1. INTRODUCTION TO OPEN-CHANNEL FLOW

1.1 Introduction

An open channel is a conduit in which water flows with a free surface. The flow of water in a conduit may be either *open channel flow* or pipe *flow*. The two kinds of flow are similar in many ways but differ in one important respect. Open-channel flow must have a *free surface,* whereas pipe flow has none, since the water must fill the whole conduit. A free surface is subject to atmospheric pressure. Pipe flow, being confined in a closed conduit exerts no direct atmospheric pressure but *hydraulic pressure* only.

The two kinds of flow are compared in Fig. 1.1. Shown on the left side is pipe flow. *Two* piezometer tubes are installed on the pipe at sections 1 and 2. The water levels in the tubes are maintained by the pressure in the pipe at elevations represented by the so called ***hydraulic grade line.*** The pressure exerted by the water in each section of the pipe is indicated in the corresponding tube by the height ***y*** of the water column above the center line of the pipe. The total energy in the flow of the section with reference to a datum line is the sum of the elevation ***z*** of the pipe-center line, the piezometric height ***y***, and the velocity head $V^2/2g$, where ***V*** is the mean velocity of flow. The energy is represented in the figure by what is called the *energy grade line* or simply the *energy line.* The loss of energy that results when water flows from section 1 to section 2 is represented by ***hf.*** A similar diagram for open channel flow is shown on the right side of Fig. 1-1. For simplicity, it is assumed that the flow is parallel and has a uniform velocity distribution and that the slope of the channel is small in this case the water surface is the hydraulic line the depth of water corresponds to piezometric height.

Despite the similarity between the two kinds of flow, it is much more difficult to solve problems of flow in open channels than in pressure pipes.

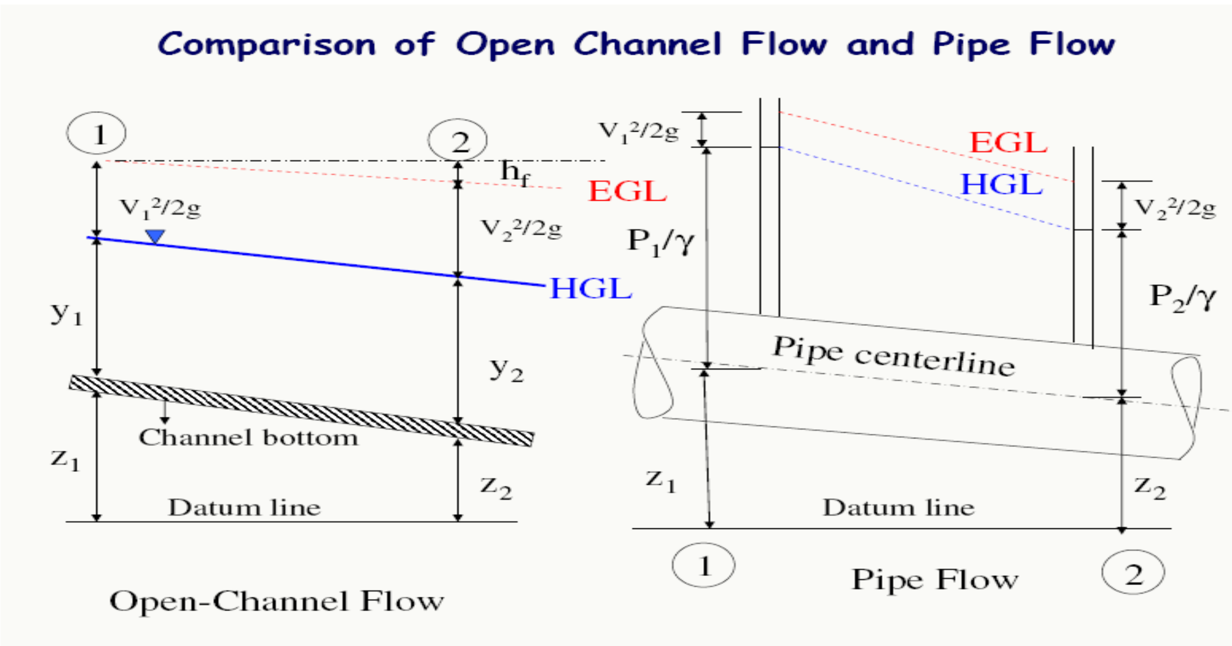


Fig 1.1 Comparison between pipe flow and open-channel flow.

