

Mechatronics

DC Motor / Tachometer Closed-Loop Speed Control System

1. Introduction

This case study is a dynamic system investigation of an important and common dynamic system: closed-loop speed control of a DC motor / tachometer system. The study emphasizes the two key elements in the study of system design:

- Integration through design of mechanical engineering, electronics, controls, and computers
- Balance between modeling / analysis / simulation and hardware implementation

The case study follows the procedure outlined in Figure 1.

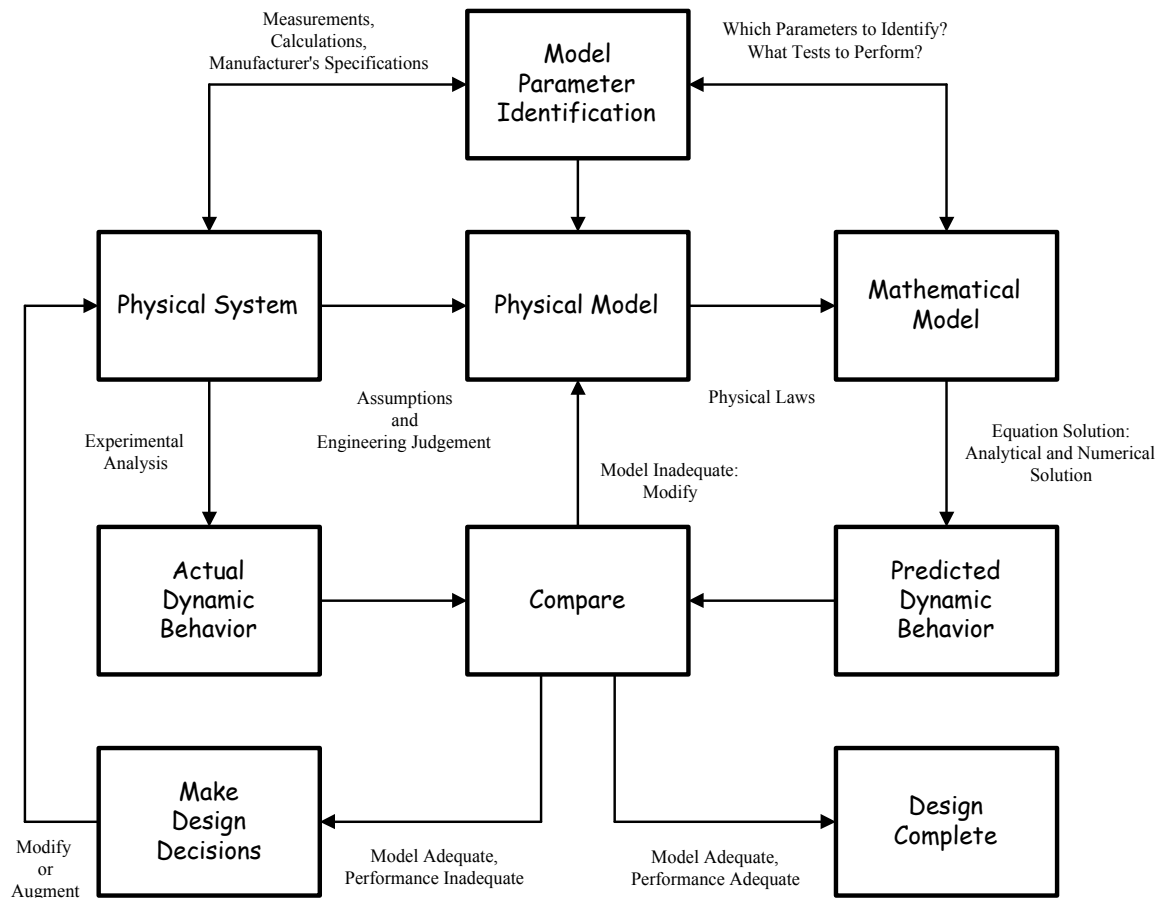


Figure 1. Dynamic System Investigation

Modern multidisciplinary products and systems depend on associated control systems for their optimum functioning. Today, control systems are an integral part of the overall system rather than afterthought "add-ons" and thus are considered from the very beginning of the design process. In order to control a dynamic system, one must be able to influence the response of the system. The device that does this is the *actuator*. Before a specific actuator is considered, one must consider which variables can be influenced. Another important consideration is which variables can physically be measured, both for control purposes and for disturbance detection. The device that does this is the *sensor*.

Considerations in *actuator selection* are:

Technology: electric, hydraulic, pneumatic, thermal, other

Functional Performance: maximum force possible, extent of the linear range, maximum speed possible, power, efficiency

Physical properties: weight, size, strength

Quality Factors: reliability, durability, maintainability

Cost: expense, availability, facilities for testing and maintenance

Considerations in *sensor selection* are:

Technology: electric or magnetic, mechanical, electromechanical, electro-optical, piezoelectric

Functional Performance: linearity, bias, accuracy, dynamic range, noise

Physical properties: weight, size, strength

Quality Factors: reliability, durability, maintainability

Cost: expense, availability, facilities for testing and maintenance

The actuator is the device that drives a dynamic system. Proper selection of actuators for a particular application is of utmost importance in the design of a dynamic system. Most actuators used in applications are continuous-drive actuators, for example, direct-current (DC) motors, alternating-current (AC) motors, hydraulic and pneumatic actuators. Stepper motors are incremental-drive actuators and it is reasonable to treat them as digital actuators. Unlike continuous-drive actuators, stepper motors are driven in fixed angular steps (increments). Each step of rotation (a predetermined, fixed increment of displacement) is the response of the motor rotor to an input pulse (or a digital command). In this manner, the step-wise rotation of the rotor can be synchronized with pulses in a command-pulse train, assuming, of course, that no steps are missed, thereby making the motor respond faithfully to the input signal (pulse sequence) in an open-loop manner. Like a conventional continuous-drive motor, the stepper motor is also an electromagnetic actuator, in that it converts electromagnetic energy into mechanical energy to perform mechanical work.

In the early days of analog control, servo-actuators (actuators that automatically use response signals from a process in feedback to correct the operation of the process) were exclusively continuous-drive devices. Since the control signals in this early generation of control systems generally were not discrete pulses, the use of pulse-driven digital actuators was not feasible in those systems. DC servo-motors and servo-valve-driven hydraulic and pneumatic actuators were the most widely used types of actuators in industrial control systems, particularly because digital was not available. Furthermore, the control of AC actuators was a difficult task at that time. Today, AC motors are also widely used as servo-motors, employing modern methods of phase-voltage control and frequency control through microelectronic drive systems and using field-feedback compensation through digital signal processing (DSP) chips. It is interesting to note that actuator control using pulse signals is no longer limited to digital actuators. Pulse-width-modulated (PWM) signals are increasingly being used to drive continuous actuators such as DC servo-motors, hydraulic and pneumatic servos, and AC motors. It is also interesting to note that electronic-switching commutation in DC motors is quite similar to the method of phase switching used in driving stepper motors.

Although the cost of sensors and transducers is a deciding factor in low-power applications and in situations where precision, accuracy, and resolution are of primary importance, the cost of actuators can become crucial in moderate-to-high-power control applications. It follows that the proper design and selection of actuators can have a significant economical impact in many applications of industrial control.

Measurement of plant outputs is essential for feedback control, and is also useful for performance evaluation of a process. Input measurements are needed in feedforward control. It is evident, therefore, that the measurement subsystem is an important part of a control system. The measurement subsystem in a control system contains sensors and transducers that detect measurands and convert them into acceptable signals, typically voltages. These voltages are then appropriately modified using signal-conditioning hardware such as filters, amplifiers, demodulators, and analog-to-digital converters. Impedance matching might be necessary to connect sensors and transducers to signal-conditioning hardware.

Accuracy of sensors, transducers, and associated signal-conditioning devices is important in control system applications for two main reasons:

- a) The measurement system in a feedback control system is situated in the feedback path of the control system. Even though measurements are used to compensate for the poor performance in the open-loop system, any errors in measurements themselves will enter directly into the system and cannot be corrected if they are unknown.
- b) It can be shown that sensitivity of a control system to parameter changes in the measurement system is direct. This sensitivity cannot be reduced by increasing the loop gain, unlike in the case of sensitivity to the open-loop components.

Accordingly, the design strategy for closed-loop (feedback) control is to make the measurements very accurate and to employ a suitable controller to reduce other types of errors.

