Chapter one

Introduction to machine elements

What are machine elements?

All machines are made up of elements or parts and units. Each element is a separate part of the machine and it may have to be designed separately and in assembly. Each element in turn can be a complete part or made up of several small pieces.

The machine elements are classified in to two main types:

Machine elements
- → *General purpose elements* (elements which are **common** to various types of machines serving different applications Eg. Bolts, nuts, Keys, axels, shafts, couplings, bearings etc)
- → *Special purpose elements* (elements which are used for **Specific** type of machines Eg. Crank shafts, push rod, Piston, turbine blades etc.)

Engineering materials

The machine elements should be made of a material which has properties suitable for the conditions of operation.

Engineering materials
- Metals
  - Ferrous
  - Non-ferrous
- Non – metals

The selection of a proper material, for engineering purposes, is one of the most difficult problem for the designer. The best material is one which serve the desired objective at the minimum cost.

Stresses in machine parts

Stress

When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as ***unit stress*** or simply a ***stress*.**

$$\sigma = \frac{p}{A}$$

Stress

- **Direct stress**
  - Tensile
    - Due to axial pull
  - compressive
    - due to axial push
  - direct shear
    - due to two equal and opposite tangential forces
- Torsional shear stress
  - Due to twisting moment

- Direct stresses governed by simple bending theory:

$$\frac{\delta}{y} = \frac{M}{I} = \frac{E}{R}$$

- Torsional shear stress analyzed on the basis of simple torsion theory:

$$\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$$

Chapter Three

Torque transmitting elements

Key

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to **prevent relative motion** between them. It is always inserted parallel to the axis of the shaft. Keys are used as **temporary fastenings** and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

The following types of keys are important from the subject point of view:

1. Sunk keys,
2. Saddle keys,
3. Tangent keys, 4.Round keys, and 5.Splines.

1. Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types:

1. ***Rectangular sunk key.*** A rectangular sunk key is shown in Fig. below. The usual proportions of this key are:
   Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$
   where d = Diameter of the shaft or diameter of the hole in the hub.
   The key has taper 1 in 100 on the top side only

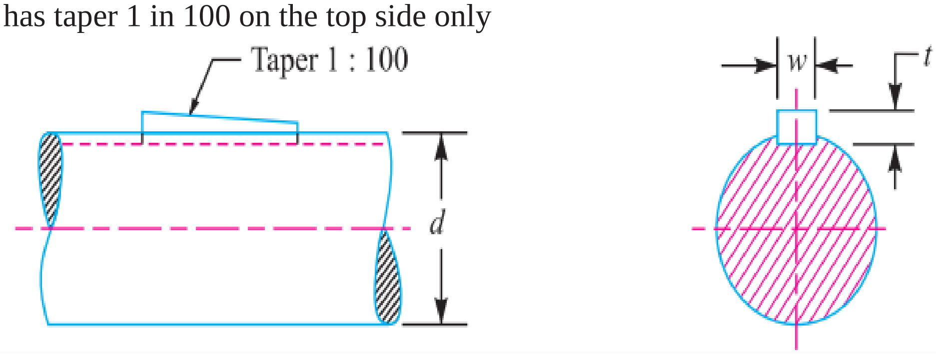


Fig. Rectangular sunk key

2. *Square sunk key.* The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

3. *Parallel sunk key*. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

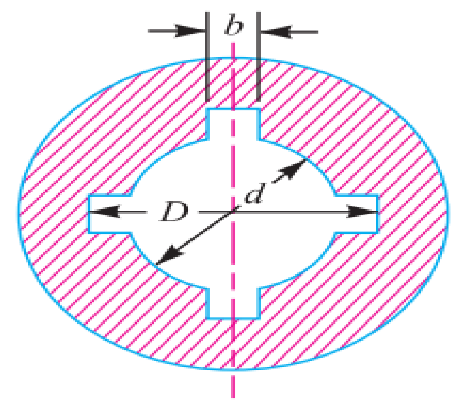
4. *Gib-head key*. It is a rectangular sunk key with a head at one end known as ***gib head*.** It is usually provided to facilitate the removal of key.

5. *Feather key*. A key attached to one member of a pair and which permits relative axial movement is known as ***feather key*.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

The feather key may be screwed to the shaft or it may have double gib heads the various proportions of a feather key are same as that of rectangular sunk key and gib head key.

Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig... These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway. The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



$D = 1.25\ d$ and $b = 0.25\ D$

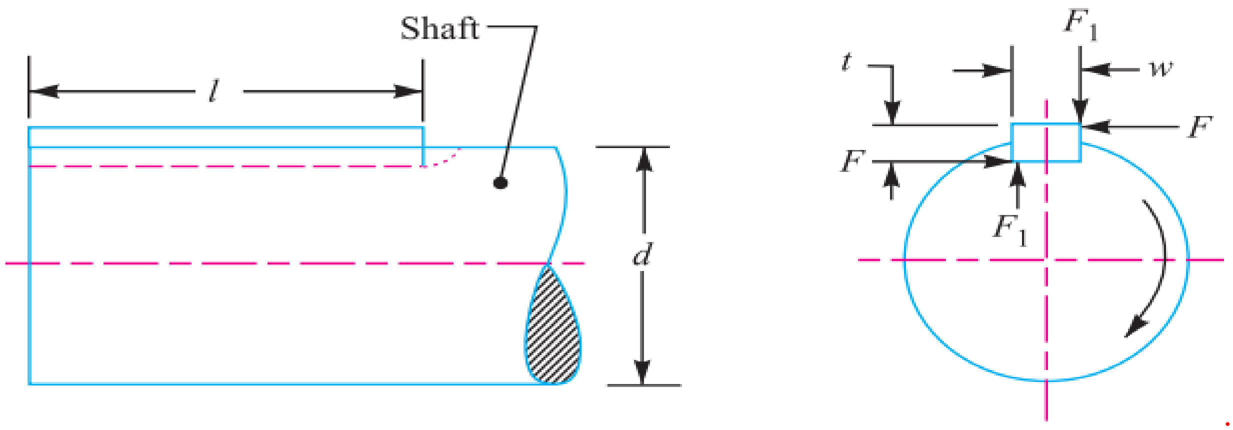
Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (F_1) **due to fit of the key in its keyway**, as in a tight fitting straight key or in a tapered key driven in place. These forces **produce compressive stresses** in the key which are difficult to determine in magnitude.