The flow in a. closed conduit is not necessarily pipe flow. It must be classified as open-channel flow if it has a free surface. The storm sewer, for example, which is a closed conduit, is generally designed for open channel flow because the flow in the sewer is expected to maintain a free surface most of the time.

Types of open channels

Depending on the channel is manmade:-

1. Natural channel
2. Artificial channel

Based on boundary characteristics

1. Rigid boundary:- lined channel no problem of sediment
2. Mobile boundary:-unlined channels where sediment problem exists

Based on cross section and slope

1. Prismatic: - Cross section and slope remain constant in the reach.
2. Non-Prismatic: - cross section and slope vary with space and time.

1. ***Prismatic channels***: - a channel in which the cross-sectional shape and size also the bottom slopes are constant is termed as prismatic channel. E.g. most of the manmade (artificial) channels. Artificial channels are those constructed or developed by human effort:- navigation channels, power canals, irrigation canals flumes, drainage ditches, trough spillways, floodways, log chutes, roadside gutters, etc., as well as model channels that are built in the laboratory for testing purpose.
2. ***Non prismatic channels:-*** all channels have a varying cross sections classified as non prismatic channels. E.g. natural channels. Natural channels include all water course that exist naturally on the earth, varying in size from tiny hillside rivulets, through brooks, steams small and large rivers, to tidal estuaries. Underground streams can water with a free surface are also considered as natural open channel.

1.2 types of open channel flow

Open-channel flow can be classified into many types and described in various ways, The following classification is made according to the change in flow depth with respect to time and space.

1. ***Steady Flow and Unsteady flow****: - Time as the Criterion,*

Flow in an open channel is said to be *steady* if the depth of flow does not change or if it can be assumed to be constant during the time interval under consideration. The flow is *unsteady* if the depth changes with time.

2. ***Uniform flow and Varied (non uniform flow)*** space *as the Criterion,*

Open channel flow is said to be *uniform* if the depth of flow is the same at every section of the channel. A uniform flow may be steady or unsteady, depending on whether or not the depth changes with time. Flow is *varied (non uniform)* if the depth of flow changes along the length of the channel. Varied flow may be either steady or unsteady.

3. ***Gradually Varied Flow (GVF) and Rapidly Varied Flow (RVF):-*** The flow is rapidly varied if the depth changes abruptly over a comparatively short distance.

4. Spatially varied flow (SVF):- in the above types of open channel it is assumed that no flow is externally added or subtracted from the channel system. But if some flow is added or subtracted from the system the resulting varied flow is known as *spatially varied flow (SVF).*

