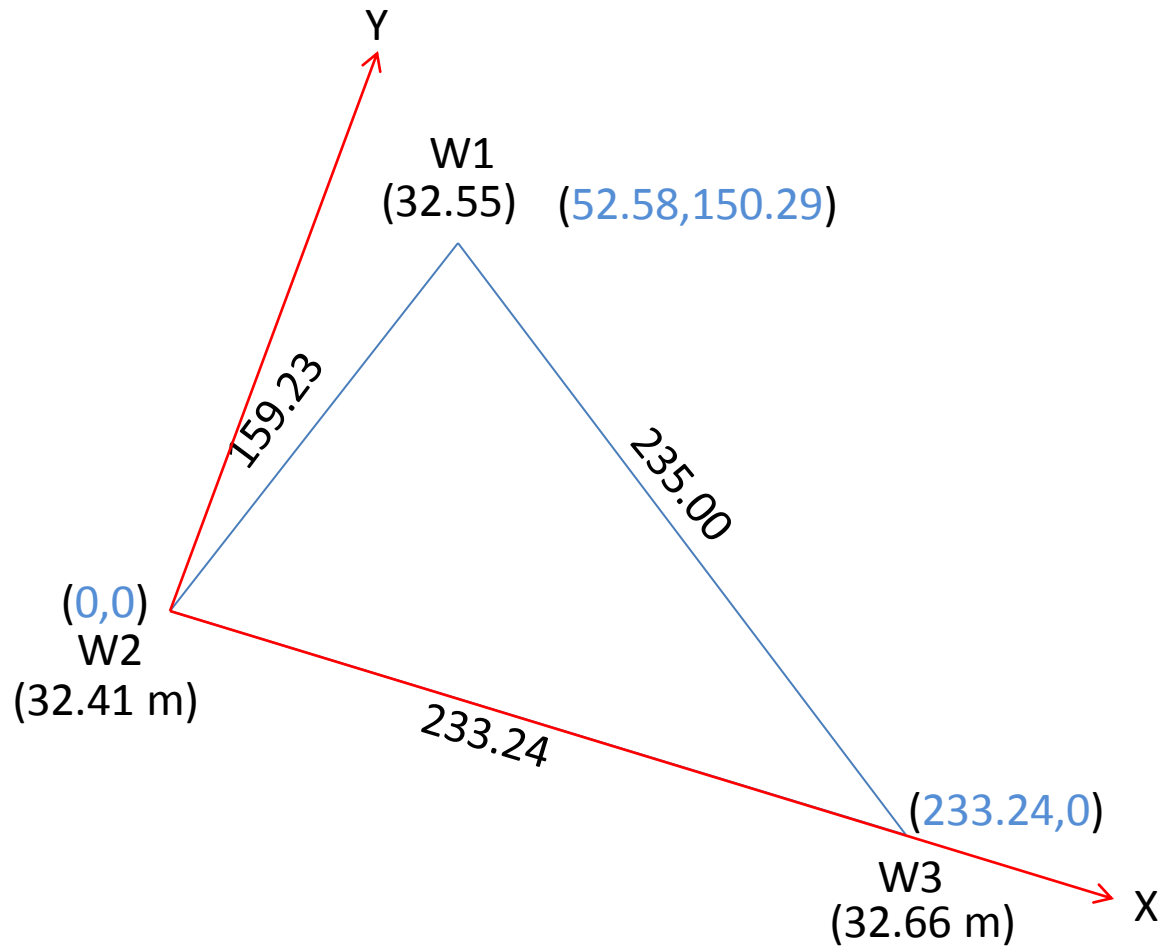
$$\begin{bmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ z \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \quad \left\{ \begin{array}{l} \\ \\ \end{array} \right. \quad \begin{bmatrix} a \\ b \\ z \end{bmatrix} = \begin{bmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

- The magnitude and the direction of the gradient are given as follows:

$$|i| = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1} \left(\frac{b}{a} \right)$$

Points 1, 2 and 3
must be selected in
counter-clockwise
direction

Compute the Gradient



Spreadsheet Array Formulas Come in Handy (MINVERSE and MMULT)

Microsoft Excel interface showing a spreadsheet with data and formulas.

Formulas:

- MINVERSE:** Used to calculate the inverse of a matrix. The range D10:D12 is highlighted, and the formula bar shows `=MINVERSE(A10:A12)`.
- MMULT:** Used to multiply two matrices. The range D14:D16 is highlighted, and the formula bar shows `=MMULT(A14:A16, B14:B16)`.

Data:

	A	B	C	D	E	F
1						
2	Set up your coordinates and heads					
3	X	Y	Z		H	
4	0	0	1		32.41	
5	233.2	0	1		32.66	
6	52.58	150.29	1		32.55	
7						
8						
9	Inverse					
10	-0.00429	0.004288	0			
11	-0.00515	-0.0015	0.006654			
12	1	0	0			
13						
14	a	0.001072				
15	b	0.000556				
16	z	32.41				
17						
18						
19	Magnitude	0.0012				
20						
21	Direction	27.43				
22						
23						

Gradient Final Thoughts

- Coordinates of wells are usually obtained using a GPS
 - Gives two angles (Lat and Long) need to project them onto the X, Y plane
 - UTM is one approach to do so (converters on the web)
 - <http://www.rcn.montana.edu/resources/tools/coordinates.aspx>
- We may have measurements at more than 3 wells
 - Reasonable to use the outermost triangle in many cases
 - Discarding information
 - Can use regression techniques to obtain a best-fit planar surface
 - Uses all available information
 - See: Devlin, J.F. 2003. A spreadsheet method of estimating best fit hydraulic gradients using head data from multiple wells. Ground Water, v. 41, no. 3, 316-320.

Gradient Final Thoughts

- The 3 well or 2D planar techniques assume the wells are screened over the same interval
 - Roughly of the same depth
- We can fit a 3D plane when the wells are not screened in the same interval
 - Need at a minimum 4 wells
 - Gradients in the X-Z plane generally greater and sensitive to errors in hydraulic head measurements
 - See: S. Silliman and G. Mantz (2000); Effects of Measurement Error on Estimating Hydraulic Gradients in Three Dimensions; *Ground Water*; vol 38; 114-120
- Planar methods better suited for confined formations than unconfined formations
 - Groundwater pumping can also reduce accuracy of the schemes