

GENERAL PRINCIPLES OF FOUNDATION DESIGN

The usual approach to a normal foundation-engineering problem is:

- ❖ To prepare a plan of the base of the structure showing the various columns, load-bearing walls with estimated loads, including dead load, live load, moments and torques coming into the foundation units.**
- ❖ To study the tentative allowable bearing pressures allocated for the various strata below the ground level, as given by the soil investigation report.**

- ❖ To determine the required foundation depth. This may be the minimum depth based on soil strength or structural requirement considerations.
- ❖ To compute the dimensions of the foundation based on the given loading and allowable bearing pressure.
- ❖ To estimate the total and differential settlements of the structure. **If these are excessive the bearing pressure will have to be reduced or the foundation taken to a deeper and less compressible stratum or the structure will have to be founded on piles or other special measures taken**

Loads on Foundation

❖ A foundation may be subjected to two or more of the following loads.

a) Dead load:

Weight of structure

- » All material permanently attached to structure
- » Static earth pressure acting permanently against the structure below ground surface.
- » Water pressure acting laterally against basement walls and vertically against slab.

b) Live load: temporary loads expected to superimpose on the structure during its useful life.

c) **Wind load**:- lateral load coming from the action of wind.

-Local building codes provide magnitude of design wind pressure.

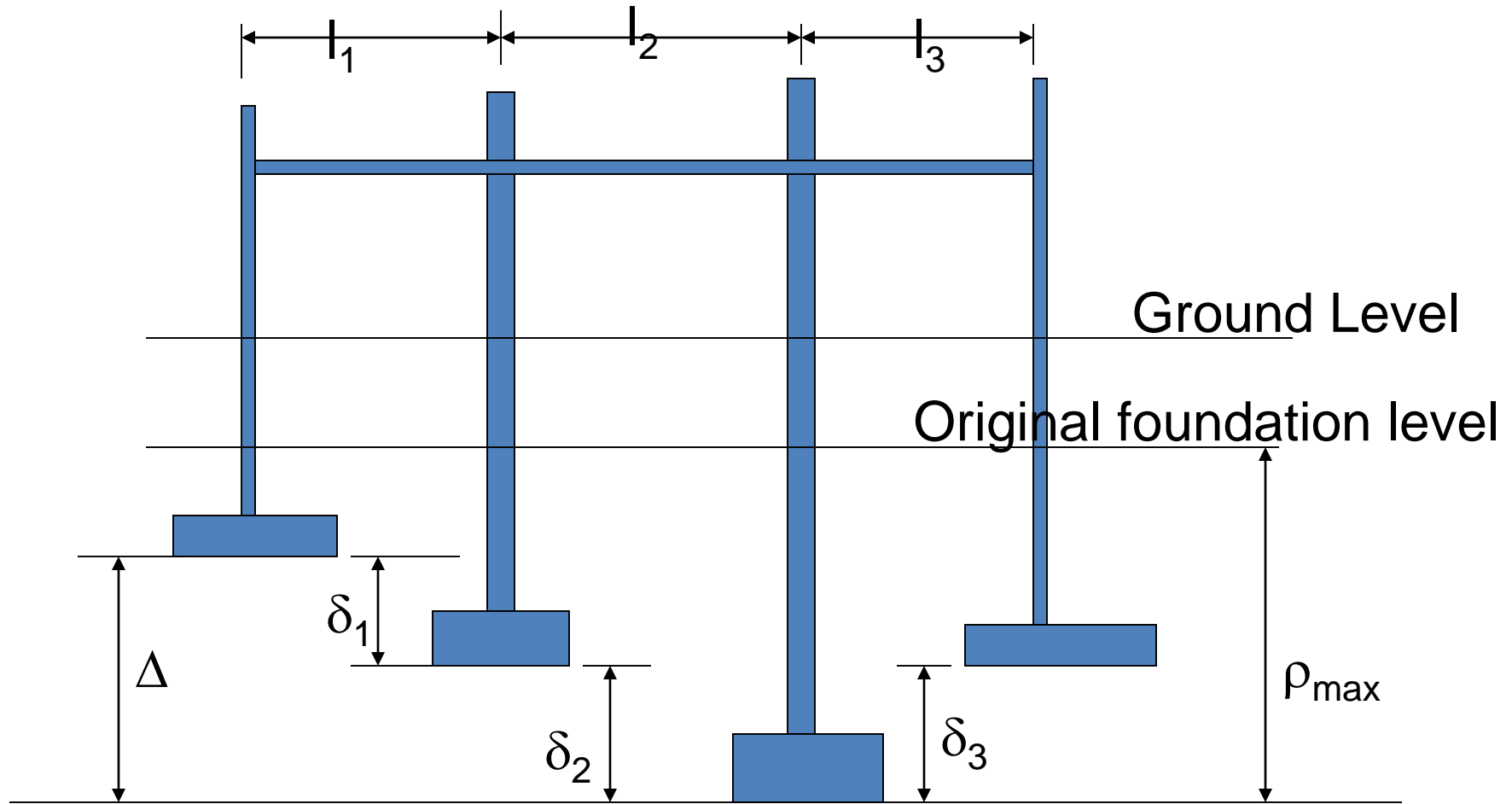
d) **Earth-quake load**:- lateral load coming from earth-quake motion.