Most sensor-transducer devices used in feedback control applications are analog components that generate analog output signals. This is the case even in real-time direct digital control systems. When analog transducers are used in digital control applications, however, some type of analog-to-digital conversion is needed to obtain a digital representation of the measured signal. The resulting digital signal is subsequently conditioned and processed using digital means. In the sensor stage, the signal being measured is felt as the response of the sensor element. This is converted by the transducer into the transmitted (or measured) quantity. In this respect, the output of a measuring device can be interpreted as the response of the transducer. In control system applications, this output is typically (and preferably) an electrical signal.

This case study is a dynamic system investigation of a DC motor using a tachometer as a speed sensor. The tachometer is integral to the DC motor used in this case study. Other candidate speed sensors are optical encoders (digital sensor), resolvers (analog and digital), and Hall-effect sensors.

2. Physical System

A DC motor converts direct-current (DC) electrical energy into rotational mechanical energy. A major fraction of the torque generated in the rotor (armature) of the motor is available to drive an external load. DC motors are widely used in numerous control applications because of features such as high torque, speed controllability over a wide range, portability, well-behaved speed-torque characteristics, and adaptability to various types of control methods. DC motors are classified as either integral-horsepower motors (≥ 1 hp) or fractional-horsepower motors (< 1 hp). Within the class of fractional-horsepower motors, a distinction can be made between those that generate the magnetic field with field windings (an electromagnet) and those that use permanent magnets. In industrial DC motors, the magnetic field is usually generated by field windings, while DC motors used in instruments or consumer products normally have a permanent magnet field.

The physical system, shown in Figure 2 and typical of commonly used motors; is a fractional-horsepower, permanent-magnet, DC motor in which the commutation is performed with brushes. The load on the motor is a solid aluminum disk with a radius $r = 1.5$ inches (0.0381 m), a height $h = 0.375$ inches (0.0095 m), and a moment of inertia about its axis of rotation $J_{load} = \frac{1}{2}mr^2$ (where mass $m = \rho\pi r^2h$ and density $\rho = 2800 \text{ kg/m}^3$) which equals $8.8 \times 10^{-5} \text{ kg-m}^2$. The system is driven by a pulse-width-modulated (PWM) power amplifier. Motor speed is measured using an analog tachometer.

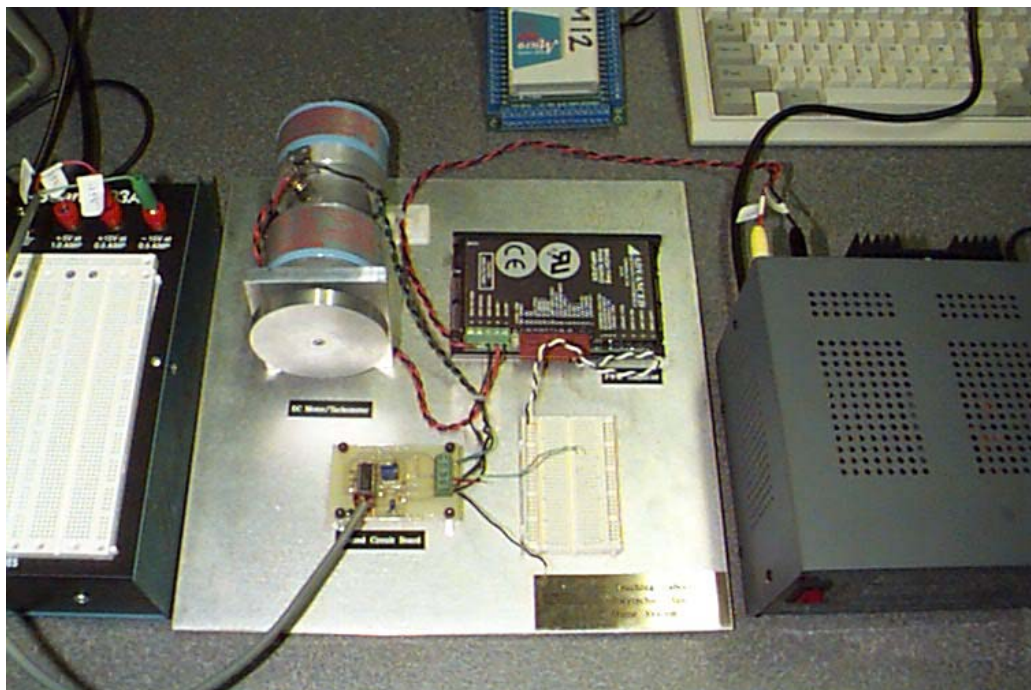


Figure 2. Physical System

The DC motor/tachometer system has a single input and a single output. The input is the voltage applied across the two motor terminals. The output is the voltage measured across the two tachometer terminals.

The principle of operation of a DC motor is illustrated in Figure 3. Consider an electric conductor placed in a steady magnetic field perpendicular to the direction of the magnetic field. The magnetic field flux density B is assumed constant. A DC current i is passed through the conductor and a circular magnetic flux around the conductor due to the current is produced. Consider a plane through the conductor parallel to the direction of flux of the magnet. On one side of this plane, the current flux and field flux are additive; on the opposite side, they oppose each other. The result is an imbalance magnetic force F on the conductor perpendicular to this plane. This force is given by

$$\vec{F} = \oint i d\vec{\ell} \times \vec{B}$$

$$F = Bi\ell$$

where

B = flux density of the original field

i = current through the conductor

ℓ = length of the conductor

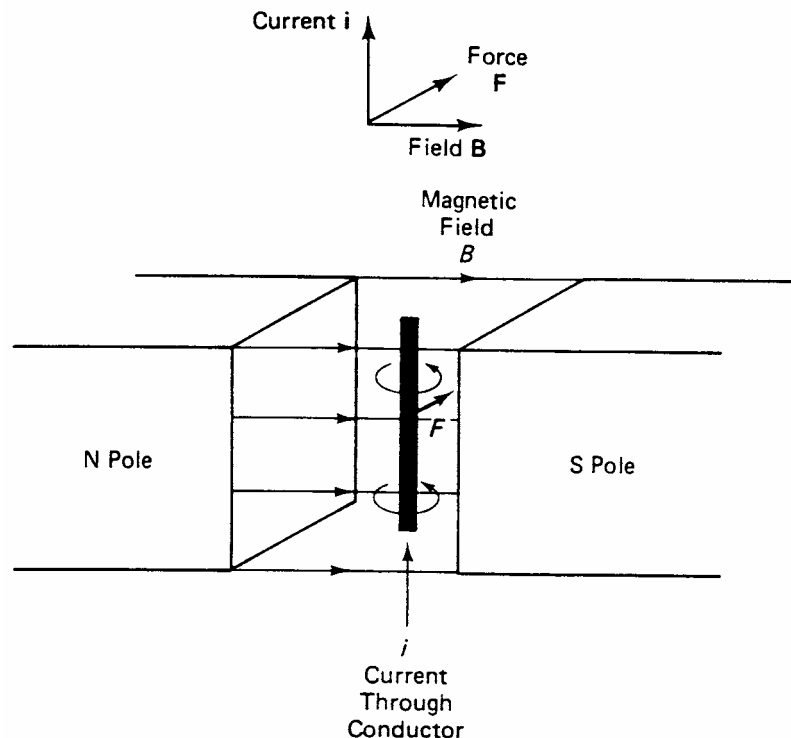


Figure 3. Operating Principle of a DC Motor

The active components of B , i , and F are mutually perpendicular and form a right-handed triad. If the conductor is free to move, the force will move it at some velocity v in the direction of the force. As a result of this motion in the magnetic field B , a voltage e_b is induced in the conductor. This voltage is known as the back electromotive force or back e.m.f. and is given by

$$e_b = \int \vec{v} \times \vec{B} \cdot d\vec{\ell} = B\ell v$$

The flux due to the back e.m.f. will oppose the flux due to the original current through the conductor (Lenz's Law), thereby trying to stop the motion. This is the cause of electrical damping in motors. Saying this another way, the back e.m.f. voltage tends to oppose the voltage which produced the original current.

