

Chapter 2 Concrete Dam

2.1 Gravity Dam

Basically, gravity dams are solid concrete structures that maintain their stability against design loads from the geometric shape and the mass and strength of the concrete. Generally, they are constructed on a straight axis, but may be slightly curved or angled to accommodate the specific site conditions. Gravity dams typically consist of a non-overflow section(s) and an overflow section or spillway.

2.1.1 Forces Acting & Load combination on dams

Loads can be classified in terms of applicability or relative importance as primary loads, secondary loads, & Exceptional loads.

- i) Primary loads: are identified as those of major importance to all dams irrespective of type. Example self weight, water & related seepage loads.
- ii) Secondary loads: are universally applicable although of lesser magnitude (e.g. Silt load) or alternatively are of major importance only to certain types of dam (e.g. thermal effects with in concrete dams).
- iii) Exceptional loads: are so designed on the basis of limited general applicability of occurrence (e.g. tectonic effects, or the inertia loads associated with seismic activity)

Gravity dam Loads

a) Primary Loads

i. Water Load

Hydrostatic distribution of pressure with horizontal resultant force P_1 (Note also a vertical component exists in the case of an u/s batter, and equivalent tail water may operate in the d/s face).

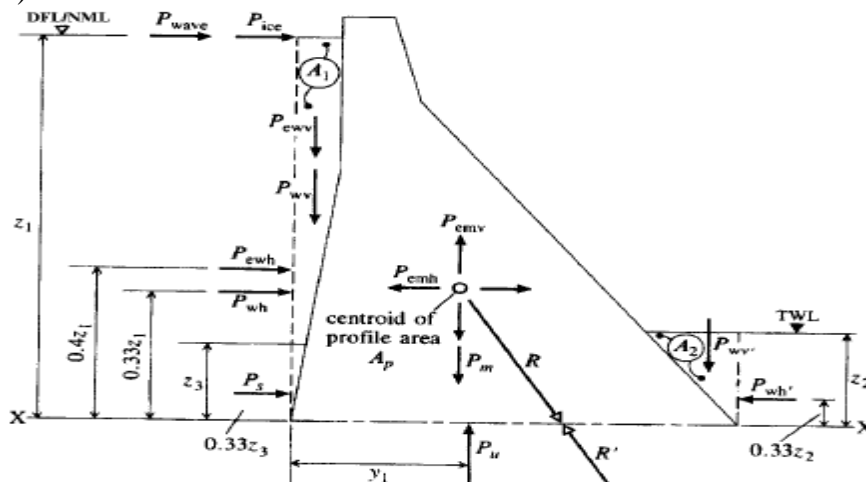


Figure 3.1 Gravity dam loading diagram. DFL_design flood level; NML_normal maximum level, i.e. maximum retention level of spillweir; TWL_tailwater level

$$P_{wh} = \tau_w \frac{Z_1^2}{2} \text{ KN / m} \quad \text{acting at } \frac{Z_1}{3}$$

Where γ_w unit weight of water = 9.81 KN/m³

$$P_{wv} = \gamma_w (\text{area } A_1) \quad \text{KN/ m}$$

Acting through centroid of A_1

Pressure of any permanent tail water above the plane considered is :

$$P_{wn'} = \frac{\gamma_w Z_2^2}{2}$$

with $P_{wv'} = \gamma_w (\text{area } A_2)$

ii. Self weight load:

Determined w.r.t an appropriate unit weight of the material

$$P_m = \gamma_c A_p \quad \text{KN/m}$$

acts through the centroid of x- sectional area A_p .
($\gamma_c \approx 23.5 \text{ KN/m}^3$)

Where crest gates & other ancillary structures of considerable weight exist they must also be considered in determining P_m & their appropriate position of line of action.

iii. Seepage & uplift load:

Equilibrium seepage patterns will establish within & under a dam eg. with resultant forces identified as P_3 & P_4 .

$$P_u = \eta A_h (U_w)_{\text{avg}}$$

$$= \eta A_h \gamma_w \left(\frac{Z_1 + Z_2}{2} \right) \quad \text{if no drain functioning.}$$

η is area reduction factor

A_h nominal plane area at a section considered.

If nod drains functioning

$$P_u \text{ acts at } Y_1 = \frac{T}{3} \frac{(2Z_2 + Z_1)}{Z_2 + Z_1} \text{ m}$$

In modern dams internal uplift is controlled by the provision of vertical relief drains close behind the u/s face. Mean effective head @ the line of drains, Z_d can be expressed as

$$Z_d = Z_2 + K_d(Z_1 - Z_2) \text{ m}$$

K_d is function of drain geometry (i.e. diameter, special & relative location with u/s face.)

$$K_d = 0.33 \quad (\text{USBR})$$

$$K_d = 0.25 \quad \text{Tennase valley Authority}$$

$K_d = 0.25-0.5$ appropriate to the site by the U.S crops of Eng's

The standard provision of deep grout curtain below the u/s face intended to limit seepage also serves to inhibit pressure within the foundation. However, less certain than efficient draw system & its effect is commonly disregarded in uplift reduction.

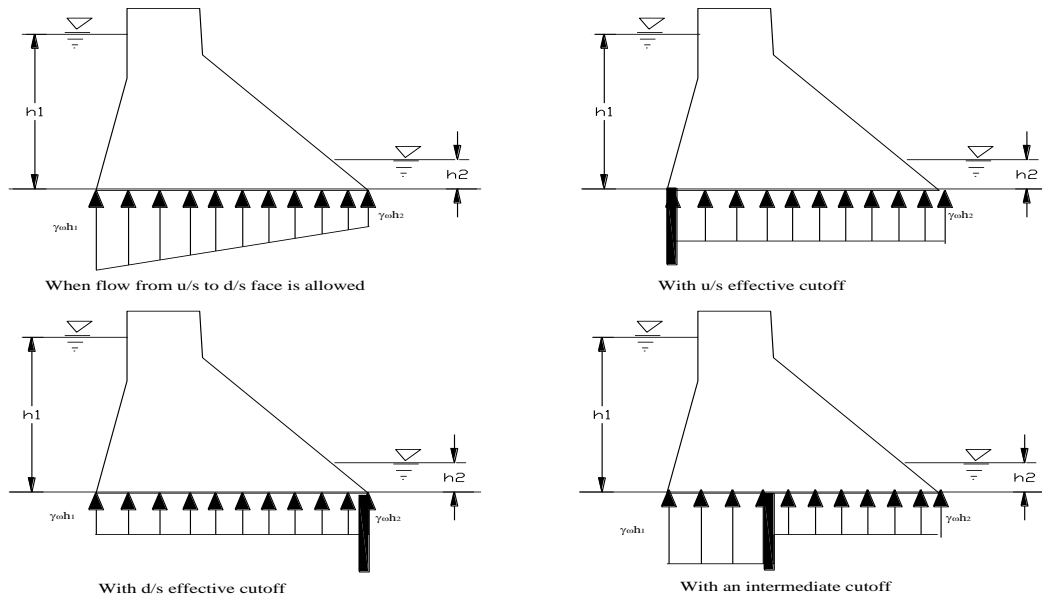


Figure 3.2 uplift pressure distribution with cut off

b. Secondary loads

i. Sediment load:

Accumulated silt etc, generates a horizontal thrust, P_s . The magnitude additional to P_{wh} is a function of sediment depth, Z_3 , submerged unit weight γ_s & active lateral pressure coefficient. K_a :

$$P_s = K_a \frac{\gamma_s^1 Z_3^2}{2} \quad \& \quad \text{acting @ } Z_3/3 \text{ above plane}$$

$$\gamma_s^1 = \gamma_s - \gamma_w \quad \text{where } \gamma_s \text{ is sediment saturated unit weight.}$$

$$K_a = \frac{1 - \sin \phi_s}{1 + \sin \phi_s} \quad \text{where } \phi_s \text{ is angle of shearing resistance.}$$

For representative values of $\gamma_s \approx 18-20 \text{KN/m}^3$
 $\phi_s \approx 30^\circ$

$$P_s \approx \frac{3 Z_3^2}{2}$$

iii. Hydrodynamic wave Load

Transient load, P_{wave} , generated by wave action against the dam. It is not normally significant & depends on the fetch & wind velocity.



$$P_{\text{wave}} = 2\gamma_w H_s^2$$

Where H_s - significant wave height (is the mean height of the highest third of the wave in train)

H_s range from $0.75 H_s$ for concrete dams to $1.3H_s$ for earth dams.

$$H = 0.32\sqrt{UF} + 0.76 - 0.24\sqrt[4]{F}$$

U= in km/hr

F= in km

- iii) **Wind load:** when the dam is full, wind acts only on the d/s side thus contribute to stability. When empty the wind can act on the u/s face but in significant compared to hydrostatic load. For buttress dams load on the exposed surface has to be considered.
- iv) **Ice load:** Not a problem in Ethiopia. It can be significant where ice sheets form to appreciable thickness & persist for lengthy periods.
 $P_{\text{ice}} = 145 \text{ KN/m}^2$ for ice > 0.6m thick, other wise neglected
- v) **Thermal & dam /foundation interaction effect:** Cooling of large pours of mass concrete following the exothermic hydration of cement & the subsequent variation in ambient & water temperatures combine to produce complex & time dependent temp. Gradients within the dam equally. Complex interaction develops as a result of foundation deformation.

C. Exceptional Loads

Seismic load: Horizontal & vertical inertia loads, are generated with respect to the dam & the retained water by seismic disturbance. Horizontal & vertical accelerations are not equal, the former being of greater in density. For design purposes both should be considered operative in the sense last favorable to stability of the dam, under reservoir full conditions the most adverse seismic loading will then occur when the ground shock is associated with.

- 1) Horizontal foundation acceleration operating u/s, and

- 2) Vertical foundation acceleration operating downwards and vice-verse for reservoir empty condition

Seismic coefficient analysis

Seismic acceleration coefficient. α_h for horizontal
 $\alpha_v = 0.5\alpha_h$ for vertical

Representative seismic coefficient applied in design

Coff. α_h	Modified mercali scale	General damage level	U.S seismic zone
0.0	-	Nil	0
0.25	VI	Minor	1
0.10	VII	Moderate	2
0.15	VIII-IX	Major	3
0.20		great	4

For more extreme circumstances eg. $\alpha_h=0.4$ has been employed for dams in high risk region in Japan, $\alpha_h = 0.5$ & $\alpha_h = 0.6-0.8$ damaged Koyna gravity dam, India (1967) & Pacima arch dam USA (1971) respectively.

Inertia forces: ;Mass of dam

$$\text{Horizontal } P_{emh} = \pm \alpha_h P_m$$

$$\text{Vertical } P_{emv} = \pm \alpha_v P_m \quad \left. \vphantom{P_{emv}} \right\} \text{operating through centroid of the dam}$$

Hydrodynamic forces: water action

Relative to any elevation @ depth Z_1 below the water surface, the pressure p_{ewh}

$$p_{ewh} = C_e \alpha_h \gamma_w Z_1 \quad \text{KN/m}$$

Z_1 = Max. Water depth

Z = the depth @ section considered

C_e = dimensionless pressure factor

= $f(Z/Z_1, \phi_u)$ where ϕ_u -inclination of u/s face to vertical

Total hydrodynamic load is given by.

$$P_{ewh} = 0.66 C_e \alpha_h Z_1 \gamma_w \sqrt{Z_1 Z_{max.}} \quad \& \text{ acts @ } 0.4 Z \text{ above section}$$

Ratio z/z_1	pressure factor C_e .	
	$\phi_u = 0^\circ$	$\phi_u = 150^\circ$
0.2	0.35	0.29
0.4	0.53	0.45
0.6	0.64	0.55
0.8	0.71	0.61
1.0	0.73	0.63

The vertical hydrodynamic load, P_{ewv} , is

$$P_{ewv} = \pm \alpha_v P_{wv}$$

Uplift load is assumed unaltered.

Resonance: results when period vibrations of the structure & earth quake period are equal. For a concrete gravity dam of triangular X- section base thickness T

$$F_n \approx \frac{600T}{h^3} \text{HZ} \text{ or } \approx \frac{\sqrt{E_{eff}}}{0.012h} \text{HZ} \quad (E_{eff} = 14GN/m^2)$$

As an example, the natural frequency of vibration of monolithic gravity profiles with nominal height of 20m & 50m are 15-25 & 6-9 HZ respectively (if major seismic shock frequency of 1-10 HZ). Thus it is only of concern for large dams & vulnerable portion of the dam.

Load combination for Design

The design of a gravity dam is based on the most adverse combination of the loads/forces acting on it, which includes only those loads having a reasonable probability of simultaneous occurrence. The combination of transient loads such as those due to maximum flood and earthquake are not considered because the probability of occurrence of each of these phenomena is quite low and hence the probability of their simultaneous occurrence is almost negligible. Thus for the design of gravity dams according to Indian Standard is specified as the following load combination:

I. *Load combination A (construction condition or empty reservoir condition):* Dam completed but no water in the reservoir and no tail water.