-The total lateral force (base shear) at the base of a structure is evaluated in accordance with local building code.

e) **Dynamic load**:- load coming from a vibrating object (machinery).

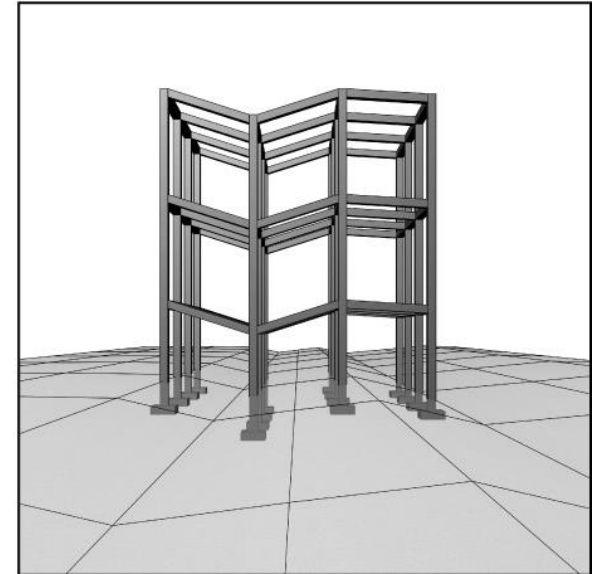
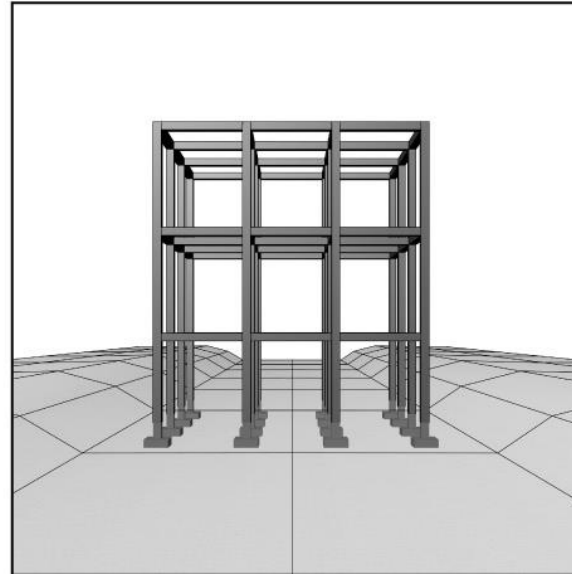
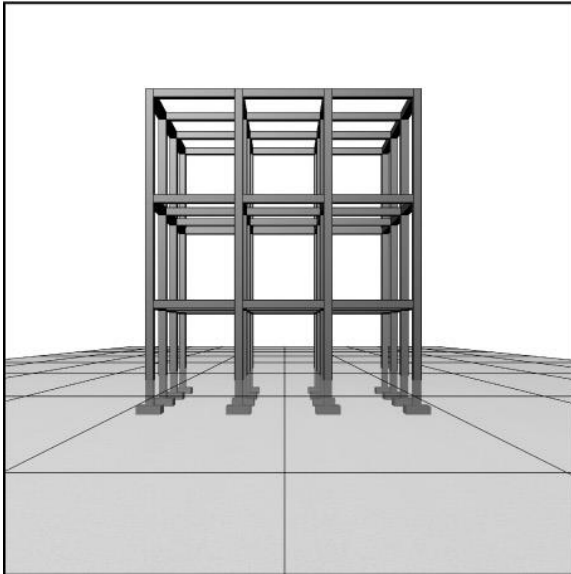
– In such case, separate foundation should be provided. The impact effect of such loads should be considered in design.