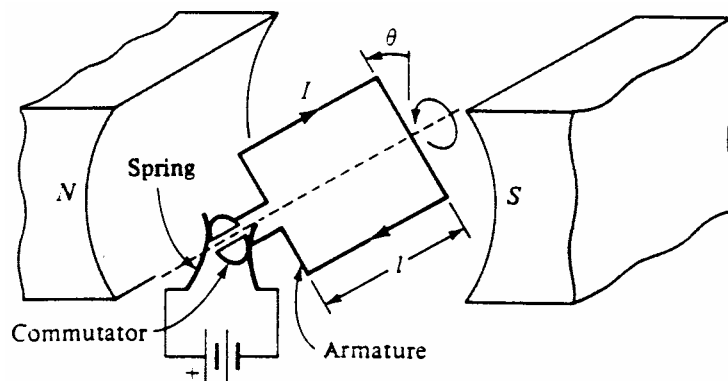


Figure 4. Elements of a Simple DC Motor

Figure 4 shows the elements of a simple DC motor. It consists of a loop, usually of many turns of wire, called an armature which is immersed in the uniform field of a magnet. The armature is connected to a commutator which is a divided slip ring. The purpose of that commutator is to reverse the current at the appropriate phase of rotation so that the torque on the armature always acts in the same direction. The current is supplied through a pair of springs or brushes which rest against the commutator. Figure 5(a) shows a rectangular loop of wire of area $A = dl$ carrying a current i and Figure 5(b) shows a cross-section of the loop.

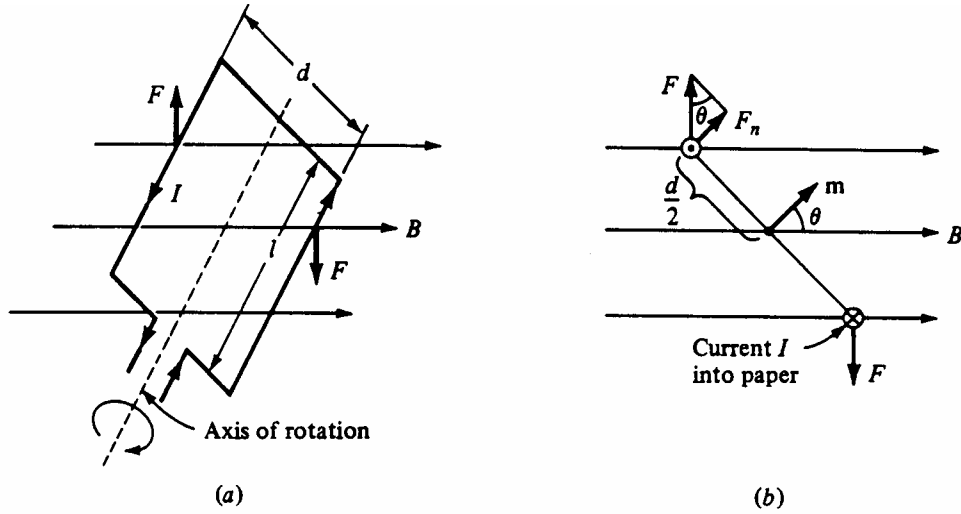


Figure 5. Rectangular Loop of Current-Carrying Wire in a Magnetic Field

From Figure 5(b), the torque of the motor is given by:

$$T = 2F_n \left(\frac{d}{2} \right) N = (iB\ell \sin \theta) dN = iABN \sin \theta = mBN \sin \theta$$

$$\vec{T} = N [\vec{m} \times \vec{B}]$$

where N is the number of turns of the armature, $A = d\ell$ is the area of the armature, $d/2$ is the moment arm of the force F_n on one side of a single turn of wire, and the magnetic moment $m = iA$, which is a vector with a direction normal to the area A (i.e., \hat{n} direction with the direction determined by the right-hand rule applied to the direction of the current).

The motor used in this system is the Honeywell 22VM51-020-5 DC Motor with Tachometer. The factory specifications are shown in Table 1.

Table 1. Specifications of the Honeywell 22VM51-020-5 DC Motor with Tachometer

Motor Characteristics	Units	Values	Parameter
Rated Voltage (DC)	volts	24	-
Rated Current (RMS)	amps	2.2	-
Pulsed Current	amps	13.4 max	-
Rated Torque	N-m	9.18E-2	-
Rated Speed	RPM	2225	-
Back EMF Constant	volts-sec/rad	0.0374	K_b
Torque Constant	N-m/amp	0.0374	K_t
Terminal Resistance	ohms	3.8	R
Rotor Inductance	henry	6.0E-4	L
Viscous Damping Coefficient	N-m-sec/rad	6.74E-6	B
Rotor Inertia (including Tach)	kg-m ²	3.18E-6	J_m
Static Friction Torque	N-m	9.18E-3	T_f
Tachometer Voltage Constant	volts-sec/rad	0.0286	K_{tach}

The power amplifier used in this system is the Advanced Motion Controls Model 25A8. It is a PWM servo-amplifier designed to drive brush-type DC motors at a high switching frequency. The factory specifications for this amplifier are shown in Table 2.

Table 2. Specifications of the Advanced Motion Controls Model 25A8 PWM Amplifier

Power Amplifier Characteristics	Values
DC Supply Voltage	20-80 V
Maximum Continuous Current	± 12.5 A
Minimum Load Inductance	200 μ H
Switching Frequency	22 KHz $\pm 15\%$
Bandwidth	2.5 KHz
Input Reference Signal	± 15 V maximum
Tachometer Signal	± 60 V maximum

Two power supplies are used to drive the system. Their specifications are shown in Table 3.

Table 3. Specifications of Power Supplies

EMCO Supply Model PSR-4/24	Values
Supply voltage	+24 Volts
Maximum Continuous Current	4 amps
Maximum Peak Current	5 amps

Proto-Board 203A	Values
Supply Voltage	$\pm 15 \text{ V @ } 0.5 \text{ A}$
Supply Voltage	$5 \text{ V @ } 1.0 \text{ A}$

A schematic diagram of a DC motor is shown in Figure 6.

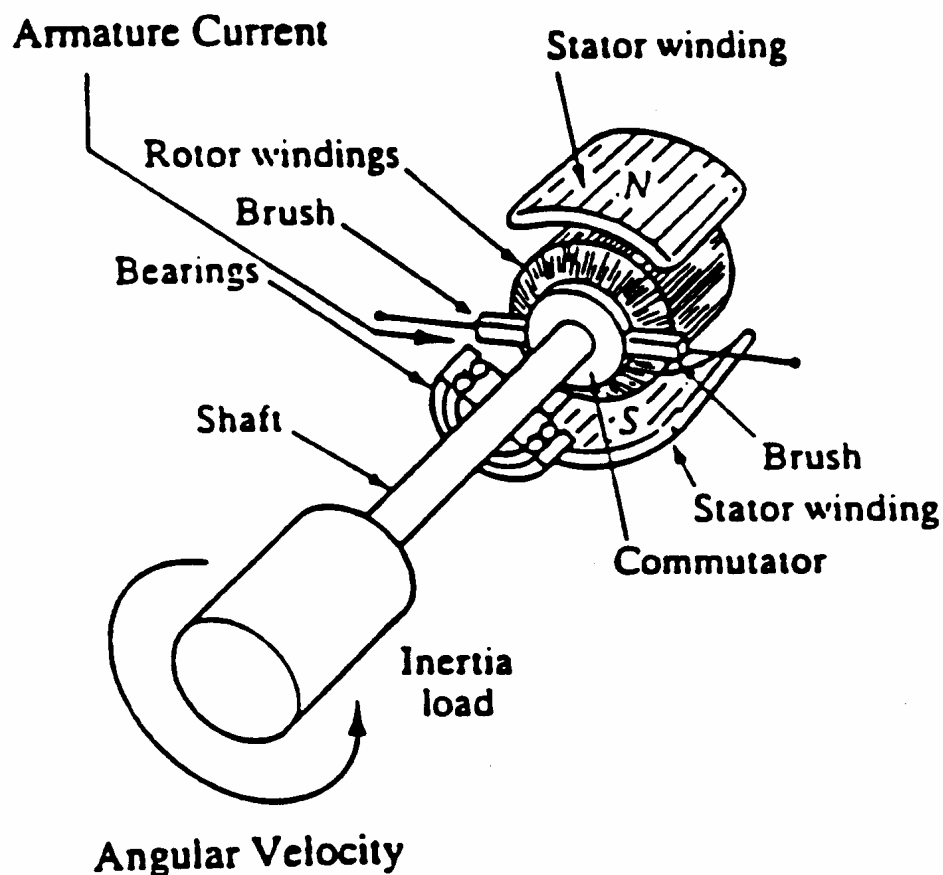


Figure 6. Schematic Diagram of a DC Motor

Motion transducers that employ the principle of electromagnetic induction are termed variable-inductance transducers. When the flux linkage (defined as the magnetic flux density times the number of turns in the conductor) through an electrical conductor changes, a voltage is induced in the conductor. This, in turn, generates a magnetic field that opposes the primary field. Hence, a mechanical force is necessary to sustain the change in flux linkage. If the change in flux linkage is brought about by relative motion, the mechanical energy is directly converted (induced) into electrical energy. This is the basis of electromagnetic induction, and it is the principle of operation of electrical generators and variable-inductance transducers. Note that in these devices, the change in flux linkage is caused by mechanical motion, and mechanical-to-electrical energy transfer takes place under near-ideal conditions. The induced voltage or change in inductance may be used as a measure of motion. Variable-inductance transducers are generally electromechanical devices coupled by a magnetic field.