II. *Load combination B (Normal operating condition):* Full reservoir elevation (or top of gates at crest), normal dry weather tail water, normal uplift, ice and uplift (if applicable)

III. *Load combination C (Flood Discharge condition):* Reservoir at maximum flood pool elevation, all gates open, tail water at flood elevation, normal uplift, and silt (if applicable)

IV. *Load combination D -* Combination A, with earthquake.

V. *Load combination E -* Combination A, with earthquake but no ice

VI. *Load Combination F -* Combination C, but with extreme uplift (drain inoperative)

VII. *Load Combination G -* Combination E, but with extreme uplift (drain inoperative)

2.2 GRAVITY DAM DESIGN AND ANALYSIS

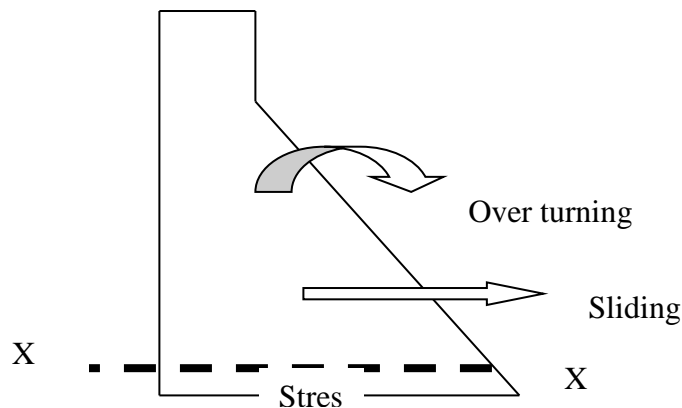
Criteria & Principles

The conditions essential to structural equilibrium & so to stability can be summarized as

$$\sum H = \sum V = 0 \quad \& \quad \sum M = 0$$

Assessed in relation to all probable conditions of loading, including reservoir empty conditions the profile must have sufficient safety factor w.r.t:

- a) Rotation & overturning.
- b) Translation & sliding and
- c) Overstress & material failure.

**a) Overturning stability**

Factor of safety against overturning, F_o , in terms of moment about the d/s toe of the dam

$$F_o = \frac{\sum M_{+ve}}{\sum M_{-ve}} \quad \sum M_{-ve} \text{ inclusive of moment generated by uplift}$$

$F_o > 1.25$ may be acceptable, but $F_o > 1.5$ is desirable.

b) sliding stability

Factor of safety against sliding, F_s , estimated using one of the three definitions:

- 1) Sliding factor, F_{SS} ;
- 2) Shear friction factor, F_{SF} or
- 3) Limit equilibrium factor, F_{LE} .

The resistance to sliding or shearing which can be mobilized across a plane is expressed through parameters C & $\tan\phi$.

- 1) *sliding factor*, F_{ss}

$$F_{SS} = \frac{u \sum V}{\sum H}$$

If the foundation plane inclined @ small angle α°

$$F_{ss} = \frac{\frac{\sum H}{\sum V} - \tan \alpha}{1 + \left(\frac{\sum H}{\sum V}\right) \tan \alpha.}$$

F_{ss} should not permitted to exceed 0.75, but under ELC up to 0.9 is acceptable.

2) **Shear friction factor, F_{sf} .** : is the ratio of total resistance to shear & sliding which can be mobilized a plane to the total horizontal load.

$$s = \frac{cA_h}{\cos \alpha (1 - \tan \phi \tan \alpha)} + \sum V \tan(\phi + \alpha) \text{ KN / m.}$$

for horizontal plane ($\alpha = 0$)

$$s = cA_h + \sum V \tan \phi.$$

$$F_{sf} = \frac{s}{\sum H} =$$

$$\therefore F_{sf} = \frac{cS_h + \sum V \tan \phi}{\sum H.}$$

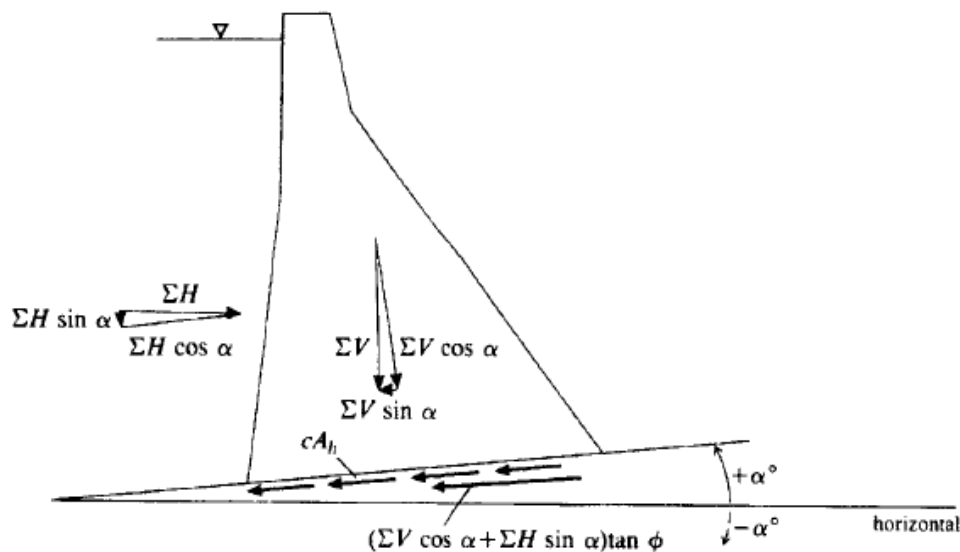


Figure: Sliding and shearing resistance: shear friction factor

In some cases it may be appropriate to include d/s passive wedge resistance, p_p , as a further component of the resistance to sliding which can be mobilized.