2. Forces (F) **due to the torque transmitted** by the shaft. These forces **produce shearing** and **compressive (or crushing) stresses** in the key. The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

In designing a key, forces **due to fit of the key are neglected** and it is assumed that the **distribution** of forces along the length of key is **uniform.**



<u>Strength of a Sunk Key</u>

A key connecting the shaft and hub is shown in Fig. above

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

and

σ_c = Shear and crushing stresses for the material of key

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress = $l \times w \times \tau$

∴ Torque transmitted by the shaft,

$$T = F * \frac{d}{2} = l.w.\tau.\frac{d}{2}$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

F = Area resisting crushing × Crushing stress = $l.\frac{t}{2} * \sigma_c$

∴ Torque transmitted by the shaft,

$$T = F * \frac{d}{2} = l.\frac{t}{2} * \sigma_c.\frac{d}{2}$$

The key is equally strong in shearing and crushing, if

$$l.w.\ \tau.\frac{d}{2} = l.\frac{t}{2} * \sigma_c.\frac{d}{2}$$

$$\frac{w}{t} = \frac{\sigma c}{2\tau} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{i}$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from equation (i), we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the **shearing strength of the key is equal to the torsional shear strength of the shaft**.

We know that the shearing strength of key,

$$T = l.w.\ \tau.\frac{d}{2}$$

And torsional shear strength of the shaft,

$$T = \frac{\pi}{16} * \tau s * d^3 \text{ (τ is shear strength of the shaft)}$$

Then equating the above equations:

$$l.w.\ \tau.\frac{d}{2} = \frac{\pi}{16} * \tau s * d^3$$

$$l = \frac{\pi}{8} * \frac{\tau s.d^2}{w.\tau} = \frac{\pi d}{2} * \frac{\tau s}{\tau} = 1.571d * \frac{\tau s}{\tau} \ldots\ldots\ldots \text{ (Taking w=d/4)}$$

When the key material is same as that of the shaft, then $\tau = \tau s$.

$l = 1.571d$

Example 1. A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Chapter 5

Springs

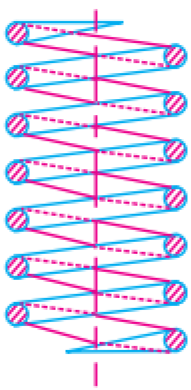
A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

- ✓ To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
- ✓ To apply forces, as in brakes, clutches and spring loaded valves.
- ✓ To control motion by maintaining contact between two elements as in cams and followers.
- ✓ To measure forces, as in spring balances and engine indicators.
- ✓ To store energy, as in watches, toys, etc.

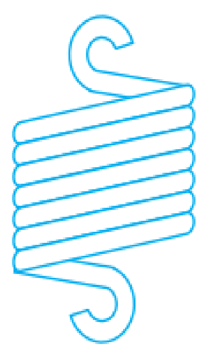
Types of springs

Though there are many types of the springs, yet the following, according to their shape:

- ✓ ***Helical springs*.** The helical springs are made up of a wire coiled in the form of a **helix** and is primarily intended for **compressive or tensile loads**. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig. (*a*) and ***tension helical spring*** as shown Fig. (b)



(a) Compression helical spring



(b) tension helical spring

- ✓ Helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are **shear stresses due to twisting**. The load applied is parallel to or along the axis of the spring.

- ✓ The helical springs have the following advantages:
  (***a***) These are easy to manufacture.
  (***b***) These are available in wide range.
  (***c***) These are reliable.
  (***d***) These have constant spring rate.
  (***e***) Their performance can be predicted more accurately.
  (***f***) Their characteristics can be varied by changing dimensions.
- ✓ ***Conical and volute springs***
- ✓ ***Torsion springs***
- ✓ ***Laminated or leaf springs***
- ✓ ***Disc or bellevile springs***
- ✓ ***Special purpose springs***

Material for Helical Springs

The material of the spring should have high **fatigue** strength, high **ductility,** high **resilience** and it should be creep resistant. It largely depends upon the service for which they are used *i.e.* **severe service, average service or light service.**

- ✓ ***Severe service*** means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.
- ✓ ***Average service*** includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.
- ✓ ***Light service*** includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

Terms used in Compression Springs

Solid length. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid.*** The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$$L_s = n'.d$$

Where n' = Total number of coils, and
d = Diameter of the wire.

Free length. The free length of a compression spring, as shown in Fig. (a), is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).
Mathematically,