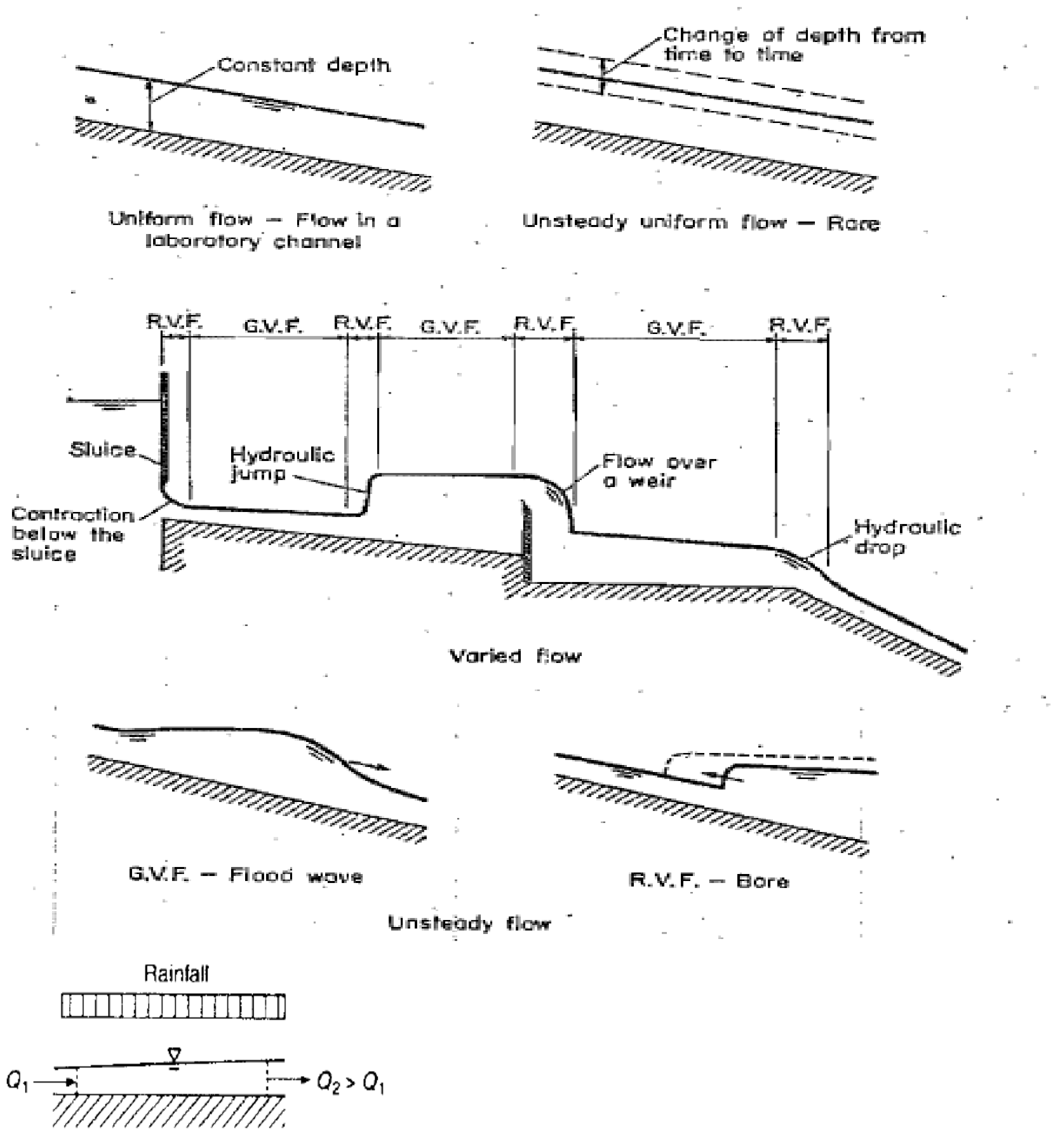
For clarity, the classification of open-channel flow is summarized as follows:-

A. Steady flow

1. Uniform Flow
2. Varied Flow
   - a. Gradually Varied Flow
   - *b.* Rapidly Varied Flow

B. Unsteady flow

1. Unsteady Uniform Flow (Rare)
2. Unsteady Flow (i.e. Unsteady Varied Flow)
   - a. Gradually Varied Unsteady Flow
   - *b.* Rapidly Varied Unsteady Flow



Spatially varied flow

Fig. 1.2 various types of open channel flow.