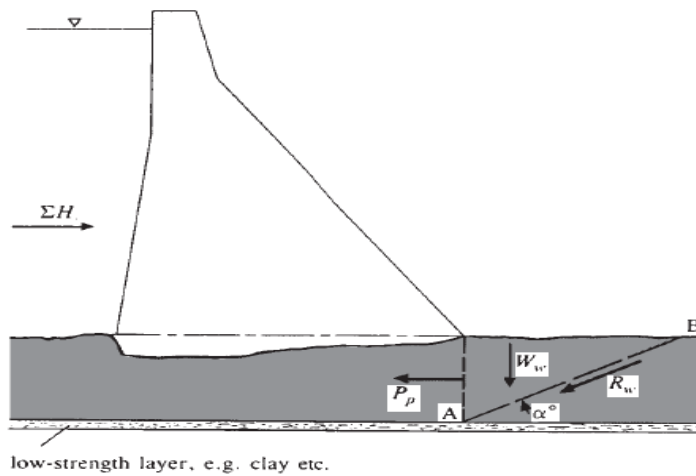


Figure: Sliding: weak seams and passive wedge resistance

W_w = weight of passive wedge.
 R_w = sliding resistance in inclined plane.
 $= CA_{AB} + (W_w \cos\alpha + \Sigma H \sin\alpha) \tan\phi$

This is affected by modifying the above equation, hence,

$$F_{sf} = \frac{s + p_e}{\sum H} \quad \text{Where} \quad p_p = \frac{CA_{as}}{\cos\alpha(1 - \tan\phi \tan\alpha)} + W_w \tan(\phi + \alpha)$$

In the presence of horizon with low shear resistance it may be advisable to make $S=0$.

Recommended shear friction factor, F_{SF} (USBR 1987)

Location of sliding plane	Load combination		
	Normal	Unusual	Extreme
Dam concrete, base interface	3.0	2.0	>1.0
Foundation rock	4.0	2.7	1.3

C. Limit equilibrium factor, F_{LE} .

This follows conventional soil mechanics logic in defining F_{LE} , as the ratio of shear strength to mean applied stress across a plane i.e

$$F_{LE} = \frac{\tau_f}{\tau}$$

τ_f is expressed by Mohr columb failure criteria, accordingly

$$F_{LE} = \frac{c + \sigma_n \tan\phi}{\sigma} \quad \sigma_n \text{ is stress acting normal to plane of sliding}$$

Referring the above figure, for single plane sliding mode.

$$F_{LE} = \frac{CA_h + \left[\sum V \cos \alpha + \sum H \sin \alpha \right] \tan \phi}{\sum H \cos \alpha - \sum V \sin \alpha}$$

Note for $\alpha = 0$ $F_{LE} = F_{SF}$.

This equation can be developed for complicated failure plane

- ❖ $F_{LE} = 2.0$ normal operation & $F_{LE} = 1.3$ under transmit condition embracing seismic activity)

C. Stress analysis in gravity method

Gravity method is useful to analyses stress in straight dams which are not geometrically complex. It is founded on 2-D elastic dam on uniformly rigid foundation & linear variation of stress from u/s to d/s .

The stresses evaluated in a comprehensive analysis are:

- 1) Vertical normal stress, σ_z , on horizontal planes.
- 2) Horizontal & vertical shear stress, τ_{zy} , & τ_{yz}
- 3) Horizontal normal stress, σ_y , on vertical planes and
- 4) Principal stress, σ_1 & σ_3 (direction & magnitude).

1. Vertical normal stress σ_z .

Analysis is based on modified beam theory which is by combining axial & bending load.

$$\sigma_z = \frac{\sum V}{A_h} \pm \frac{\sum M^* y^1}{I}$$

where, $\sum V$ - resultant vertical load above the plane considered exclusive of uplift.

$\sum M^*$ - summation of moments expressed w.r.t the centroid of the plane.

y^1 - distance from the centroid to point of considerations

I - second moment of area of the plane w.r.t centroid.

For 2-D plane section of unit width Parallel to the dam axis, & with thickness T normal to the axis:

$$\sigma_z = \frac{\sum V}{B} \pm 12 \frac{\sum V e y^1}{B^3} \quad \text{and at } y^1 = B/2$$

$$\sigma_z = \frac{\Sigma v}{B} \left(1 \pm \frac{6e}{B} \right)$$

For reservoir full condition resultant is near the toe, s

- At the u/s face (heel) $\sigma_{zu} = \frac{\Sigma v}{B} \left[1 - \frac{6e}{B} \right]$
- At the d/s face (toe) $\sigma_{zd} = \frac{\Sigma v}{B} \left[1 + \frac{6e}{B} \right]$

For reservoir empty condition, resultant is near the heel, so compressive stress produced at heel.

- At the u/s face (heel) $\sigma_{zu} = \frac{\Sigma v}{B} \left[1 + \frac{6e}{B} \right]$
- At the d/s face (toe) $\sigma_{zd} = \frac{\Sigma v}{B} \left[1 - \frac{6e}{B} \right]$

Where e is the eccentricity of the resultant load, R, which must intersect the plane d/s of its centroid for the reservoir full condition

$$e = \frac{\Sigma M^*}{\Sigma V} \quad \text{Where } \Sigma v \text{ - excludes uplift}$$

For $e > T/6$, at u/s face -ve stress is developed, i.e. tensile stress. In design, tensile stress has to be prohibited, but difficult to totally eliminate low tensile stress in gravity dam. Total vertical stresses at either face are obtained by the addition of external hydrostatic pressure.

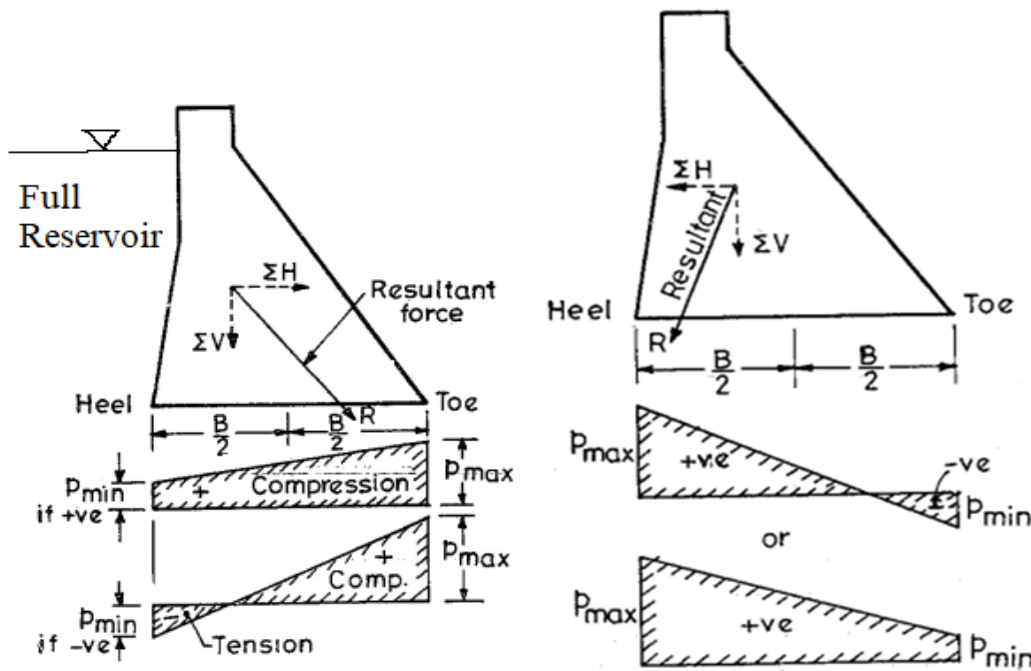
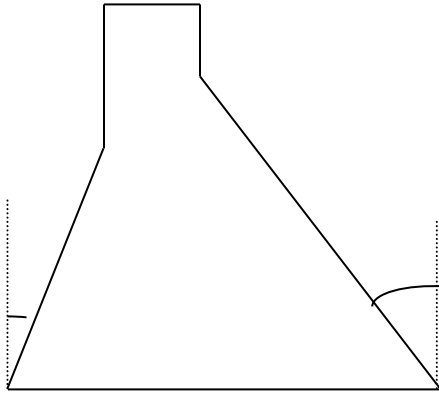