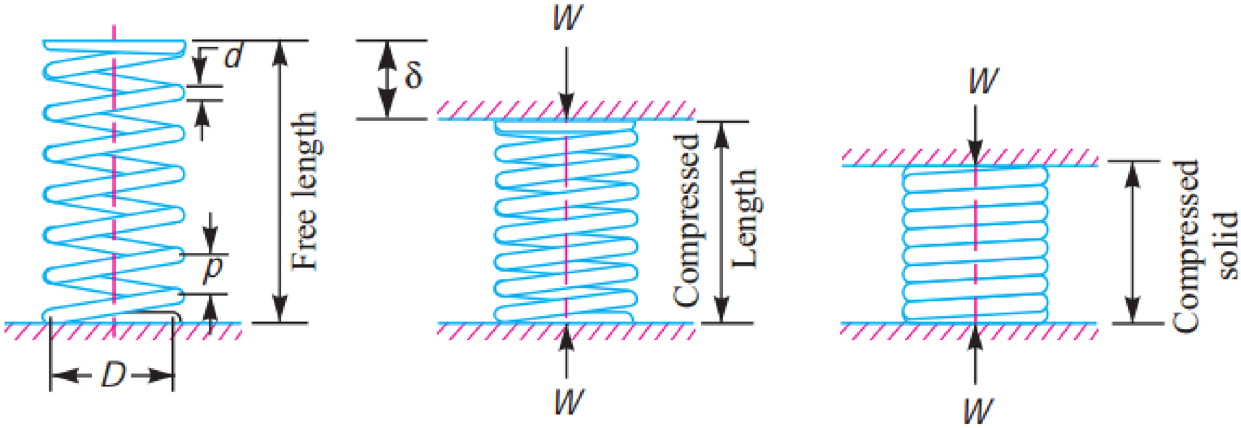


Fig. Compression spring nomenclature

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta max + 0.15\ \delta max$

The following relation may also be used to find the free length of the spring, *i.e.*

$L_F = n'.d + \delta max + (n' - 1) \times 1\ mm$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

Where D = Mean diameter of the coil, and
d = Diameter of the wire.

Spring rate. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

Where W = Load, and

δ= Deflection of the spring

Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $$P = \frac{free\ length}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

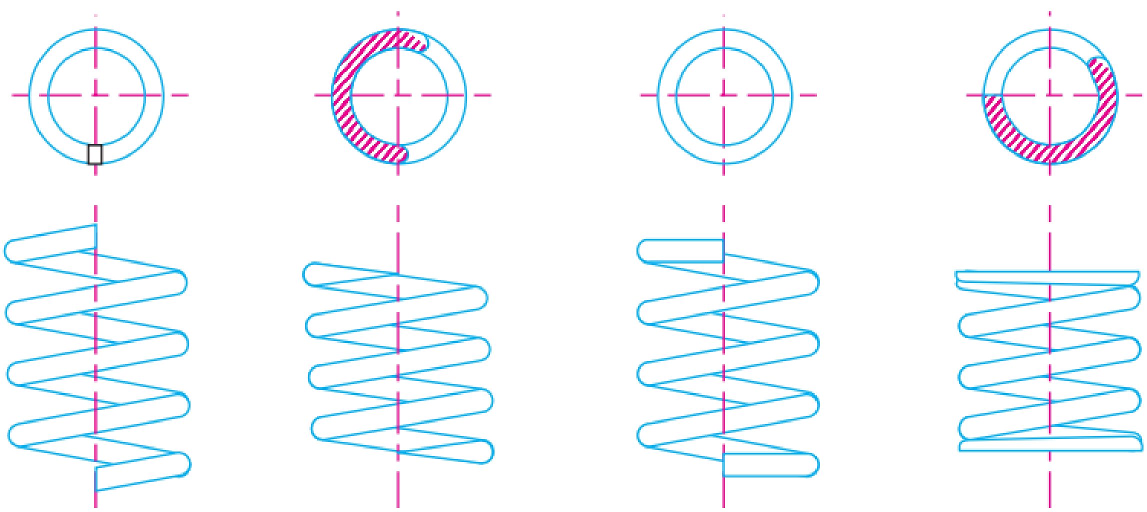
Pitch of the coil, $$P = \frac{Lf - Ls}{n'} + d$$

Where d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :
(*a*) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.
(*b*) The spring should not close up before the maximum service load is reached

End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in Fig. below



a) Plain end b) ground end c) square end d) square and ground end

Table (a) Total number of turns, solid length and free length for different types of end connections

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\, d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\, d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\, d$	$p \times n + 2d$

Where n = Number of active turns,
p = Pitch of the coils, and
d = Diameter of the spring wire.

Stresses in Helical Springs of Circular Wire

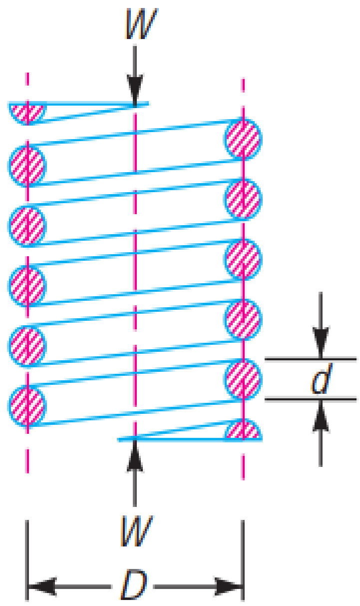
Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. (*a*).

Let D = Mean diameter of the spring coil,
d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring

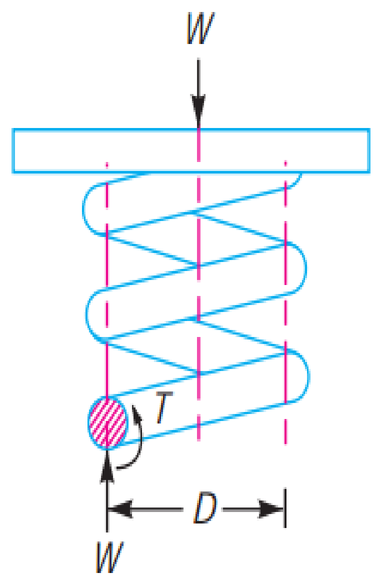
τ= Maximum shear stress induced in the wire,
C = spring index

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W



a) Axially loaded helical spring



b) Free body diagram showing that wire is subjected to torsional shear and a **direct shear.**