1.3 Geometric Elements of Channel Section

Geometric elements: - are properties of a channel section that can be defined entirely by the geometry of the section and the depth of flow. These elements are very important and are used extensively in flow computations. These elements are:-

1. ***Depth of Flow(y):-*** is the vertical distance of the lowest point of a channel section from the free surface. This term is often used interchangeably with the *depth of flow section d.* Depth of flow section is measured perpendicular to the channel bottom. The relationship between d and y is $d=y\ cos\theta$. For most manmade and natural channels $\cos\theta=1.0$, and therefore y=d. The two terms are used interchangeably.
2. ***Stage (Z):-*** is the elevation or vertical distance of the free surface above a datum. If the lowest point of the channel section is chosen as the datum, the stage is identical with the depth of flow.
3. ***Top Width (T):-*** is the width of channel section at the free surface.
4. ***Water Area (A):-*** is the cross sectional area of the floe normal to the direction of flow.
5. ***Wetted Perimeter (P):-*** is the length of the line of intersection of the channel wetted surface with a cross-sectional plane normal to the direction of flow.
6. ***Hydraulic Radius(R):-*** *is* the ratio of the water area to its wetted perimeter.

$$R = \frac{A}{P}$$

7. ***Hydraulic Mean Depth (D_m):-*** is the ratio of water area to the top width.

$$D_m = \frac{A}{T}$$

8. ***Section Factor (Z):-*** is the product of the water area and the square root of the Hydraulic mean depth.

$$Z = A\sqrt{\quad} = A\ \frac{-}{-}$$

9. ***Bottom slope (S0):-*** Longitudinal slope of the channel bottom. most of the time The bottom slope is less than (*10%*) and θ becomes small and cos $\theta \approx 1.0$

   There for: - $S_o = \tan\theta \approx \sin\theta$

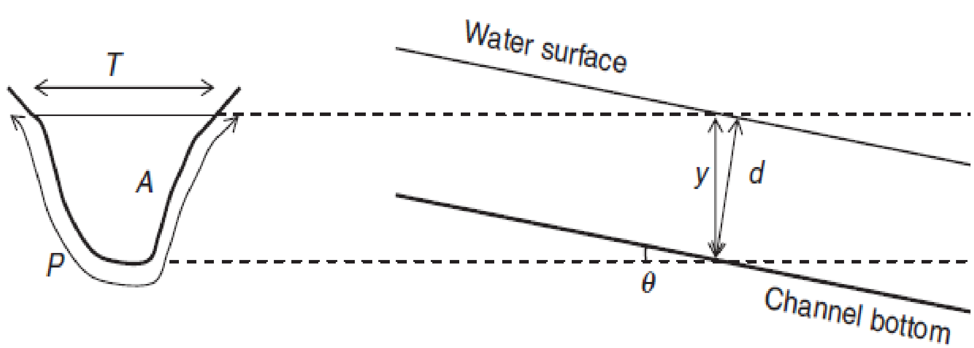
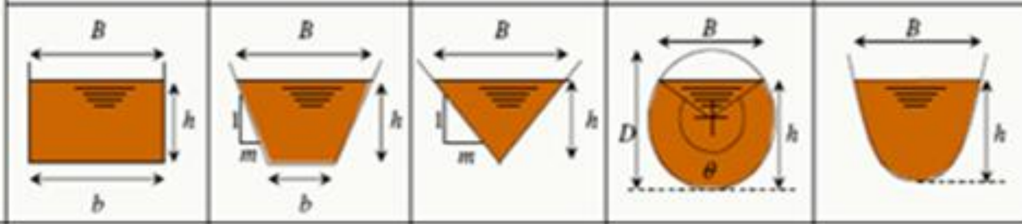