Figure: stress distribution in both reservoir full and empty condition

2. Horizontal & vertical shear stresses

Numerically equal & complementary horizontal (τ_{zy}) & vertical (τ_{yz}) shear stresses are generated @ any point as a result of variation of vertical normal stress over a horizontal plane.



For u/s & d/s face angle ϕ_u & ϕ_d respectively & P_w hydrostatic pressure @ u/s end

$$\tau_u = (P_w - \sigma_{zu}) \tan \phi_u$$

&

$$\tau_d = \sigma_{zd} \tan \phi_d$$

The variation b/n u/s & d/s stress is parabolic, & depend on rate of change of variation of normal stress

3. Horizontal normal stress, σ_y

It can be determine by consideration of the equilibrium of the horizontal shear force operating above & below a hypothetical horizontal element through the dam. The difference in shear forces is balanced by the normal stresses on vertical planes.

$$\sigma_{yu} = P_w + (\sigma_{zu} - P_w) \tan^2 \phi_u$$

$$\sigma_{yd} = \sigma_{zd} \tan^2 \phi_d$$

4. Principal stresses

σ_1 & σ_3 may be determined from knowledge of σ_z & σ_y & construction of Mohr's circle diagram to represent stress conditions at a point, or by application of the equation given below.

$$\text{Major Principal Stress } \sigma_1 = \frac{\sigma_z + \sigma_y}{2} + \tau_{\max}$$

$$\text{Minor principal stress } \sigma_3 = \frac{\sigma_z + \sigma_y}{2} - \tau_{\max}$$

$$\text{Where } \tau_{\max} = \sqrt{\frac{\sigma_z - \sigma_y}{2} + \tau^2}$$

The boundary values, σ_1 & σ_3 are determined by:

For upstream face

$$\left\{ \begin{array}{l} \sigma_{1u} = \sigma_{zu} (1 + \tan^2 \phi_u) - P_w \tan^2 \phi_u \\ \sigma_{3u} = P_w \end{array} \right.$$

For downstream face assuming no tail water

$$\begin{cases} \sigma_{1d} = \sigma_{zd} (1 + \tan 2\phi_d) \\ \sigma_{3d} = 0 \end{cases}$$

Permissible stresses & cracking

The following table gives permissible compression stresses factor of safety for gravity dam body & rock foundations. (USBR 1976)

load combination	Minimum factor of safety on compressive strength	
	F_c (concrete)	F_r , (rock)
Normal	3.0 (σ_{\max} & 10 MN/m ²)	4.0
Unusual	2.0 (σ_{\max} & 15 MN/m ²)	2.7
Extreme	1.0 max <u>m</u> allowable stress	1.3

Horizontal cracking assumed to occur if $\sigma_{zu \min}$ (without uplift) below limit set by

$$\sigma_{zu \min} = k_d \frac{\gamma_w z - \sigma_t'}{F_t'}$$

$$K_d = 0.4 \text{ if drains are effective} \\ = 1.0 \text{ if no drains.}$$

σ_t' = tensile bond strength of concrete.

F_t' = Factor of Safety [$F_t' = 3$ for NLC,
= 2 for ULC, &
= 1.0 for ELC]

5 Rules Governing the Design of Gravity Dams

The following are basic assumptions that should be considered relative to the design of important masonry/concrete dams.

1. The rock that constitutes the foundation and abutments at the site is strong enough to carry the forces imposed by the dam with stresses well below the elastic limit at all places along the contact planes.
2. The bearing power of the geologic structure along the foundation and abutments is great enough to carry the total loads imposed by the dam without rock movements of detrimental magnitude.
3. The rock formations are homogeneous and uniformly elastic in all directions, so that their deformations may be predicted satisfactorily by calculations based on the theory of elasticity, by laboratory measurements on models constructed of elastic materials, or by combinations of both methods.
4. The flow of the foundation rock under the sustained loads that result from the construction of the dam and the filling of the reservoir may be adequately allowed for by using a somewhat lower modulus of elasticity than would otherwise be adopted for use in the technical analyses.
5. The base of the dam is thoroughly keyed into the rock formations along the foundations and abutments.

6. Construction operations are conducted so as to secure a satisfactory bond between the concrete and rock materials at all areas of contact along the foundation and abutments.
7. The concrete in the dam is homogeneous in all parts of the structure.
8. The concrete is uniformly elastic in all parts of the structure, so that deformations due to applied loads may be calculated by formulae derived on the basis of the theory of elasticity or may be estimated from laboratory measurements on models constructed of elastic materials.
9. Effects of flow of concrete may be adequately allowed for by using a somewhat lower modulus of elasticity under sustained loads than would otherwise be adopted for use in technical analyses.
10. Contraction joints are properly grouted under adequate pressures, or open slots are properly filled with concrete, so that the dam may be considered to act as a monolith.
11. Sufficient drains are installed in the dam to reduce such uplift pressures as may develop along areas of contact between the concrete and rock materials.
12. Effects of increases in horizontal pressures caused by silt contents of flood waters usually may be ignored in designing high storage dams, but may require consideration in designing relatively low diversion structures.
13. Uplift forces adequate for analyzing conditions at the base of the dam are adequate for analyzing conditions at horizontal concrete cross sections above the base.
14. Internal stresses caused by natural shrinkage and by artificial cooling operations may be adequately controlled by proper spacing of contraction joints.
15. Internal stresses caused by increases in concrete temperature after grouting are beneficial.
16. Maximum pressures used in contraction joint grouting operations should be limited to such values as may be shown to be safe by appropriate stress analyses.
17. No section of the Ethiopia may be assumed to be entirely free from the occurrence of earthquake shocks.
18. Assumptions of maximum earthquake accelerations equal to one tenth of gravity are adequate for the design of important masonry dams without including additional allowances for resonance effects.
19. Vertical as well as horizontal accelerations should be considered, especially in designing gravity dams.
20. During the occurrence of temporary abnormal loads, such as those produced by earthquake shocks, some increases in stress magnitudes and some encroachments on usual factors of safety are permissible.
21. Effects of foundation and abutment deformations should be included in the technical analyses.