Now consider a part of the compression spring as shown in Fig. (*b*). the load *W* tends to rotate the wire due to the twisting moment (*T)* set up in the wire. Thus torsional shear stress is induced in the wire. A little consideration will show that part of the spring, as shown in Fig. (*b*), is in equilibrium under the action of two forces *W* and the twisting moment *T*. We know that the twisting moment,

$$T = W * \frac{D}{2} = \frac{\pi}{16} * \tau_1 * d^3, \text{ Thus}$$

$$\tau_1 = \frac{8WD}{\pi d^3} \quad \ldots\ldots\ldots\ldots 1$$

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire:

- ✓ **Direct shear** stress **due to the load *W***, and
- ✓ **Stress due to curvature of wire**

We know that direct shear stress due to the load *W*,

$$\tau_2 = \frac{load}{cross - sectional\ area}$$

$$\tau_2 = \frac{4W}{\pi d^2}$$

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2$$

$$\tau = \frac{8WD}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8WD}{\pi d^3}\left(1 + \frac{d}{2D}\right), \text{ Substituting } \frac{D}{d} = c$$

$$= \frac{8WD}{\pi d^3}\left(1 + \frac{1}{2C}\right)$$

Where K_s = shear stress factor = $1 + \frac{1}{2C}$

It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (*K*) introduced. Wahl may be used,

∴ Maximum shear stress induced in the wire,

$$\tau = k * \frac{8WD}{\pi d^3}$$

Where $K = \frac{\mathbf{4\,C - 1}}{4C - 4} + \frac{0.615}{C}$

Note: The Wahl's stress factor (*K*) may be considered as composed of two sub-factors, K_s and *K*c, such that

$K = K_s \times K_c$

Where

K_s = Stress factor due to shear, and

K_c= Stress concentration factor due to curvature.

Deflection of Helical Springs of Circular Wire

Total active length of the wire,

l = Length of one coil × No. of active coils = $\pi D * n$

Let θ = Angular deflection of the wire when acted upon by the torque *T*.

∴ Axial deflection of the spring,

$$\delta = \theta * \frac{D}{2}$$

But, from simple torsion theory:

$$\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}, \text{ Where R=D/2}$$

$$\theta = \frac{TL}{GJ} = \frac{\left(W*\frac{D}{2}\right)*\pi D*n}{\frac{\pi}{32}d^4 G} = \frac{16WD^2.n}{Gd^4}$$

Substituting the above value in the equation of deflection,

$$\delta = \frac{16WD^2.n}{Gd^4} * \frac{D}{2} = \frac{8WD^3.n}{Gd^4} = \frac{8WC^3.n}{Gd\cdot} \quad \text{... } (C = D/d)$$

And the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{Gd}{8WC^3.n} = \text{constant}$$

Example 1: A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 KN/mm2, find the axial load which the spring can carry and the deflection per active turn.

Example 2: Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 KN/mm2.

Example 3: Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 KN/mm2. Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Chapter four

Shafts

A shaft is a rotating machine element which is **used to transmit power from one place to another.** The power is delivered to the shaft by some **tangential force** and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In other words, we may say that a **shaft is used for the transmission of torque and bending moment.** The various members are mounted on the shaft by means of **keys** or **splines.**

An ***axle,*** though similar in shape to the shaft, is a **stationary machine element and is used for the transmission of bending moment only.**

A ***spindle*** is a short shaft that **imparts motion** either to a cutting tool (*e.g*. drill press spindles) or to a work piece (*e.g*. lathe spindles).

Types of Shafts

There are two types of shafts:

1. Transmission shafts. These shafts **transmit power between the source and the machines absorbing power**. The counter shafts, line shafts, overhead shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. Machine shafts. These shafts form an **integral part of the machine itself**. The crank shaft is an example of machine shaft.

Stresses in Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to **combined torsional and bending loads**.

Design of Shafts

The shafts may be designed on the basis of
1. Strength, and **2.** Rigidity and stiffness.
In designing shafts on the basis of strength, the following cases may be considered :
(*a*) Shafts subjected to twisting moment or torque only,
(*b*) Shafts subjected to bending moment only,
(*c*) Shafts subjected to combined twisting and bending moments, and
(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.
We shall now discuss the above cases, in detail, in the following pages.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that from simple torsion theory:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{\tau}{r} = \frac{T}{J}$$

We know that for round solid shaft, polar moment of inertia:

$$J = \frac{\pi}{32} d^4, \text{ and } R = d/2$$

So, substituting the above results on the relation of simple torsion theory leads or gives us:

$$T = \frac{\pi}{16} * \tau_1 * d^3$$

From this equation, we may determine the diameter of round solid shaft (d). We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32} ({d_o}^4 - {d_i}^4), \text{ and } r_o = d_o/2$$

Substituting these values in equation of simple torsion theory:

$$T = \frac{\pi}{16} * \tau_1 * \left[\frac{{d_o}^4 - {d_i}^4}{d_o}\right] = \frac{\pi}{16} * \tau_1 * {d_o}^3 (1 - k^4) \text{ Where } k = \frac{d_i}{d_o} \ldots \textbf{(1)}$$

N.B. The twisting moment (*T*) may be obtained by using the following relation:

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi . N . T}{60}$$

$$T = \frac{Px60}{2\pi . N}$$

Example.1. A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Example.2.Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{\delta}{y} = \frac{M}{I} = \frac{E}{R}$$

$$\frac{\delta}{y} = \frac{M}{I}$$

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64}\, d^4 \text{ And } y = \frac{d}{2} \text{ so}$$

$$\frac{\delta}{\frac{d}{2}} = \frac{M}{\frac{\pi}{64} d^4} \text{ Or } M = \frac{\pi}{32} * \delta * d^3$$

We also know that for a hollow shaft, moment of inertia,

$I = \frac{\pi}{64}({d_o}^4 - {d_i}^4) = \frac{\pi}{64} * {d_o}^4(1 - k^4)$…where k = $\frac{d_i}{d_o}$ so then substituting the results we can calculate the bending moment induced in the shaft by using the following equation:

$$M = \frac{\pi}{32} * \delta * {d_o}^3(1 - k^4) \ldots\ldots \textbf{(2)}$$

Example 3. A pair of wheels of a railway wagon carries a load of 50 KN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