Settlement of Foundations



$\delta_1, \delta_2, \delta_3 =$ Differential sett., $\Delta =$ Greatest differential sett.

$\rho_{\max} =$ maximum total sett., $l_1, l_2, l_3 =$ Bay width, $\delta/l =$ angular distortion



NO SETTLEMENT

TOTAL SETTLEMENT

DIFFERENTIAL SETTLEMENT

Uniform settlement is usually of little consequence in a building, but differential settlement can cause severe structural damage

Design of Isolated or Spread Footings

I. Depth of footing

The depth of embedment must be at least large enough to accommodate the required footing thickness. This depth is measured from the lowest adjacent ground surface to the bottom of the footing.

Footings should be carried below

- a) zone of high volume change due to moisture fluctuation
- b) top (organic) soil
- c) peat and muck
- d) unconsolidated (or fill) material

According to EBCS-7

minimum depth of footing should be 50cm for footings on sloping sites, minimum depth of footing should be 60cm and 90cm below ground surface on rocky and soil formations, respectively.

Footing at different elevations: -

When adjacent footings are to be placed at different levels, the distance between the edges of footings shall be such as to prevent undesirable overlapping of stresses in soils and disturbance of the soil under the higher footing due to excavation for the lower footing. A minimum clear distance of half the width of the footing is recommended.

II. Proportioning of footing

The required area of the footing and subsequently the proportions will be determined using presumptive allowable soil pressure and the soil strength parameters ϕ and c as discussed previously.

III. Structural Design

i) Punching shear:- This factor generally controls the depth of footings. It is the normal practice to provide adequate depth to sustain the shear stress developed without reinforcement. The critical section that is to be considered is indicated in Fig.