22. In monolithic straight gravity dams, some proportions of the loads may be carried by twist action and beam action at locations along the sloping abutments, as well as by the more usually considered gravity action.

23. Detrimental effects of twist and beam action in straight gravity dams, such as cracking caused by the development of tension stresses, may be prevented by suitable construction procedure.

24. In monolithic curved gravity and arch dams, some proportions of the loads may be carried by tangential shear and twist effects, as well as by the more usually considered arch and cantilever actions.

25. The distribution of loads in masonry dams may be determined by bringing the calculated deflections of the different systems of load transference into agreement at all conjugate points in the structure.

The aforementioned assumptions are rephrased as rule/guideline for design of concrete gravity dam as described below:

Rule1: Location of the resultant: No tension in any joint of the dam under all loading conditions (i.e. for full and empty reservoir). Thus, resultant of all forces (including uplift) must intersect the joint within the middle third.

Rule2a: Resistance to sliding when shear is neglected: the tangent of the angle between the vertical and the resultant (including uplift) above horizontal plane shall be less than the allowable coefficient of frictional force „ f “. If empirical values are taken, factor of safety, $S_f = 2$.

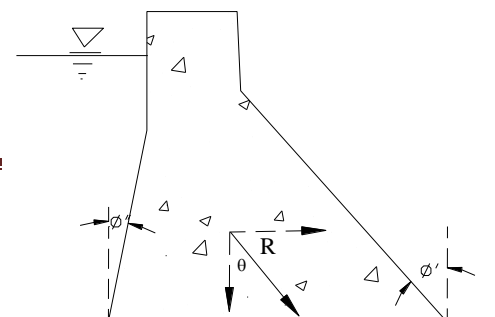
Table 2 Some values of Coefficient of friction f

Surface	f
Masonry on masonry or masonry on good rock or concrete on concrete	0.75
Concrete or masonry on gravel	0.50
Concrete or masonry on sand	0.40
Concrete or masonry on clay	0.30

Rule 2b: Resistance to sliding when shear is considered

The total friction resistance to sliding on any joint plus the ultimate shearing strength of the joint must exceed

Hydraulic structure I



the total horizontal force above the joint by a safe margin, i.e.

$$\Sigma P \leq \frac{f\Sigma W + r.S_n.A}{S_{sf}}$$

Where: S_n – ultimate shearing strength of material

S_{sf} = shear friction factor of safety

A = cross sectional area of joints

r = ratio of average to maximum shearing strength

Recommended values $S_{sf} = 5$, $r = 0.5$

$$rS_{sf} = 200 \text{ to } 500 \text{t/m}^2$$

While analyzing resistance to sliding, first compute $\tan\phi$ and if $\tan\phi > f$ apply Rule 2b. In that case, S_{sf} should equal or exceed the allowable value.

Rule 3: Governing compressive stresses: P'_v , or P''_v (maximum vertical stresses) are not the maximum stresses in the structure. The maximum stresses occur at the end joints, or inclined planes, normal to the face of the dam.

Maximum stress for downstream face, reservoir full:

$$P'_i = P'_v(1 + \tan^2 \phi')$$

Maximum stress for upstream face, reservoir full

$$P''_i = P''_v(1 + \tan^2 \phi'')$$

The inclined compressive stresses in the dam and foundation shall not exceed the allowable values.

Ultimate stress, $\sigma'_c = 14 \text{ to } 31 \text{ MPa}$ (after 28 days curing)

Working stress $\sigma_c = \sigma'_c/6$

For foundation materials some indications for allowable stress are:

Limestone -----200 to 350 t/m³

Granite -----250 to 300t/m³

Rule 4: Governing internal tension: The dam shall be designed and constructed in such a manner as to avoid or adequately provide for tension on interior planes, inclined, vertical or horizontal.

Rule 5: Margin of safety: all assumptions of forces acting on the dam shall be unquestionably on the safe side, all unit stresses adopted in design should provide an ample margin of safety against rupture and the shear-factors shall be considered.

Rule 6: Detail of design and methods of construction: all details shall support and confirm to the assumptions used in design; masonry should be of quality suited to the stresses adapted, protection against overflowing water shall be ample.

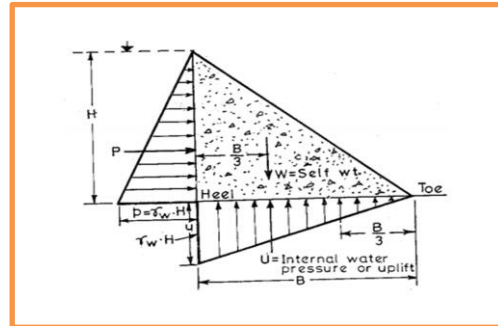
Theoretical (Elementary profile) of a dam

Considering only the three major forces acting on the dam, i.e. the weight of the dam, uplift force and the hydrostatic water pressure, the required section of the dam for its stability will be a triangle of base width,

$$B = \frac{H}{\sqrt{s}}$$

Where: H = depth of water

s = specific gravity of concrete



When the reservoir is empty

The only single force acting on it is the self- weight of the dam and it acts at the distance of B/3 from the heel. This is the maximum possible inner most position of the resultant for no tension to develop. The vertical stress distribution at the base, when the reservoir is empty

- $\sigma_{max} = \frac{\sum F_V}{B} \left(1 + \frac{6e}{B} \right)$, $\sum F_V = W$, Resultant (R) acts at B/3 from the heel, so $e = B/6$.
- $\sigma_{max} = \frac{\sum F_V}{B} \left(1 + \frac{6e}{B} \right) = \frac{W}{B} \left(1 + \frac{6(B/6)}{B} \right) \rightarrow \sigma_{max} = 2W/B$
- $\sigma_{min} = 0$
- i.e. elementary profile of gravity dam is subjected to maximum stress of $2W/B$ and this σ_{max} should not exceed the allowable stress (3000 KN/M^2).