According to maximum shear stress theory, the maximum shear stress in the shaft;

$$\tau_{max} = \frac{1}{2}\sqrt{\delta^2 + 4\tau^2}$$

Then substituting the values of δ and τ from equation 2 and 1 respectively to the above equation we have,

$$\tau_{max} = \frac{1}{2}\sqrt{\delta^2 + 4\tau^2} = \frac{1}{2}\sqrt{(\frac{32M}{\pi d^3})^2 + 4(\frac{16T}{\pi d^3})^2} = \frac{16}{\pi d^3}[\sqrt{M^2 + T^2}]$$

Implies

$$\frac{\pi}{16} * \tau_{max} * d^3 = \sqrt{M^2 + T^2} \; \textbf{... (3)}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation ***(3)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} * \tau_{.} * d^3$$

From this expression, diameter of the shaft (d) may be evaluated. Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$\sigma_{max} = \frac{1}{2}\sigma + \frac{1}{2}\sqrt{\delta^2 + 4\tau^2}$, Substituting the results for both the shear and bending stress values we will get the following value,

$$\sigma_{max} = \frac{1}{2}\frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{(\frac{32M}{\pi d^3})^2 + 4(\frac{16T}{\pi d^3})^2}$$

$$= \frac{32}{\pi d^3}[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)]$$

$$\frac{\pi}{32} * \sigma_{max} * d^3 = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

The expression $\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ) as the actual bending moment.** By limiting the maximum normal stress [σ_b (*max*)] equal to the allowable bending stress (σ_b), then the equation may be written as,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} * \sigma_{max} * d^3$$

Example 4. A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 meters. It carries two pulleys each weighing 1500 N supported at a distance of 1 meter from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.

Chapter 7

Couplings and Clutches

Types of Shaft Couplings

A **coupling** is termed as a device used to make **permanent** or **semi-permanent** connection where as a **clutch** permits rapid **connection or disconnection** at the will of the operator.

Shafts are usually available up to 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling. The primary purposes of shaft couplings are:

- ✓ To provide for the **connection** of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
- ✓ To provide for **misalignment** of the shafts or to introduce mechanical flexibility.
- ✓ To **reduce** the transmission of **shock loads** from one shaft to another.
- ✓ To introduce protection against **overloads.**

Shaft couplings are divided into two main groups as follows:

1. ***Rigid coupling*****.** It is used to connect two shafts which are **perfectly aligned**. Following types of rigid coupling are important from the subject point of view :
   (*a*) Sleeve or muff coupling.
   (*b*) Clamp or split-muff or compression coupling, and
   (*c*) Flange coupling.
2. ***Flexible coupling*****.** It is used to connect two shafts having both lateral and angular **misalignment.** Following types of flexible coupling are important from the subject point of view:
   (*a*) Bushed pin type coupling,
   (*b*) Universal coupling, and
   (*c*) Oldham coupling.

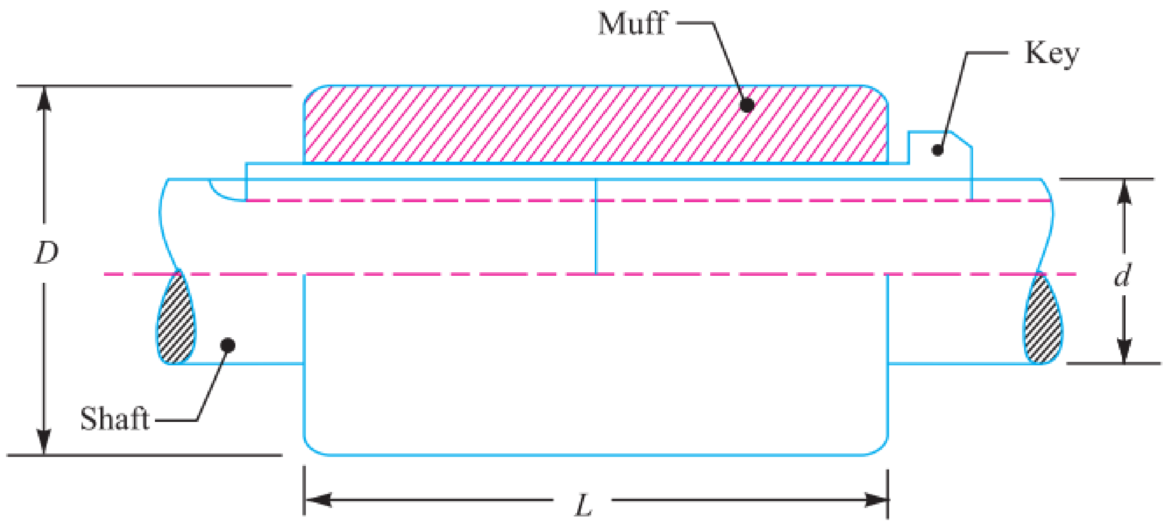
Sleeve or Muff-coupling

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose **inner diameter is the same as that of the shaft**. It is fitted over the ends of the two shafts by means of a **gib head key**, as shown in Fig. below. The power is transmitted from one shaft to the other shaft by means of a **key** and a **sleeve**. It is, therefore, necessary that all the elements must be strong enough to transmit the **torque**. The usual proportions of a cast iron sleeve coupling are as follows:

Outer diameter of the sleeve, $D = 2d + 13$ mm
and length of the sleeve, $L = 3.5\,d$
where d is the diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.

1. ***Design for sleeve***
   The sleeve is designed by considering it as a hollow shaft.



Let T = Torque to be transmitted by the coupling, and

τ = Permissible shear stress for the material of the sleeve which is cast iron.