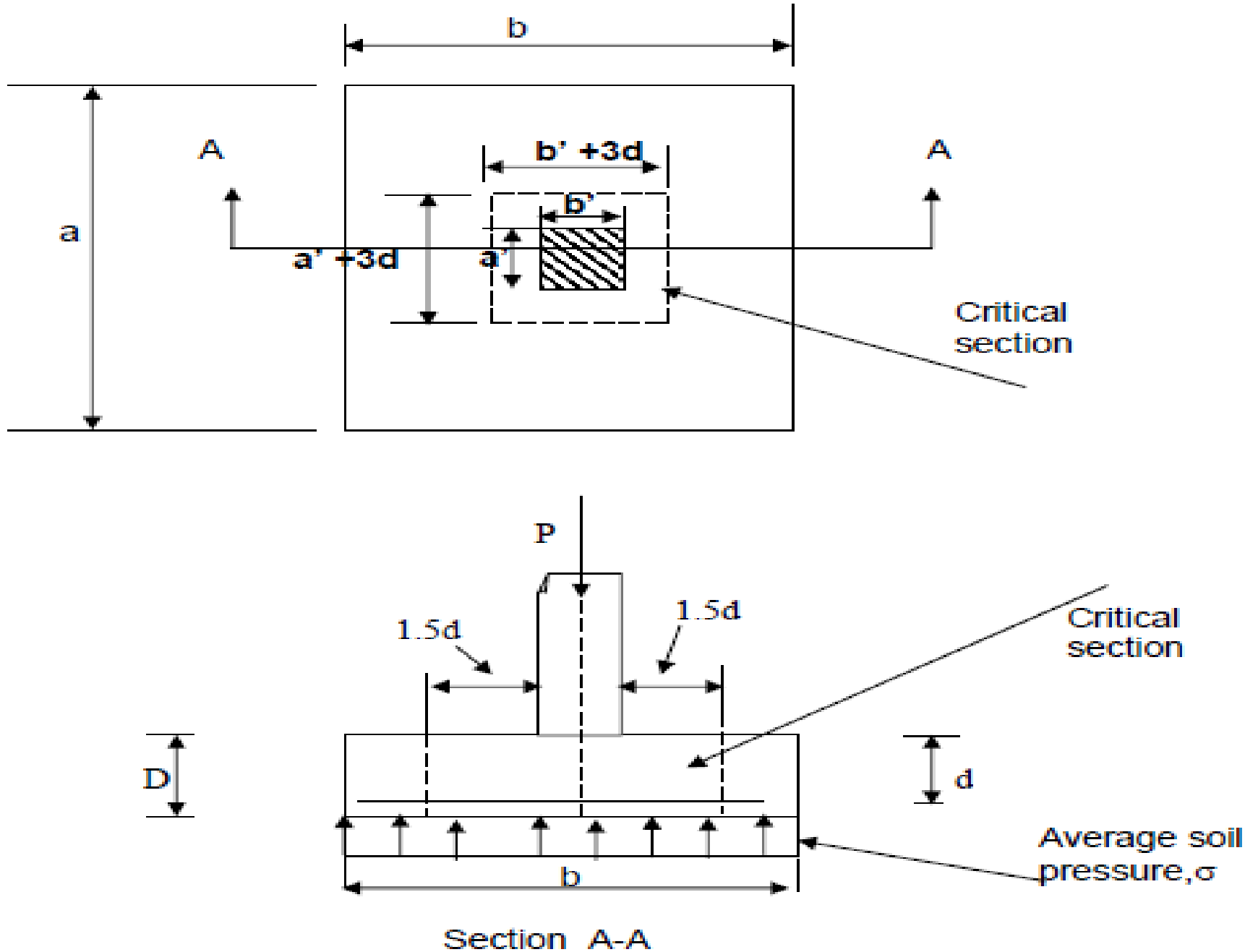


Fig. Critical section for punching shear

From the figure it is apparent the concrete shear resistance along the perimeter according to EBCS would be

$$2(a' + 3d + b' + 3d) dV_{up}$$

Where V_{up} = punching shear resistance

The net force on the perimeter due to the soil pressure would be

$$\{a * b - [(a' + 3d)(b' + 3d)]\} \sigma_{ult}$$

From equilibrium consideration, above two Eqn. should be equal

$$2(a' + 3d + b' + 3d) dV_{up} = \{a * b - [(a' + 3d)(b' + 3d)]\} \sigma_{ult}$$

For square columns $a' = b'$ and round columns with diameter a' , above Eqn. would be

$$d^2 (12V_{up} + 9\sigma_{ult}) + d(2V_{up} + 3\sigma_{ult})(2a') = (A_{footing} - A_{column})\sigma_{ult}$$

ii) Diagonal Tension (wide beam shear)

The selected depth using the punching shear criterion may not be adequate to withstand the diagonal tension developed. Hence one should also check the safety against diagonal tension. The critical sections that should be considered are given in Fig.