Reservoir full condition

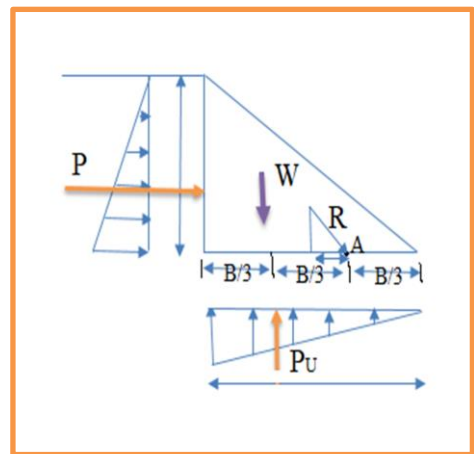
The resultant of all forces (P,W and U) pass through the outer most middle third point(lower middle third point). To prevent tension development the resultant should be within the middle third, but for the most economic section the resultant force shall lie at B/6 from the center of the section, i.e. the outer middle third point (point A in fig below).

- Taking moments of all forces at A,
- $$W(B/3) - P(H/3) - Pu(B/3) = 0$$
- ϕ = uplift intensity factor
 - S = specific gravity of concrete

$$\frac{1}{2} * \gamma_w * S * B * H(B/3) - \frac{1}{2} * \gamma_w H^2 (H/3) - \frac{1}{2} \gamma_w \phi * B * H(B/3) = 0$$

$$B^2(S - H) = H^2$$

$$B = \frac{H}{\sqrt{S-\phi}} \dots\dots\dots (a)$$



If $B \geq \frac{H}{\sqrt{S-\phi}}$, no tension developed at heel when reservoir is full

For no sliding, the force resisting sliding must be greater than the force causing sliding and in limiting case these forces must be equal.

$$F_s = \frac{\mu \sum V}{\sum H}$$

- In limiting case, $\mu \sum V = \sum H$

$$\mu(W - P_U) = P$$

$$\mu(\frac{1}{2} * \gamma_w * S * B * H - \frac{1}{2} \gamma_w * \phi * B * H) = \frac{1}{2} * \gamma_w H^2$$

$$\mu B(S - \phi) = H$$

$$B = \frac{H}{\mu(S - \phi)} \dots\dots\dots (b)$$

The minimum base width to be provided for elementary profile of gravity dam should be greater of base width given in equation (a) and (b).

Vertical stress distribution in reservoir full case

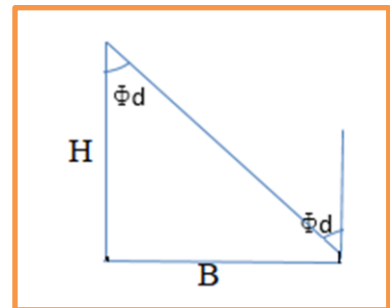
Limiting height of elementary profile the normal stress at toe for reservoir full condition is

$$\sigma_{max} = \frac{\sum F_V}{B} \left(1 + \frac{6e}{B} \right). \quad \sum F_V = W - P_U = \frac{1}{2} * \gamma_w * B * H(S - \phi)$$

- $\sigma_{max} = \frac{\frac{1}{2} * \gamma_w * B * H(S - \phi)}{B} * \left(1 + \frac{6e}{B} \right)$
- In limiting case of no tension at any point in the base of the dam, $e = B/6$ and hence;
- $\sigma_{max} = \gamma_w H(S - \phi) \dots\dots\dots$ at toe
- $\sigma_{min} = 0 \dots\dots\dots$ at heel

The principal stress at the toe of the dam is,

- $(s_1)_{toe} = s_{toe} \sec^2 F_d$ (for no tail water)
- $(s_1)_{toe} = s_{toe} (1 + \tan^2 F_d) = \gamma_w H(S - \phi) (1 + \tan^2 F_d)$
- $\tan^2 F_d = \frac{B^2}{H^2}$
- $(s_1)_{toe} = \gamma_w H(S - \phi) \left(1 + \frac{B^2}{H^2} \right)$
- But, for no tension --- $B = \frac{H}{\sqrt{S - \phi}} \rightarrow \frac{B}{H} = \frac{1}{\sqrt{S - \phi}} \rightarrow \frac{B^2}{H^2} = \frac{1}{S - \phi}$
- $(s_1)_{toe} = \gamma_w H(S - \phi) \left(1 + \frac{1}{S - \phi} \right)$
- $(s_1)_{toe} = \gamma_w H(S - \phi + 1)$
- But for safe design, σ_1 should not be greater than allowable compressive strength of concrete, σ_{all} .



In limiting case, $\sigma_1 = \sigma_{all}$

$$\sigma_{all} = \gamma_w H(S - \phi + 1)$$

$$H_{crit} = \sigma_{allowable} / (\gamma_w (S - \phi + 1))$$

This is the maximum height of the dam for safe and economical case without exceeding allowable working stress.

Low dam is the one in which H is less than that given by above equation and maximum compressive stress is not more than allowable stress. ($H \leq H_{crit}$)

High dam is the one in which H is greater than that given by above equation and for this dams extra slopes are provided at u/s and d/s sides below the limiting height to bring the compressive stress within the limit. ($H > H_{crit}$)

For this section, the resultant will pass through the upper middle third point of the base when reservoir is empty and through the lower middle third point when the reservoir is full.

Practical section:

- i. The pointed crest of the theoretical dam is unstable to resist shock due to floating objects.
- ii. There is need for a free board
- iii. There is also need for top width for a roadway

For practical section

- i. Crest of the dam shall be a certain thickness depending on the height of the dam. For non-overflow dams, most economical crest width $\approx 14\%$ of the height (10 – 15 %) is normal.
- ii. Free board is provided and usually 3-4% of the dam height is used as a maximum height of the free board.

Design procedure of gravity dams

Design methods

The various methods used for the design of concrete gravity dams are as follows:

1. Stability analysis method
 - a. Gravity method (two dimensional method)
 - b. Three dimensional (analyzed by software applications).
2. Zoned (multiple-step) method of determining profile of dam
3. Single step method

Two procedures of design will be discussed in this course: – multiple-step method and single-step method.