The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section.

$$T = \frac{\pi}{16} * \tau_1 * \left[\frac{{d_o}^4 - {d_i}^4}{d_o}\right] = \frac{\pi}{16} * \tau_1 * {d_o}^3 (1 - k^4) \text{ ... (k = d/D)}$$

From this expression, the induced shear stress in the sleeve may be checked.

2. ***Design for key***

The key for the coupling may be designed in the similar way as discussed in key design. The width and thickness of the coupling key is obtained from the proportions. The length of the coupling key is at least equal to the length of the sleeve (*i.e.* 3.5 *d*). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l.w.\ \tau.\frac{d}{2}$$ ... (Considering shearing of the key)

$$T = l.\frac{t}{2} * \sigma_c$$ ... (Considering crushing of the key)

Example1. Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Clutches

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft.

The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

Types of Clutches

Following are the two main types of clutches commonly used in engineering practice:

1. Positive clutches, and **2.** Friction clutches

Positive Clutches

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a ***jaw*** or ***claw clutch*****.** The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. It consists of two halves, one of which **is permanently fastened** to the driving shaft by a sunk key. The other half of the clutch is **movable and it is free to slide axially** on the driven shaft, but it is prevented from turning relatively to its shaft by means of feather key.

Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces.

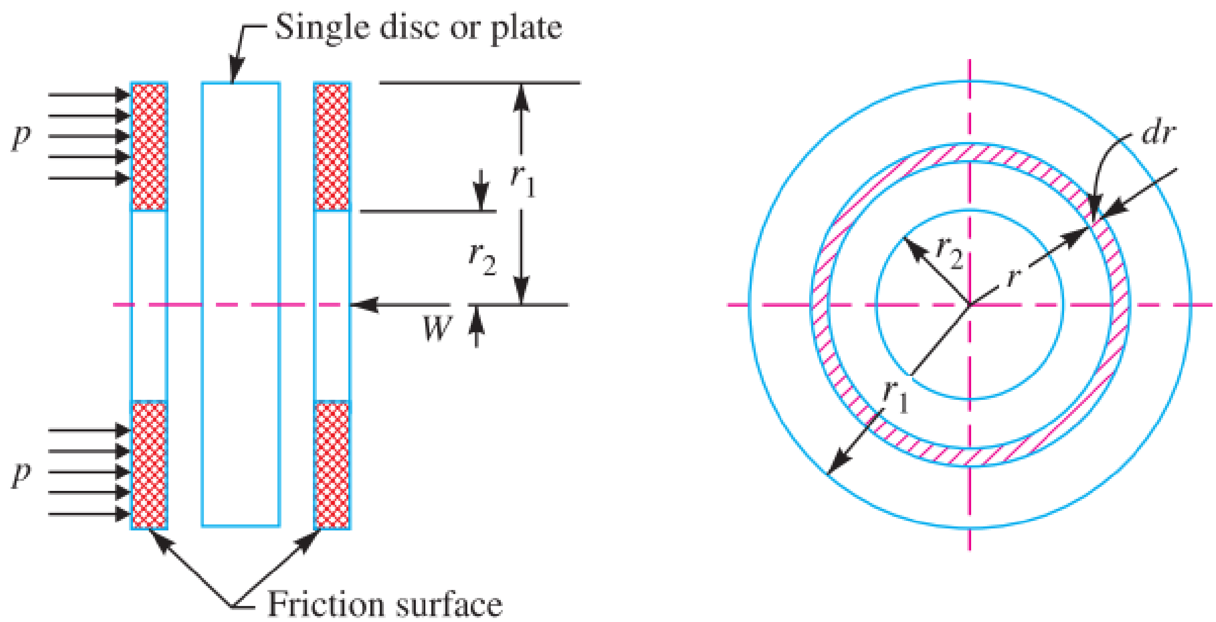
Types of Friction Clutches

Though there are many types of friction clutches, yet the following are important from the subject point of view:

1. Disc or plate clutches (single disc or multiple disc clutch),
2. Cone clutches, and
3. Centrifugal clutches.

Design of Single Disc or Plate Clutch

Consider two friction surfaces maintained in contact by an axial thrust (*W*) as shown in the figure below,



Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces,

r = Mean radius of the friction face, and

μ = Coefficient of friction.

Consider an elementary ring of radius *r* and thickness *dr* as shown in Fig. (*b*).
We know that area of the contact surface or friction surface

$$= 2\pi r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi r.dr$$

And the frictional force on the ring acting tangentially at radius *r*,

$$f_r = \mu.\mathrm{d}W = \mu.p \times 2\pi r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = f_r \times r = \mu.p \times 2\pi r.dr \times r = 2\mu\pi p.r^2.dr$$

We shall now consider the following two cases:
1. when there is a uniform pressure, and
2. when there is a uniform axial wear.

1. ***Considering uniform pressure.*** When the pressure is uniformly distributed over the entire
area of the friction face as shown in Fig. (*a*), then the intensity of pressure,

$$P = \frac{W}{\pi({r_1}^2 - {r_2}^2)}$$

Where W = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is,

$$T_r = 2\mu\pi p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total friction torque,

$$T_r = \int_{r_2}^{r_1} 2\mu\pi p.r^2.dr = 2\mu\pi P.[\frac{r^3}{3}]_{r_2}^{r_1}$$

Then substituting the value of the pressure intensity the final equation of the torque will be,

$$T_r = \frac{2}{3}\mu\pi.\frac{W}{\pi({r_1}^2 - {r_2}^2)}.[{r_1}^3 - {r_2}^3] = \frac{2}{3}\mu W\left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right] = \mu W R$$

Where $R = \frac{2}{3}\left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right]$ is mean radius of the friction surface.