There are three primary types of variable-inductance transducers:

- a) Mutual-inductance transducers, e.g., linear variable differential transformer (LVDT), rotary variable differential transformer (RVDT), mutual-induction proximity probe, resolver, synchro-transformer.
- b) Self-induction transducers, e.g., self-induction proximity sensor.
- c) Permanent-magnet transducers, e.g., permanent-magnet DC velocity sensors (DC tachometers), AC permanent-magnet tachometers, AC induction tachometers.

The permanent-magnet DC velocity sensor (DC tachometer) is a variable-inductance transducer. It has a permanent magnet to generate a uniform and steady magnetic field. A relative motion between the magnetic field and an electrical conductor induces a voltage that is proportional to the speed at which the conductor crosses the magnetic field. In some designs, a unidirectional magnetic field generated by a DC supply (i.e., an electromagnet) is used in place of a permanent magnet.

The principle of electromagnetic induction between a permanent magnet and a conducting coil is used in speed measurement by permanent-magnet transducers. Depending on the configuration, either rectilinear speeds or angular speeds can be measured. Schematic diagrams of the two configurations are shown in Figure 7. These are passive transducers because the energy for the output signal v_o is derived from the motion (measured signal) itself. The entire device is usually enclosed in a steel casing to isolate it from ambient magnetic fields.

In the rectilinear velocity transducer, the conductor coil is wrapped on a core and placed centrally between two magnetic poles, which produce a cross-magnetic field. The core is attached to the moving object whose velocity must be measured. The velocity v is proportional to the induced voltage v_o . A moving-magnet and fixed-coil arrangement can also be used, thus eliminating the need for any sliding contacts (slip rings and brushes) for the output leads, thereby reducing mechanical loading error, wearout, and related problems.

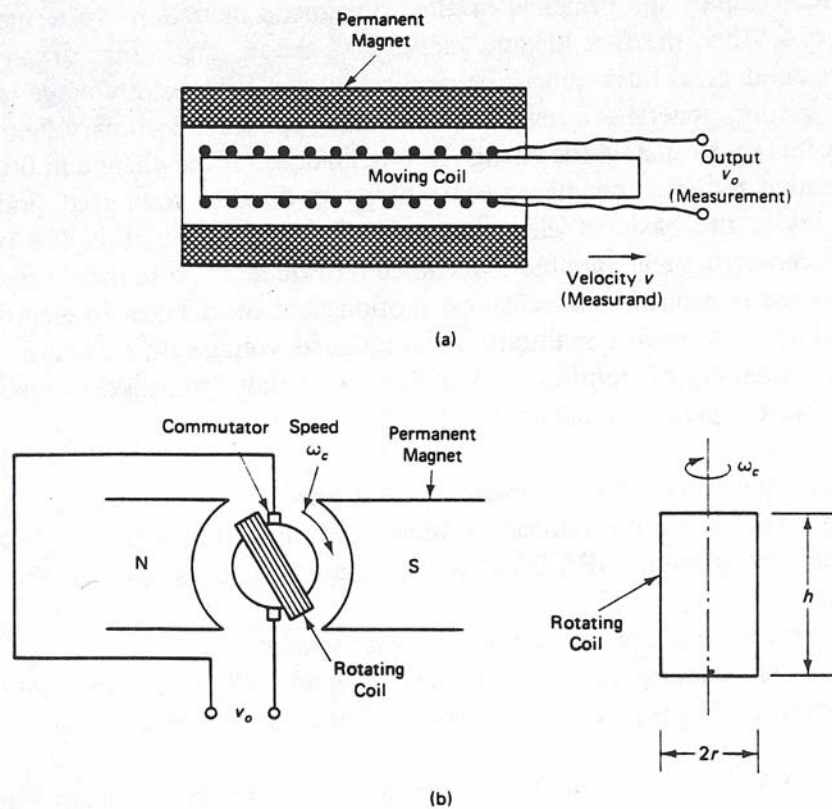


Figure 7. Permanent-Magnet Transducers:
(a) rectilinear velocity transducer; (b) DC tachometer-generator

The tachometer-generator is a very common permanent-magnet device. The rotor is directly connected to the rotating object. The output signal that is induced in the rotating coil is picked up as DC voltage v_o using a suitable commutator device - typically consisting of a pair of low-resistance carbon brushes - that is stationary but makes contact with the rotating coil through split slip rings so as to maintain the positive direction of induced voltage throughout each revolution.

The induced voltage is given by

$$v_o = B\ell v = B(2hn)(r\omega_c) = K_{\text{tach}}\omega_c$$

for a coil of height h , radius r , and n turns, moving at an angular speed ω_c in a uniform magnetic field of flux density B . This proportionality between v_o and ω_c is used to measure the angular speed ω_c .

3. Physical Model

The challenges in physical modeling are formidable:

- Dynamic behavior of many physical processes is complex.
- Cause and effect relationships are not easily discernible.
- Many important variables are not readily identified.
- Interactions among the variables are hard to capture.

The first step in physical modeling is to specify the system to be studied, its boundaries, and its inputs and outputs. One then imagines a simple physical model whose behavior will match sufficiently closely the behavior of the actual system. A physical model is an imaginary physical system which resembles the actual system in its salient features but which is simpler (more "ideal") and is thereby more amenable to analytical studies. It is not oversimplified, not overly complicated - it is a slice of reality. The astuteness with which approximations are made at the outset of an investigation is the very crux of engineering analysis. The ability to make shrewd and viable approximations which greatly simplify the system and still lead to a rapid, reasonably accurate prediction of its behavior is the hallmark of every successful engineer. This ability involves a special form of carefully developed intuition known as *engineering judgment*. Table 4 lists some of the approximations used in the physical modeling of dynamic systems and the mathematical simplifications that result. These assumptions lead to a physical model whose mathematical model consists of linear, ordinary differential equations with constant coefficients.

Table 4. Approximations Used in Physical Modeling

Approximation	Mathematical Simplification
Neglect small effects	Reduces the number and complexity of the equations of motion
Assume the environment is independent of system motions	Reduces the number and complexity of the equations of motion
Replace distributed characteristics with appropriate lumped elements	Leads to ordinary (rather than partial) differential equations
Assume linear relationships	Makes equations linear; allows superposition of solutions
Assume constant parameters	Leads to constant coefficients in the differential equations
Neglect uncertainty and noise	Avoids statistical treatment

Let's briefly discuss these assumptions:

Neglect Small Effects

Small effects are neglected on a relative basis. In analyzing the motion of an airplane, we are unlikely to consider the effects of solar pressure, the earth's magnetic field, or gravity gradient. To ignore these effects in a space vehicle problem would lead to grossly incorrect results!

Independent Environment

Here we assume that the environment, of which the system under study is a part, is unaffected by the behavior of the system, i.e., there are no loading effects. In analyzing the vibration of an instrument panel in a vehicle, for example, we assume that the vehicle motion is independent of the motion of the instrument panel. If loading effects are possible, then either steps must be taken to eliminate them (e.g., use of buffer amplifiers), or they must be included in the analysis.

Lumped Characteristics

In a lumped-parameter model, system dependent variables are assumed uniform over finite regions of space rather than over infinitesimal elements, as in a distributed-parameter model. Time is the only independent variable and the mathematical model is an ordinary differential equation. In a distributed-parameter model, time and spatial variables are independent variables and the mathematical model is a partial differential equation. Note that elements in a lumped-parameter model do not necessarily correspond to separate physical parts of the actual system. A long electrical transmission line has resistance, inductance, and capacitance distributed continuously along its length. These distributed properties are approximated by lumped elements at discrete points along the line.