Fig 1.2 Definition sketch for section elements

Table 1.1 Geometric elements of channel sections



	rectangular	*trapezoidal*	*triangular*	*circular*	*parabolic*
flow area A	bh	$(b+mh)h$	mh^2	$\frac{1}{8}(\theta-\sin\theta)D^2$	$\frac{2}{3}Bh$
wetted perimeter P	$b+2h$	$b+2h\sqrt{1+m^2}$	$2h\sqrt{1+m^2}$	$\frac{1}{2}\theta D$	$B+\frac{8}{3}\frac{h^2}{B}$ *
hydraulic radius R_h	$\frac{bh}{b+2h}$	$\frac{(b+mh)h}{b+2h\sqrt{1+m^2}}$	$\frac{mh}{2\sqrt{1+m^2}}$	$\frac{1}{4}\left[1-\frac{\sin\theta}{\theta}\right]D$	$\frac{2B^2h}{3B^2+8h^2}$ *
top width B	b	$b+2mh$	$2mh$	$(\sin\theta/2)D$ or $2\sqrt{h(D-h)}$	$\frac{3}{2}Ah$
hydraulic depth D_h	h	$\frac{(b+mh)h}{b+2mh}$	$\frac{1}{2}h$	$\left[\frac{\theta-\sin\theta}{\sin\theta/2}\right]\frac{D}{8}$	$\frac{2}{3}h$

1.4 Velocity Distribution in a Channel Section

At any point in an open channel, the flow may have velocity components in all *three directions*. For the most part, however, open-channel flow is assumed to be *one-dimensional*, and the flow equations are written in the main flow direction. Therefore, by velocity we usually refer to the velocity component in the main flow direction. The velocity varies in a channel section due to the *friction forces* on the boundaries and the presence of the *free-surface*. We use the term point velocity to refer to the velocity at different points in a channel section. Figure 1.3a shows a typical distribution of point velocity, v, in a trapezoidal channel.

The volume of water passing through a channel section per unit time is called the flow rate or discharge. Referring to Figure 1.3b, the incremental discharge, dQ, through an incremental area, dA, is

$$dQ = udA \quad \ldots\ldots (1.1)$$

Where: - v = point velocity.
Then by definition,

$$Q = \int \qquad A \quad \ldots\ldots (1.2)$$

Where:- Q=discharge.

In most open-channel flow applications we use the cross-sectional average velocity V, defined as

$$V = - \quad - \qquad dA \quad \ldots\ldots (1.3)$$

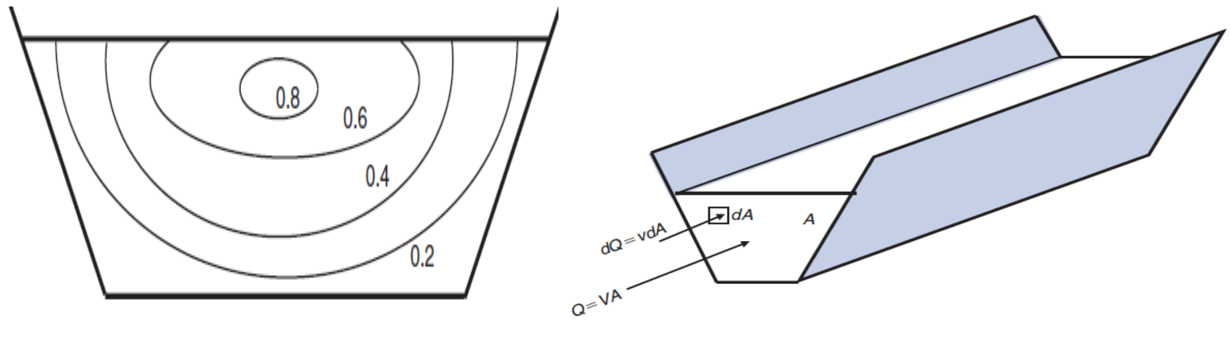


Fig.1.3 A. *Velocity Distribution in a Trapezoidal Channel Section* **B**. *Definition of discharge*

The velocity *V* is zero at the solid boundaries and gradually increases with distance from the boundary. The maximum velocity of the *X*-section occurs at a certain distance below the free surface.