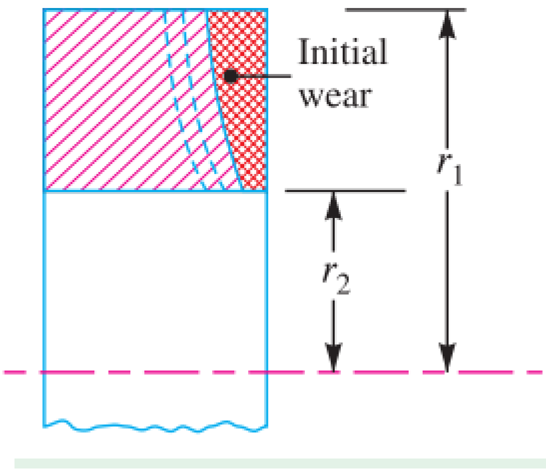
2. ***Considering uniform axial wear.*** The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (p) and the sliding velocity (V). Therefore,

Normal wear ∝ Work of friction ∝ $P.V$

Or P.V = K (a constant) or $p = K/V$

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product *p.V* is constant over the entire surface.

Let *p* be the normal intensity of pressure at a distance *r* from the axis of the clutch. Since the intensity of pressure varies inversely with the distance,



Therefore $p.r = C$ (a constant) or $p = C / r$

And the normal force on the ring,

$$dW = p.2\pi r.d = \frac{c}{r} \times 2\pi r.dr = 2\pi c.dr$$

∴ Total force acing on the friction surface,

$$w = \int_{r_2}^{r_1} 2\pi c.dr = 2\pi c.[r]_{r_2}^{r_1} = 2\pi c(r_1 - r_2)$$

Or $$c = \frac{W}{2\pi(r_{1-r_2})}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\mu\pi p.r^2.dr = 2\pi\mu.\frac{c}{r} \times r^2.dr = 2\mu\pi.C.r.dr \qquad \text{... } (p = C/r)$$

∴ Total frictional torque acting on the friction surface (or on the clutch),

$$T_r = \int_{r_2}^{r_1} 2\mu\pi.C.r.dr = 2\mu\pi.C.\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\mu\pi.C.\left[\frac{r_1{}^2 - r_2{}^2}{2}\right] = \mu\pi.\frac{W}{2\pi(r_{1-r_2})} \times [r_1{}^2 - r_2{}^2]$$

$$= \frac{1}{2} \times \mu.W.(r_1 + r_2) = \mu.W.R$$

Where $R = \frac{r_1 + r_2}{2}$ = Mean radius of the friction surface.

Notes: In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by,

$$T_r = n.\mu.W.R$$

Where n = Number of pairs of friction (or contact) surfaces.

- ✓ For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (*i.e.* $n = 2$).
- ✓ Since the intensity of pressure is maximum at the inner radius ($r2$) of the friction or contact surface, therefore equation may be written as,

  $$P_{max} \times r2 = C \text{ or } P_{max} = C / r2$$

- ✓ Since the intensity of pressure is minimum at the outer radius ($r1$) of the friction or contact surface, therefore equation may be written as

  $$P_{min} \times r1 = C \text{ or } P_{min} = C / r1$$

- ✓ The average pressure (P_{av}) on the friction or contact surface is given by,

  $$P_{av} = \frac{total\ force\ on\ frictional\ surface}{cross - sectional\ area\ on\ the\ frictional\ surface} = \frac{W}{\pi[r_1{}^2 - r_2{}^2]}$$

- ✓ In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch, the uniform wear theory is more approximate.
- ✓ The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

Example 2 A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner diameters of frictional surface if the coefficient of friction is 0.255, ratio of diameters is 1.25 and the maximum pressure is not to exceed 0.1 N/mm2. Also, determine the axial thrust to be provided by springs. Assume the theory of uniform wear.

Example 3 A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 kW at 1250 r.p.m. The outer diameter of the contact surfaces is to be 300 mm. The coefficient of friction is 0.4.

(a) Assuming a uniform pressure of 0.17 N/mm2; determine the inner diameter of the friction surfaces.
(b) Assuming the same dimensions and the same total axial thrust, determine the maximum torque that can be transmitted and the maximum intensity of pressure when uniform wear conditions have been reached.

Example 4 A single dry plate clutch is to be designed to transmit 7.5 kW at 900 r.p.m. Find:

1. Diameter of the shaft,
2. Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4,
3. Outer and inner radii of the clutch plate, and
4. Dimensions of the spring, assuming that the number of springs are 6 and spring index = 6.

The allowable shear stress for the spring wire may be taken as 420 MPa.

Chapter 8

Brakes

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. The brake absorbs either kinetic energy of the moving member. The energy absorbed by brakes is **dissipated in the form of heat**.

The design or capacity of a brake depends upon the following factors:

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,
4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

Notice: The major functional difference between a clutch and a brake is that a **clutch** is used to **keep the driving and driven member moving together**, whereas **brakes** are used to **stop** a **moving member or to control its speed.**

Types of Brakes

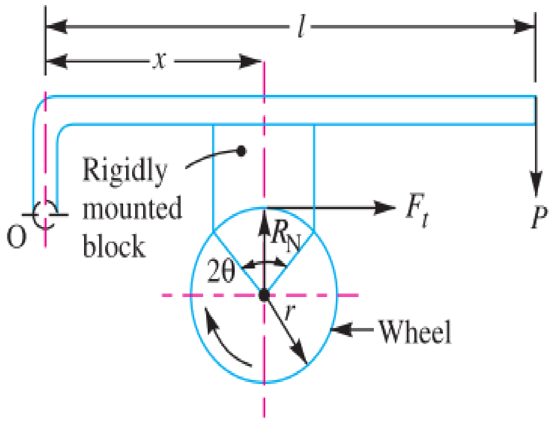
The brakes, according to the means used for transforming the energy by the braking element,are classified as :
1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,
2. Electric brakes *e.g.* generators and eddy current brakes, and
3. Mechanical brakes.