Linear Relationships

Nearly all physical elements or systems are inherently nonlinear if there are no restrictions at all placed on the allowable values of the inputs. If the values of the inputs are confined to a sufficiently small range, the original nonlinear model of the system may often be replaced by a linear model whose response closely approximates that of the nonlinear model. When a linear equation has been solved once, the solution is general, holding for all magnitudes of motion. Linear systems also satisfy the properties of superposition and homogeneity. The superposition property states that for a system initially at rest with zero energy, the response to several inputs applied simultaneously is the sum of the individual responses to each input applied separately. The homogeneity property states that multiplying the inputs to a system by any constant multiplies the outputs by the same constant.

Constant Parameters

Time-varying systems are ones whose characteristics change with time. Physical problems are simplified by the adoption of a model in which all the physical parameters are constant.

Neglect Uncertainty and Noise

In real systems we are uncertain, in varying degrees, about values of parameters, about measurements, and about expected inputs and disturbances. Disturbances contain random inputs, called noise, which can influence system behavior. It is common to neglect such uncertainties and noise and proceed as if all quantities have definite values that are known precisely.

The assumptions made in developing a physical model for the DC motor / tachometer system are as follows:

1. The copper armature windings in the motor are treated as a resistance and inductance in series. The distributed inductance and resistance is lumped into two characteristic quantities, L and R .
2. The commutation of the motor is neglected. The system is treated as a single electrical network which is continuously energized.
3. The compliance of the shaft connecting the load to the motor is negligible. The shaft is treated as a rigid member. Similarly, the coupling between the tachometer and motor is also considered to be rigid.
4. The total inertia J is a single lumped inertia, equal to the sum of the inertias of the rotor, the tachometer, and the driven load.
5. There exists motion only about the axis of rotation of the motor, i.e., a one-degree-of-freedom system.
6. The parameters of the system are constant, i.e., they do not change over time.
7. The damping in the mechanical system is modeled as viscous damping B , i.e., all stiction and dry friction are neglected.
8. Neglect noise on either the sensor or command signal.
9. The amplifier dynamics are assumed to be fast relative to the motor dynamics. The unit is modeled by its DC gain, K_{amp} .
10. The tachometer dynamics are assumed to be fast relative to the motor dynamics. The unit is modeled by its DC gain, K_{tach} .

The physical model for the DC motor based on these assumptions is shown in Figure 7.

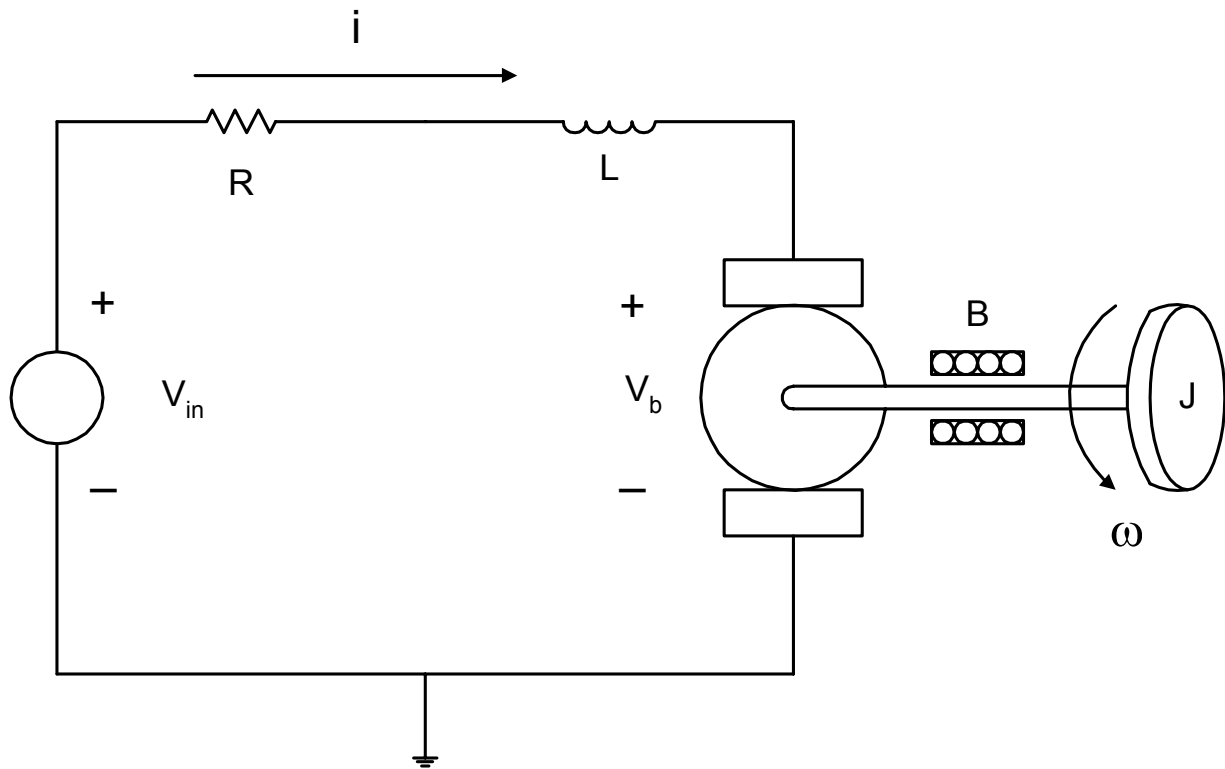


Figure 7. Physical Model of a DC Motor

The analog DC tachometer is a permanent-magnet DC velocity sensor. The assumptions made in developing a physical model for the analog tachometer are similar to the assumptions made for the DC motor model, as the analog tachometer is a DC generator - a DC motor in reverse.

4. Mathematical Model

The steps in mathematical modeling are as follows:

- Define System, System Boundary, System Inputs and Outputs
- Define Through and Across Variables
- Write Physical Relations for Each Element
- Write System Relations of Equilibrium and/or Compatibility
- Combine System Relations and Physical Relations to Generate the Mathematical Model for the System

Let's look at each step more closely and then apply these steps to our physical model.

- *Define System, System Boundary, System Inputs and Outputs:* A system must be defined before equilibrium and/or compatibility relations can be written. Unless physical boundaries of a system are clearly specified, any equilibrium and/or compatibility relations we may write are meaningless. System outputs to the environment and system inputs from the environment must be also clearly defined.
- *Define Through and Across Variables:* Precise physical variables (velocity, voltage, pressure, flow rate, etc.) with which to describe the instantaneous state of a system, and in terms of which to study its behavior must be selected. Physical Variables may be classified as:
 1. *Through Variables* (one-point variables) measure the transmission of something through an element, e.g., electric current through a resistor, fluid flow through a duct, force through a spring.
 2. *Across Variables* (two-point variables) measure a difference in state between the ends of an element, e.g., voltage drop across a resistor, pressure drop between the ends of a duct, difference in velocity between the ends of a damper.
- *Write Physical Relations for Each Element:* Write the natural physical laws which the individual elements of the system obey, e.g., mechanical relations between force and motion, electrical relations between current and voltage, electromechanical relations between force and magnetic field, thermodynamic relations between temperature, pressure, etc. These relations are called constitutive physical relations as they concern only individual elements or constituents of the system. They are relations between the through and across variables of each individual physical element and may be algebraic, differential, integral, linear or nonlinear, constant or time-varying. They are purely empirical relations observed by experiment and not deduced from any basic principles. Also write any energy conversion relations, e.g., electrical-electrical (transformer), electrical-mechanical (motor or generator), mechanical-mechanical (gear train). These relations are between across variables and between through variables of the coupled systems.