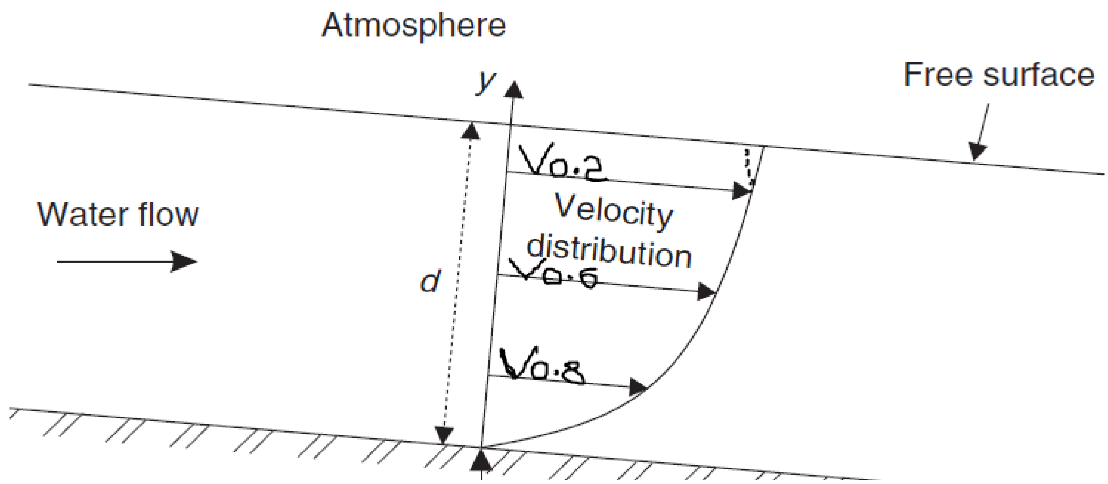


Fig 1.4 typical velocity profile

The average velocity at any vertical V_{av} occurs at a level of 0.6y_0 from the free surface.

It is found that:- ———…………………………………………….(1.4)

Where: - y_0 = depth of flow,

= velocity at a depth of 0.2 y_0 from the free surface

$V_{0.8}$ = velocity at a depth of 0.8 y_0 from the free surface

The surface velocity Vs is related to the average velocity V_{av} as:-

$V_{av} = kV_s$ ………………………………………………………… (1.5)

k = a constant with value between 0.8 and 0.95

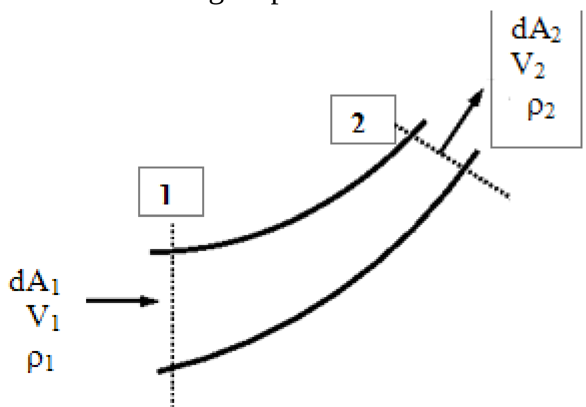
It should be noted that the above methods are simple and approximate. For precise measurements more elaborate methods must be used, which are beyond the scope of this course.

1.5 Continuity Equations

The continuity equation is a mathematical statement of the principle of conservation of mass. Consider the following fixed region with flowing fluid. Since fluid is neither created nor destroyed within the region it may be stored that the rate of increase of mass contained within the region must be equal to the differences b/n the rate at which the fluid mass enters the region & the rate of which it leaves the region.

However, if the flow is steady, the rate of increase of the fluid mass within the region is equal to zero; then the rate at which fluid mass enters the region is equal to the rate at which the fluid mass leaves the region.