The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel. The mechanical brakes, according to the direction of acting force, may be divided into the following two groups:

(a)Radial brakes**.** In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into ***external brakes*** and ***internal brakes*****.** According to the shape of the friction element, these brakes may be block or shoe brakes and band brakes.
(*b*) ***Axial brakes*****.** In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches.

Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. below. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum.



(a) Clockwise rotation of brake wheel (b) anti-clockwise rotation of brake wheel

This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel.

Let P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ= Angle of contact surface of the block,

μ= Coefficient of friction, and

Ft = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel.

$$F_t = \mu . R_N$$

And the braking torque,

$$T_B = r \times F_t = \mu . R_N \times r$$

Let us now consider the following three cases:

Case 1. When the line of action of **tangential** braking force (*Ft*) passes through the **fulcrum** *O* of
the lever, and the brake wheel rotates clockwise as shown in Fig. (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

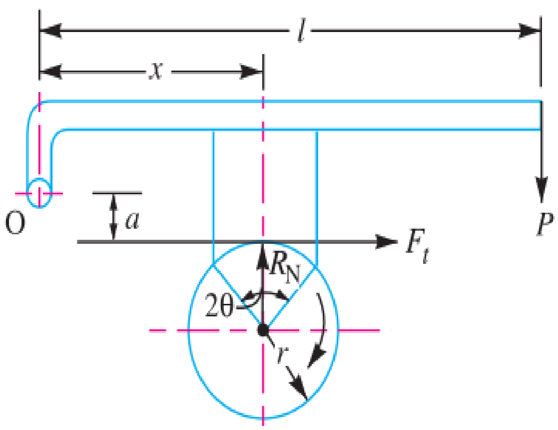
$$R_N.x = P \times l$$

$$R_N = \frac{P \times l}{x}$$

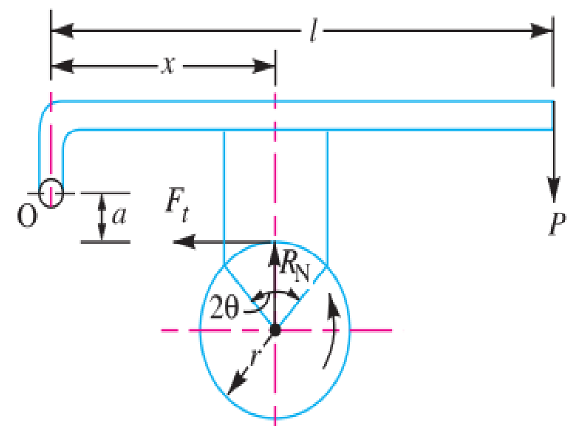
∴ Braking torque, $T_B = \mu.R_N \times r = \mu \times \frac{P \times l}{x} \times r = \frac{N.P.l.r}{x}$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig.(*b*), then the braking torque is same.

Case 2. When the line of action of the **tangential** braking force (*Ft*) passes through a **distance** '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. below (*a*), then for equilibrium, taking moments about the fulcrum *O*,



(a) Clockwise rotation of brake wheel



(b) anti-clockwise rotation of brake wheel

When the brake wheel rotates anticlockwise, as shown in Fig. (*b*), then for equilibrium,

$$R_N.x = F_t.a + P \times l = \mu.R_N.a + Pl$$

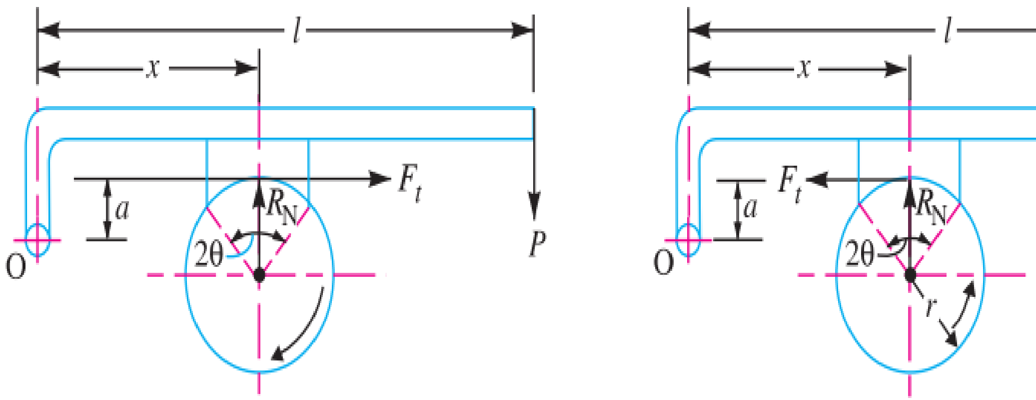
$$R_N(x - \mu a) = P.l$$

$$R_N = \frac{P.l}{(x - \mu a)}$$

And braking torque,

$$T_B = \mu . R_N \times r = \mu . \frac{P.l}{(x - \mu a)} \times r = \frac{\mu . p . l . r}{(x - \mu a)}$$

Case 3. When the line of action of the tangential braking force passes through a distance '*a*' above the fulcrum, and the brake wheel rotates clockwise as shown in Fig. below (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have



(a) Clockwise rotation of brake wheel (b) anti-clockwise rotation of brake wheel

$$R_N . x = F_t . a + P \times l = \mu . R_N . a + Pl$$
$$R_N (x - \mu a) = P . l$$
$$R_N = \frac{P.l}{(x - \mu a)}$$

When the brake wheel rotates anticlockwise as shown in Fig. above (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N . x + F_t . a = P \times l$$
$$R_N . x + \mu . R_N . a = Pl$$
$$R_N (x + \mu a) = P . l$$
$$R_N = \frac{P.l}{(x + \mu a)}$$