- *Write System Relations of Equilibrium and/or Compatibility:* Dynamic equilibrium relations describe the balance of forces, of flow rates, of energy which must exist for the system and its subsystems. Equilibrium relations are always relations among through variables, e.g., Kirchhoff's Current Law (at an electrical node), continuity of fluid flow, equilibrium of forces meeting at a point. Compatibility relations describe how motions of the system elements are interrelated because of the way they are interconnected. These are inter-element or system relations. Compatibility relations are always relations among across variables, e.g., Kirchhoff's Voltage law around a circuit, pressure drop across all the connected stages of a fluid system, geometric compatibility in a mechanical system.
- *Combine System Relations and Physical Relations to Generate the Mathematical Model for the System:* The mathematical model can be an input-output differential equation or state-variable equations. A state-determined system is a special class of lumped-parameter dynamic system such that: (i) specification of a finite set of n independent parameters, *state variables*, at time $t = t_0$ and (ii) specification of the system inputs for all time $t \geq t_0$ are necessary and sufficient to uniquely determine the response of the system for all time $t \geq t_0$. The *state* is the minimum amount of information needed about the system at time t_0 such that its future behavior can be determined without reference to any input before t_0 . The *state variables* are independent variables capable of defining the state from which one can completely describe the system behavior. These variables completely describe the effect of the past history of the system on its response in the future. Choice of state variables is not unique and they are often, but not necessarily, physical variables of the system. They are usually related to the energy stored in each of the system's energy-storing elements, since any energy initially stored in these elements can affect the response of the system at a later time. The *state-space* is a conceptual n -dimensional space formed by the n components of the state vector. At any time t the state of the system may be described as a point in the state space and the time response as a trajectory in the state space. The number of elements in the state vector is unique, and is known as the *order* of the system. The *state-variable* equations are a coupled set of first-order ordinary differential equations where the derivative of each state variable is expressed as an algebraic function of state variables, inputs, and possibly time.

Let's apply these steps to the DC motor physical model:

- *Define System, System Boundary, System Inputs and Outputs*
Figure 7 shows the physical model of the physical system, identifying the system and system boundary. The input to the system is the command voltage supplied to the motor, V_{in} , and the output of the system is the absolute angular velocity of the motor shaft, ω , as measured by the DC tachometer.

- *Define Through and Across Variables*

The through variables for this system are the current i in the electrical circuit flowing through the resistance R and inductance L , and the torques (i.e., electromechanical, inertial, damping) applied to the inertia J , where $J = J_{\text{motor}} + J_{\text{tachometer}} + J_{\text{load}}$. The across variables are the voltages across the resistance R and inductance L , and the angular velocity ω and angular acceleration α of the inertia J . The state variables for the system are the current i through the inductance L , as energy is stored in the magnetic field of the inductor ($E = \frac{1}{2}Li^2$), and the angular velocity ω of the inertia J , as energy is stored in the kinetic energy of the inertia ($E = \frac{1}{2}J\omega^2$).

- *Write Physical Relations for Each Element*

$$\begin{aligned} V_L &= L \frac{di_L}{dt} & V_R &= Ri_R & T_B &= B\omega & T_J &= J\alpha = J\dot{\omega} & J &\equiv J_{\text{motor}} + J_{\text{tachometer}} + J_{\text{load}} \\ T_m &= K_t i_m & V_b &= K_b \omega \\ P_{\text{out}} &= T_m \omega = K_t i_m \omega & P_{\text{in}} &= V_b i_m = K_b \omega i_m \\ \frac{P_{\text{out}}}{P_{\text{in}}} &= \frac{K_t}{K_b} \end{aligned}$$

For an ideal energy conversion element (no losses), $P_{\text{out}} = P_{\text{in}}$ $K_t = K_b \equiv K_m$

- *Write System Relations of Equilibrium and/or Compatibility*

Compatibility Equation:

$$\text{Kirchoff's Voltage Law: } V_{\text{in}} - V_R - V_L - V_b = 0$$

Equilibrium Equations:

$$\text{D'Alembert's Principle: } T_m - T_B - T_J = 0$$

$$\text{Kirchoff's Current Law: } i_R = i_L = i_m \equiv i$$

- *Combine System Relations and Physical Relations to Generate the Mathematical Model for the System*

Equations of Motion:

$$V_{\text{in}} - Ri - L \frac{di}{dt} - K_b \omega = 0 \qquad J\dot{\omega} + B\omega - K_t i = 0$$

State-Variable Equations:

$$\begin{bmatrix} \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -B/J & K_t/J \\ -K_b/L & -R/L \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{\text{in}}$$

5. Predicted Dynamic Behavior

Take the Laplace transform of the equations of motion, assuming zero initial conditions:

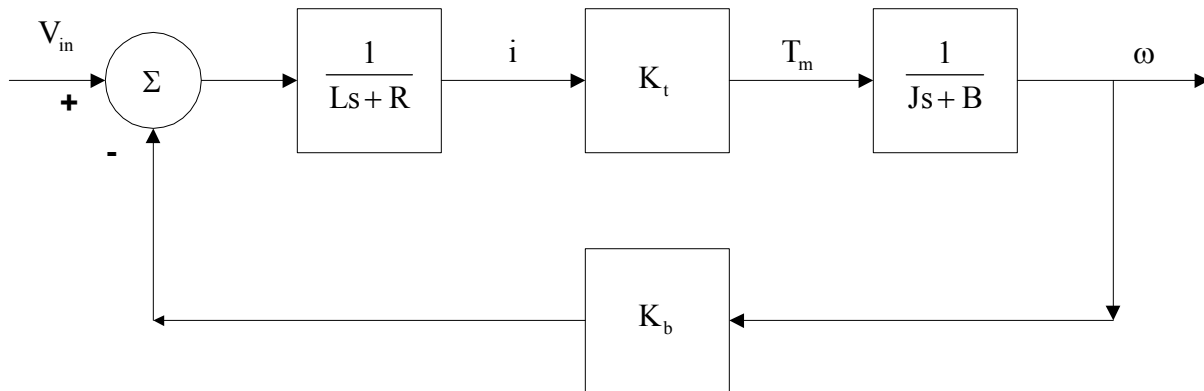
$$V_{in} - Ri - L \frac{di}{dt} - K_b \omega = 0 \quad J\dot{\omega} + B\omega - K_t i = 0$$

$$V_{in}(s) - (Ls + R)I(s) - K_b \Omega(s) = 0 \quad (Js + B)\Omega(s) - K_t I(s) = 0$$

Combining these equations results in the following transfer function:

$$\begin{aligned} \frac{\Omega(s)}{V_{in}(s)} &= \frac{K_t}{(Js + B)(Ls + R) + K_t K_b} = \frac{K_t}{JLs^2 + (BL + JR)s + (BR + K_t K_b)} \\ &= \frac{\frac{K_t}{JL}}{s^2 + \left(\frac{B}{J} + \frac{R}{L}\right)s + \left(\frac{BR}{JL} + \frac{K_t K_b}{JL}\right)} \\ &= \frac{6.8363 \times 10^5}{s^2 + (6.3334 \times 10^3)s + (2.6036 \times 10^4)} = \frac{6.8363 \times 10^5}{(s + 4.11)(s + 6329.3)} \end{aligned}$$

A block diagram for this open-loop system is shown below:



The mechanical time constant τ_m and the electrical time constant τ_e of the motor are defined below. We see that $\tau_m \gg \tau_e$.

$$\tau_m = \frac{J_{\text{motor}}}{B} = \frac{3.18 \times 10^{-6} \text{ kg} \cdot \text{m}^2}{6.74 \times 10^{-6} \frac{\text{N} \cdot \text{m} \cdot \text{sec}}{\text{rad}}} = 472 \text{ msec}$$

$$\tau_e = \frac{L}{R} = \frac{6.0 \times 10^{-4} \text{ H}}{3.8 \Omega} = 0.158 \text{ msec}$$

Since $\tau_m \gg \tau_e$, motor armature inductance is small, and therefore we may neglect the inductance and write:

$$V_{in} - Ri - K_b \omega = 0 \quad J\dot{\omega} + B\omega - K_t i = 0$$

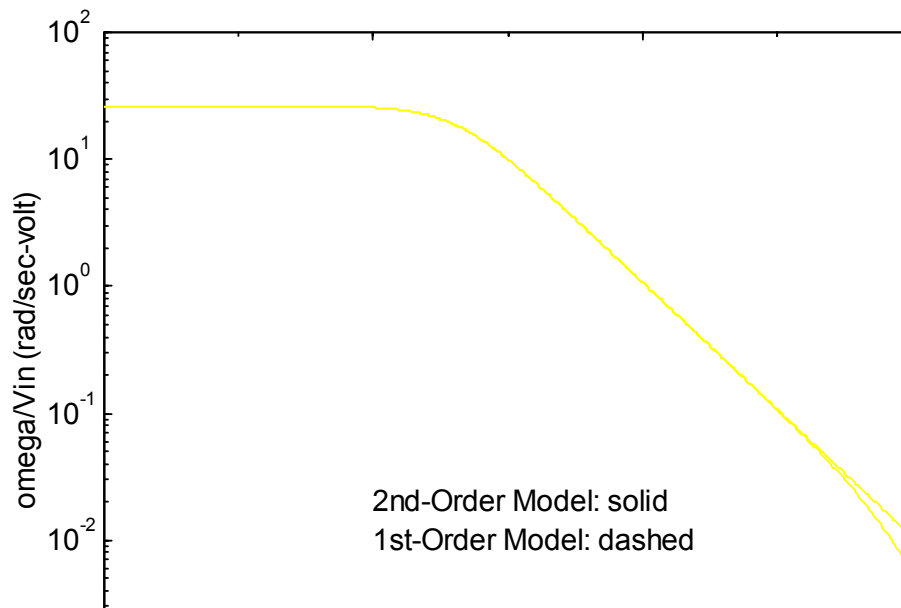
The following differential equation results:

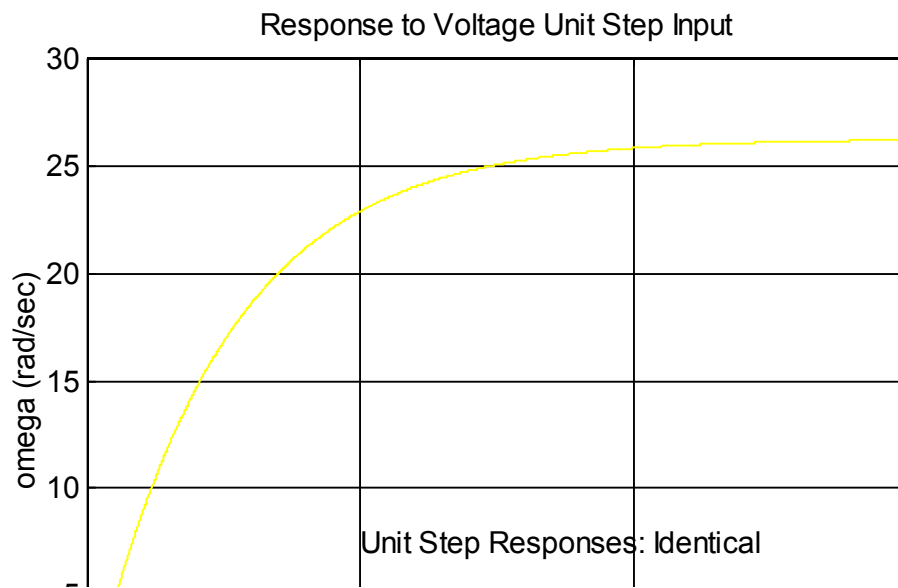
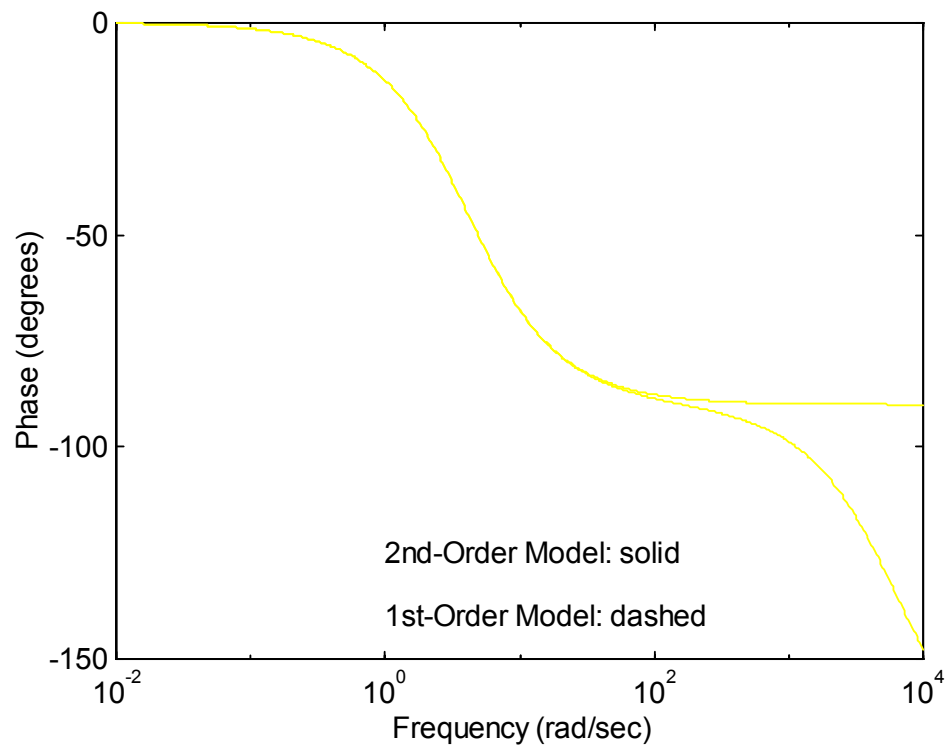
$$\begin{aligned} J\dot{\omega} + B\omega &= K_t i = K_t \left(\frac{1}{R} (V_{in} - K_b \omega) \right) = \frac{K_t}{R} (V_{in} - K_b \omega) \\ \dot{\omega} + \left(\frac{K_t K_b}{RJ} + \frac{B}{J} \right) \omega &= \frac{K_t}{RJ} V_{in} \\ \dot{\omega} + (4.11) \omega &= (107.94) V_{in} \end{aligned}$$

The resulting differential equation is a first-order ordinary differential equation with a time constant:

$$\frac{1}{\tau} = \frac{K_t K_b}{RJ} + \frac{B}{J} = 4.11 \text{ sec}^{-1}$$

Let's compare 2nd-order and 1st-order models in both the time and frequency domains.





6. Experimental Verification of the Model

In order to determine if the proposed model adequately represents the important dynamics of the physical system, experimental measurements of the parameter values and dynamic responses must be taken and compared to the predicted results from the analytical models.

6.1 Design Experiments to Determine Parameter Values

In Section 5 we determined the system could be adequately represented by a first-order transfer function. In performing that analysis, the frequency response and step response of the system were used to characterize the dynamic performance of the model. Two experiments are proposed to experimentally generate the open-loop frequency response and step response of the physical system. These experimental results should be compared with the analytical predictions to assess the performance of the model.

In the first experiment, the open-loop frequency response is determined by applying a sinusoidal voltage with a DC bias to the amplifier and measuring the voltage signal across the tachometer terminals. The ratio of the peak-to-peak amplitudes of the output and input signals is the gain of the system at the evaluation frequency. The phase shift between the output and input signals is the phase lag of the system. The bias voltage on the input signal is used to ensure the system does not encounter dry friction or stiction during the experiment. The bias should, therefore, be large enough so that the motor does not stop or reverse direction during the experiment. The experiment should be conducted over a range of frequencies that encompasses the anticipated design domain, taking into account the dynamics that have been identified during the mathematical analysis. Plotting the magnitude and phase of the system as a function of frequency yields a dynamic response plot that can be directly compared to the analytical results.

In the second experiment, a square wave with a DC bias voltage is applied to the amplifier, and the voltage produced by the tachometer is measured. The resulting response (scaled by the amplitude of the square wave) can be directly compared to the analytical unit step response. It is important to select the frequency of the square wave such that the tachometer response achieves steady state.

6.2 Conduct Experiments

With the objective understood, experiments should be conducted and comparisons made. The gain and dynamic response of the amplifier need to be determined and both it and the tachometer voltage constant need to be included in the analysis.