Considers flow through a portion of a stream tube:-



Mass of fluid per unit time flowing past section-1 = $\rho_1 * dA_1 * V1$ [kg/s]

Mass of fluid flowing per unit of time past section 2 = $\rho_2 * dA_2 * V_2$ [kg/s]

For steady flow, by the principle of conservation of mass

$$\rho_1 \, dA_1 \, V_1 = \rho_2 \, d \, A_2 \, V_2$$

For the entire area of the stream tube:

$$\int_{A1} \rho_1 \, dA_1 V_1 = \int_{A2} \rho_2 \, dA_2 V_2 = \text{constant}$$

If ρ_1 and ρ_2 are average densities at section (1) and (2), then

$$\rho_1 \int_{A1} V_1 dA_1 = \rho_2 \int_{A2} V_2 dA_2 = \rho VA = \text{constant}$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \rho VA = \text{constant} \quad \text{……………………………………………}(1.6)$$

This is equation of continuity applicable to steady, one-dimensional flow of compressible as well as incompressible ($\rho_1 = \rho_2$) flow.

For incompressible flow, ρ = constant and doesn't vary from point to point, $\rho_1 = \rho_2$

$A_1V_1 = A_2V_2 = Q = \text{constant}$

This is continuity equation for steady incompressible flow.

Q is the discharge (or volumetric flow rate or flow) defined as

$Q = AV\ [m^2 m/s = m^3/s = \text{Volume/time}]$

$Q = A_1V_2 = A_2V_2$ --- $V_1 = \dfrac{Q}{A_1}$, $V_2 = \dfrac{Q}{A_2}$

Hence, the velocity of flow is inversely proportional to the area of flow section. This is useful for most engineering application.

1.6 Energy and Momentum Coefficients (α and β)

Energy coefficient (α):- As a result of non uniform distribution of velocities over a channel section the velocity head of an open channel flow is generally greater than the value computed according to the expression $\boldsymbol{V^2/2g}$, where *V* is the mean velocity. When the, energy Principle is used in computation, the true velocity head may be expressed as $\boldsymbol{\alpha V^2/2g}$, where *a* is known as the *energy coefficient* or *Coriolis coefficient.* An expression for α can be obtained as flows:-

For an elementary area dA, the flux of kinetic energy through it is equal to :-

(——) (—) = —— = ρudA—

For the total area, the kinetic energy flux

$= \int \frac{\rho}{\ } u^3 dA = \alpha \frac{\rho}{\ } V^3 A$ ……………………………………………….(1.7)

From which: - $\alpha = \frac{\ }{\ }$ …………………………………………(1.8)

The cross-section of a natural river, comprise the main river channel and the flood plains. The flow velocity in the flood plain is usually very low, compared to the main section. In addition the variation of low velocity in the sub-section is small. Therefore, each sub-section may be assumed to have the same flow through out. In such a case the integration of various terms of equation (1.8) may be replaced by summation as follows.

$$\alpha = \frac{v_1^3 A_1 + v_2^3 A_2 + v_3^3 A_3}{v_m^3 (A_1 + A_2 + A_3)}$$ ……………………………………………………………..(1.9)

In which $$v_m = \frac{V_1 A_1 + V_2 A_2 + V_3 A_3}{A_1 + A_2 + A_3}$$ ………………………………………………..(1.10)

In general $$\alpha = \frac{\Sigma u^3 \Delta A}{V^3 A}$$ ……………………………………………(1.11)

The Kinetic Energy per unit weight of fluid can be written as $\alpha V^2/2g$.