Multiple step method of determining profile of gravity dam

This method deals with designing the dam joint by joint (block by block) beginning at the top and making each joint conform to all gravity dam design requirements. The procedure results in a dam with polygonal face that may be smoothed up for appearance with no appreciable change in stability or economy. The multiple-step method is almost always used for the final design of dams with a height that does not encroach greatly on Zone V

Zoning of high non-overflow dams

A high gravity dam may be divided into seven zones according to design and stability requirements. The characteristics and limits of these zones are described below.

Zone I: is a rectangular section from the top of the dam to the water surface. The resultant force passes through the mid-point of the base.

Zone II: is also a rectangular section and extends to a depth where the resultant in the reservoir full condition reaches the outer middle third point of the base.

Zone III: upstream face of the dam is vertical but the downstream face is gradually inclined so that the resultant in the reservoir full condition has exactly at the outer middle third point of the base. This zone extends to a depth where the resultant in the reservoir empty condition reaches the inner middle third point of the base.

Zone IV: in this zone both the upstream and downstream faces are inclined so that the resultant both in the reservoir full and empty conditions lie at the middle third point. The zone extends to a point where maximum permissible compressive stress is reached at the *toe* of the dam.

Zone V: the slope of the downstream face is further increased to keep the principal stresses within permissible limits. Resultant in the reservoir full condition is kept well within the middle third section. The resultant in the reservoir empty condition follows the upper middle third section. This zone extends to a depth where the stress at the *heel* of the section reaches the permissible limits in the reservoir empty case.

Zone VI: the slope of the upstream face is rapidly increased so as to keep the principal stress at the heel within the permissible limits in the reservoir empty condition. The inclination of the downstream face should also be adjusted so that the principal stress at the toe does not exceed the maximum allowable stress. The resultants in both reservoir empty and full conditions lie *within* the middle third section. This zone extends to a point where the slope of the *downstream* face reaches 1:1. This normally happens when the dam is 80 to 90 meters high.

Zone VII: in this zone the inclination of both upstream and downstream faces increase with the height of the dam. Consequently, at some plane the value of $(1 + \tan^2\alpha)$ may become so great that the principal stress at the downstream face may exceed the allowable limit. If one reaches this zone during design, it is better to avoid it and start again with a fresh design with increased crest width and/or better quality concrete.

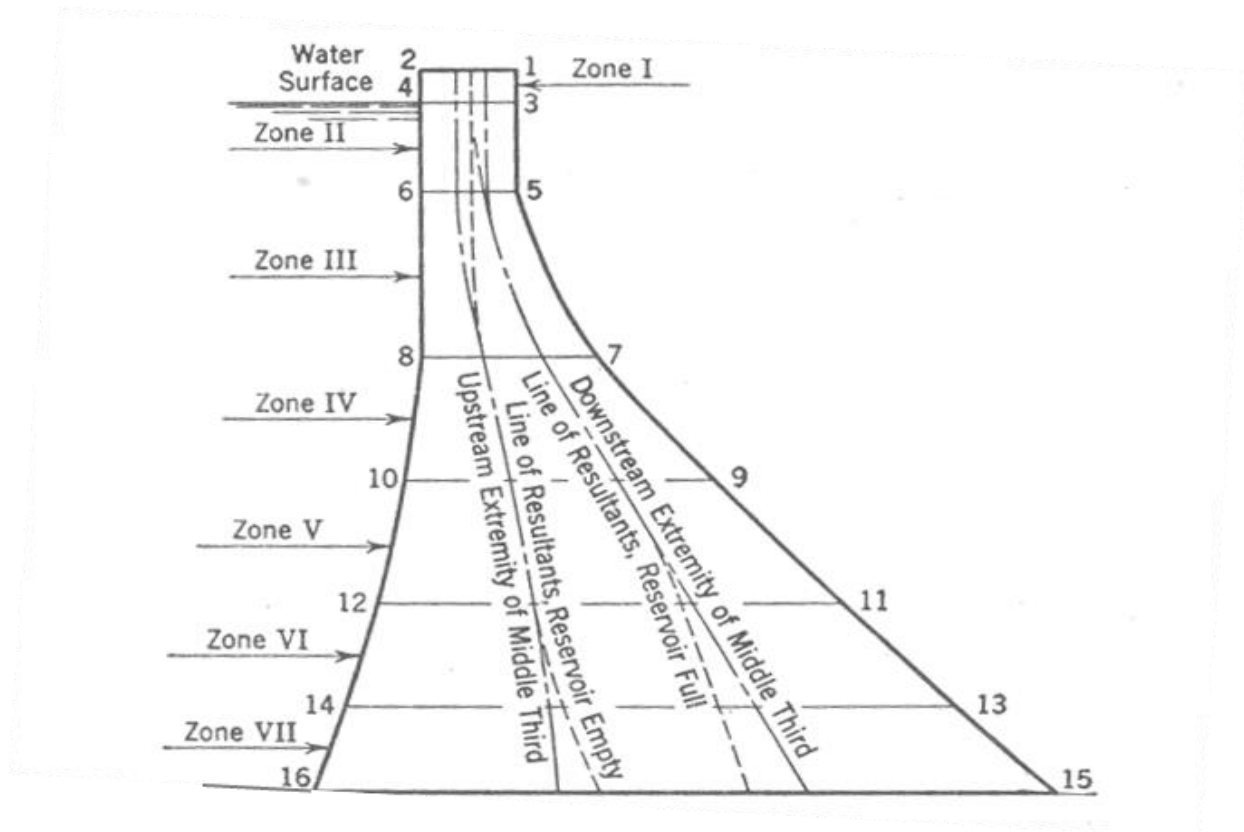


Figure: Multiple step method for design of dam

Single Step Method

This method considers the whole dam as a single block. It is used for final design of very high dams that extend well beyond zone V. It can also be used with an accuracy of 2 to 4% on the safe side; for preliminary designs to obtain the area of the maximum section of the dam.

The dam designed by single step method has a straight downstream face. When extended it intersects upstream face at the headwater surface.

Consider the sketch given:

$$L = 10-15\% \text{ of } h_1$$

$$H_{10} = 2L \text{ (when earthquake is considered)}$$

$$= 3L \text{ (when earthquake is not considered)}$$

$$H_6 = 1.33L$$

When designing (analyzing) a dam in the single step method, the dam is considered as a single block; and dam dimensions are determined in such a way that rules of Zone IV are satisfied.