6.3 Frequency-Response Experiments

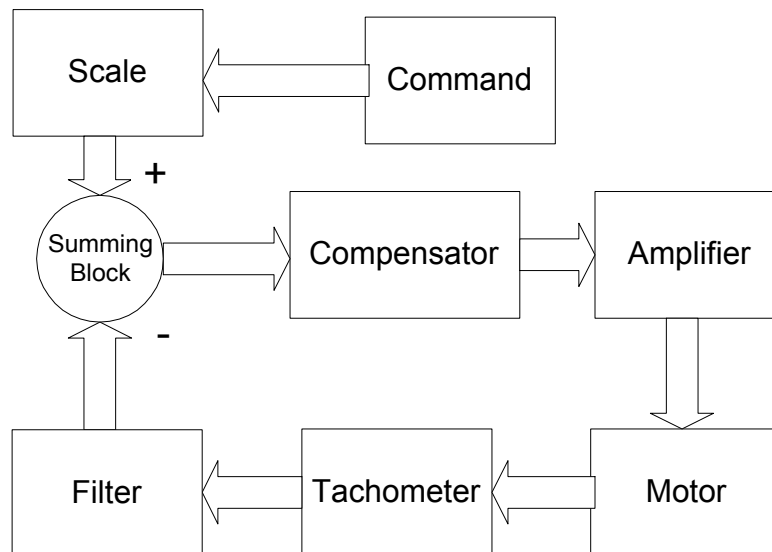
6.4 Time-Response Experiments

7. Control Design

The objective of the exercise is to control the angular speed of the load by regulating the command voltage applied to the power amplifier. It is desired to have the load rotate at speeds varying between ± 500 RPM as the command voltage varies between ± 5 volts. The bandwidth of the command signal should always be less than 20 Hz. The system should have a zero steady-state error to a step voltage command, a rise time ≤ 6 msec, a 1% settling time ≤ 30 msec, and a percent overshoot $\leq 20\%$.

To meet these specifications, a feedback control system must be designed. This involves control system design and development, command and sensor signal conditioning, power amplification, and compensator implementation.

A block diagram of the proposed control system is shown below.



It is desired to command the system with a ± 5 volt signal, linearly corresponding to a load velocity of ± 500 RPM. The tachometer gain is specified to be 3.0 volts/kRPM, which yields ± 1.5 volts from the tachometer at the peak operating speeds of ± 500 RPM. The input must therefore be scaled by $1.5/5 = 0.3$ to ensure the difference between the two signals goes to zero at the desired velocity.

The models for the amplifier, motor and tachometer have already been discussed in the previous sections. The signal coming from the tachometer may be noisy and, thus, may require low-pass filtering. This is a common problem for a system using brushed, DC motors and/or brushed tachometers.

First, the specifications must be translated into root-locus criteria. This is done using the method of mapping time-domain specifications into the root-locus domain.

In order to achieve zero steady-state error to a step voltage input, the open-loop system must have a pole at or very near the origin. The rise-time requirement translates to a specification that all the closed-loop eigenvalues must have a frequency greater than or equal to 300 radians/second. The settling-time requirement translates to a specification that the real part of the closed-loop poles must be less than or equal to -153 radians/second. The percent-overshoot specification translates to a requirement that the damping ratio of the closed-loop eigenvalues must be greater than or equal to 0.46.

Fortunately, the open-loop eigenvalue of the first-order model is at -4.11 radians/second which should be sufficiently close to the origin to allow for compliance with the zero steady-state error requirement. Thus, we should be able to meet the specifications using only proportional feedback on the first-order plant. In order to comply with the other specifications, the closed-loop pole must be placed in the left-half plane with a magnitude greater than or equal to 300 radians/second.

From the root locus of the first-order system we determine that by using proportional, positive feedback, this requirement can be met providing the proportional gain is greater than or equal to $\frac{1}{K}$. It is necessary to use positive feedback because the amplifier gain is negative. This is equivalent to having negative feedback with a negative proportional gain as will be shown when we discuss the implementation of this proportional feedback compensator. For a proportional gain of $\frac{1}{K}$, the closed-loop pole will be at $\frac{1}{K}$ radians/second.

7.1 Frequency-Domain Analysis

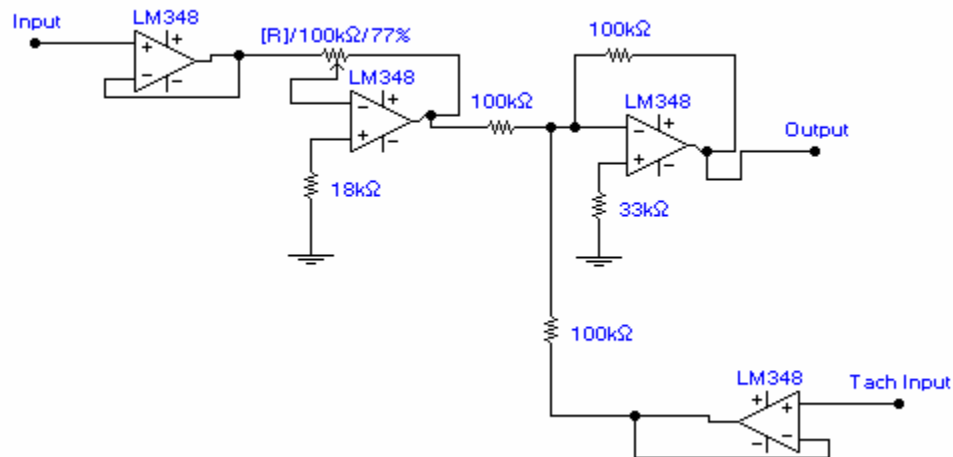
7.2 Time-Domain Analysis

8. Implementation

The next step is to develop circuitry that will implement the control design proposed.

8.1 Analog Control Implementation

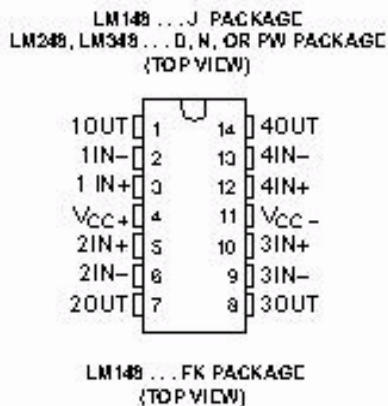
The circuit implemented to perform command scaling and feedback summation is shown in the following diagram.



Command Scaling and Feedback Summation Circuit

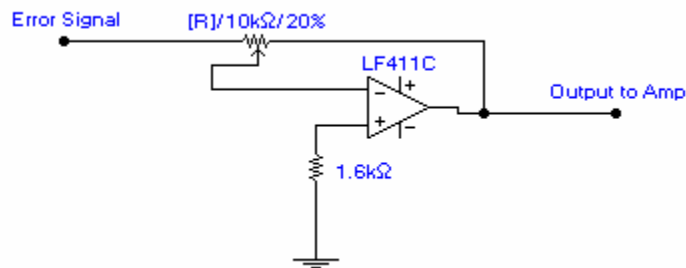
The input, shown on the left side of the schematic, is the command voltage. The output, shown on the right, is the error signal. The tachometer input is shown on the bottom right of the diagram. The circuit is implemented using a quad operational amplifier, the LM 348. This amplifier is an industry standard commonly used for buffering and low-gain amplification. Working from the input to the output in the diagram, the first stage amplifier is configured as a non-inverting, unity-gain follower, which buffers the input signal and prevents upstream circuit loading. The second amplifier has a gain scaling configurable by adjusting the potentiometer value. This allows the command to be scaled by the desired factor of 0.3. The third amplifier is a unity-gain summer, which adds the scaled command signal to the tachometer signal which has been buffered by the fourth amplifier, configured as a non-inverting, unity-gain follower. This circuit is provided as a printed circuit board and provides an error signal for control design based on the command and tachometer signals. The test point on this printed circuit board is the output of the second amplifier, which allows the command scaling gain to be set.

The pin-out for the LM 348 is shown below.



LM 348 Pinout

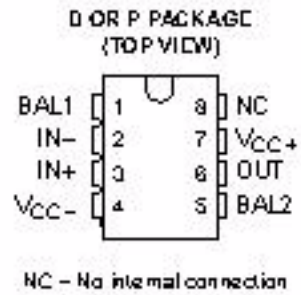
The proportional gain is implemented using the circuit shown schematically below.



Proportional Compensator

The compensator shown is a simple inverting amplifier. The circuit is implemented using a LF 411C operational amplifier, which is a standard single-channel device. The gain is selected via a potentiometer that varies the ratio of the feedback and input resistance. The resistance on the positive leg of the amplifier should be chosen as the parallel combination of these resistances, which, for the control gain of five, is a 1.6 k Ω resistor. The input signal is the error signal generated by the command-scaling, printed-circuit board. The output is the signal sent to the power amplifier.

The pinout for the LF 411C amplifier is shown below.



Pinout of LF 411C Operational Amplifier

9. Testing of the Closed-Loop System and Comparison with Predicted Response

The final stage in the control development process is to experimentally determine the performance of the final, closed-loop system. The experimental testing is performed in both the frequency domain and the time domain in order to determine the closed-loop system response. Comparisons are then made with the predicted responses.

9.1 Frequency-Domain Testing

9.2 Time-Domain Testing