Momentum Coefficient (β):- The non uniform distribution of velocities also affects the computation of momentum in open-channel flow. From the principle of mechanics, the momentum of the fluid passing through a channel section per unit time is expressed by ***βwQV/g*** *where **β*** is known as the *momentum coefficient* or *Boussinesq coefficient.*
The flux of momentum also expressed in terms of V and correction factor ***β.***
For an elementary area dA the flux of momentum in the longitudinal direction is:-

= (—— × ***velocity***) =***(ρudA)(u)***

For the total area the momentums flux:-

$$=\int \quad dA = \beta\rho V^2 A \ldots\ldots(1.10)$$

Which gives:-

$$\beta = \text{——} \qquad \text{for discrete value } \beta = \text{——}\ldots\ldots(1.11)$$

Values of α and β:-

- ***α = β =1** for uniform velocity distribution.*
- *For any other variation* ***α > β >1.0.***
- *The higher the non uniformity of the velocity distribution, the greater will be the values of **α and β.***

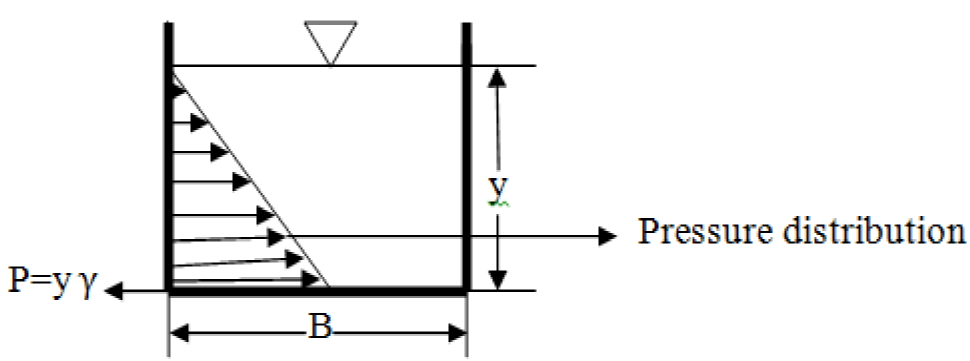
1.7 Pressure distribution in an open channel

The intensity of pressure for liquid at its free surface is equal to that of surrounding atmosphere. Since the atmospheric pressure is commonly taken as reference and value equal to zero, the free surface of the liquid is thus a surface of zero pressure.

1. Hydrostatic pressure distribution

The normal acceleration will be zero.
U=0 there is no motion
Since there is no force in the direction of flow, the component of the resultant force in the flow direction is zero that is:

=0 pdA =W = dA*Y*γ
P = y γ



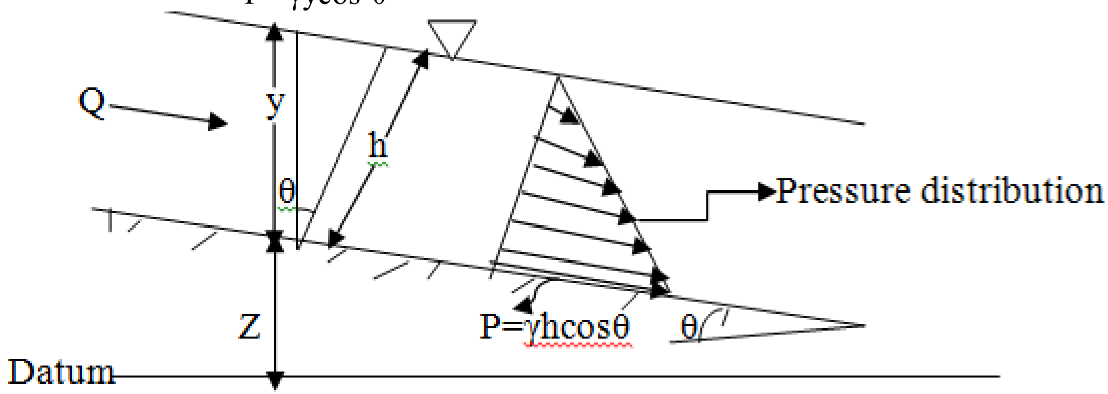
2. Pressure distribution in sloping channels

There is no acceleration in the direction of the flow and flow velocity is uniform at a channel cross-section and is parallel to the channel section.

=0 $pdA = W\cos\theta = dA\gamma h\cos\theta$

$P = \gamma h\cos\theta$

$P = \gamma y\cos^2\theta$



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