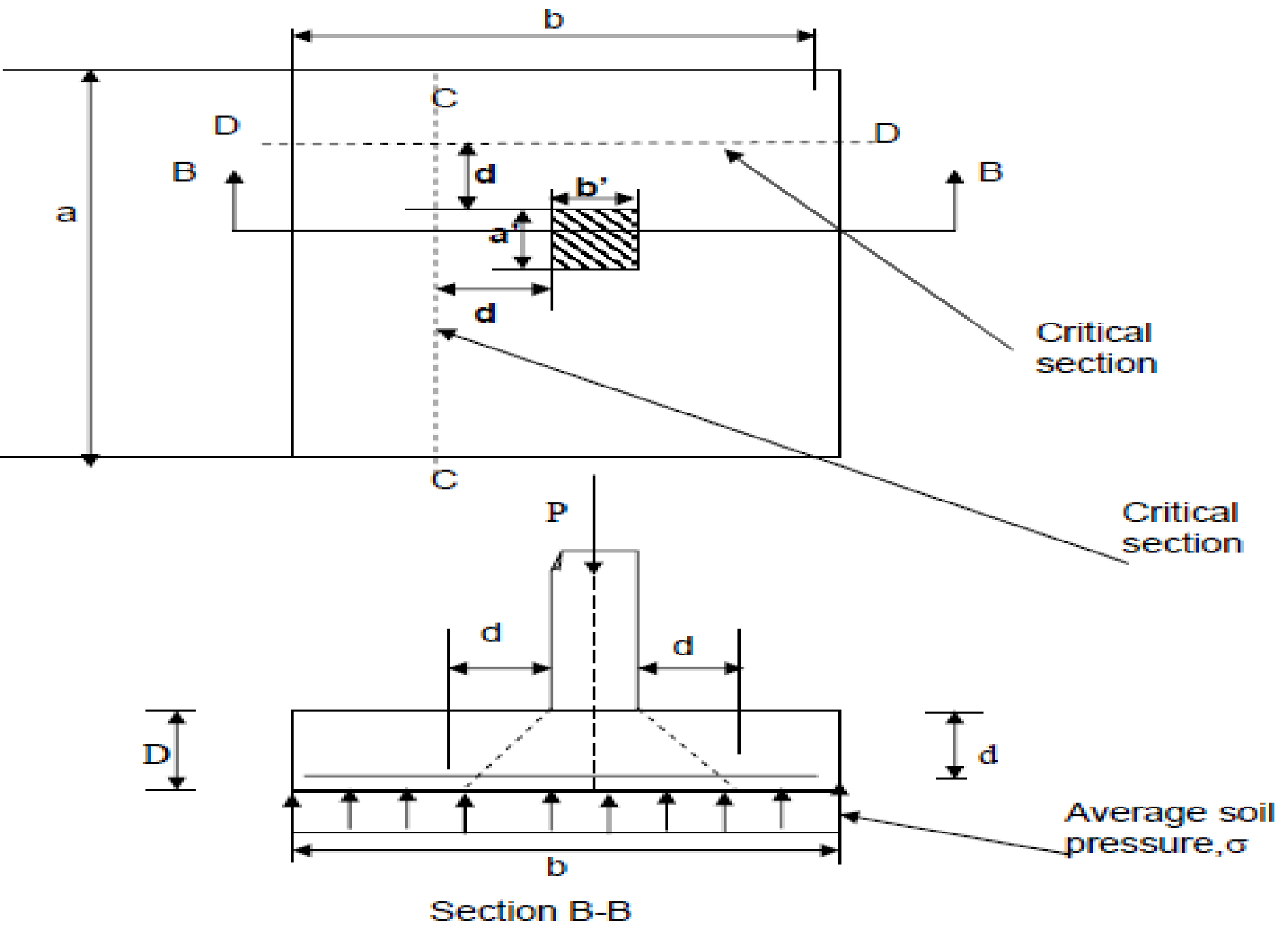


Fig. Critical section for diagonal tension

The shear forces are calculated along the plane C-C and D-D

$$V_{C-C} = (b/2 - d - b'/2) a \sigma_{ult}.$$

$$V_{D-D} = (a/2 - d - a'/2) b \sigma_{ult}$$

The actual shear stress is then calculated from These

$$v_{C-C} = \frac{V_{C-C}}{ad}$$

$$v_{D-D} = \frac{V_{D-D}}{bd}$$

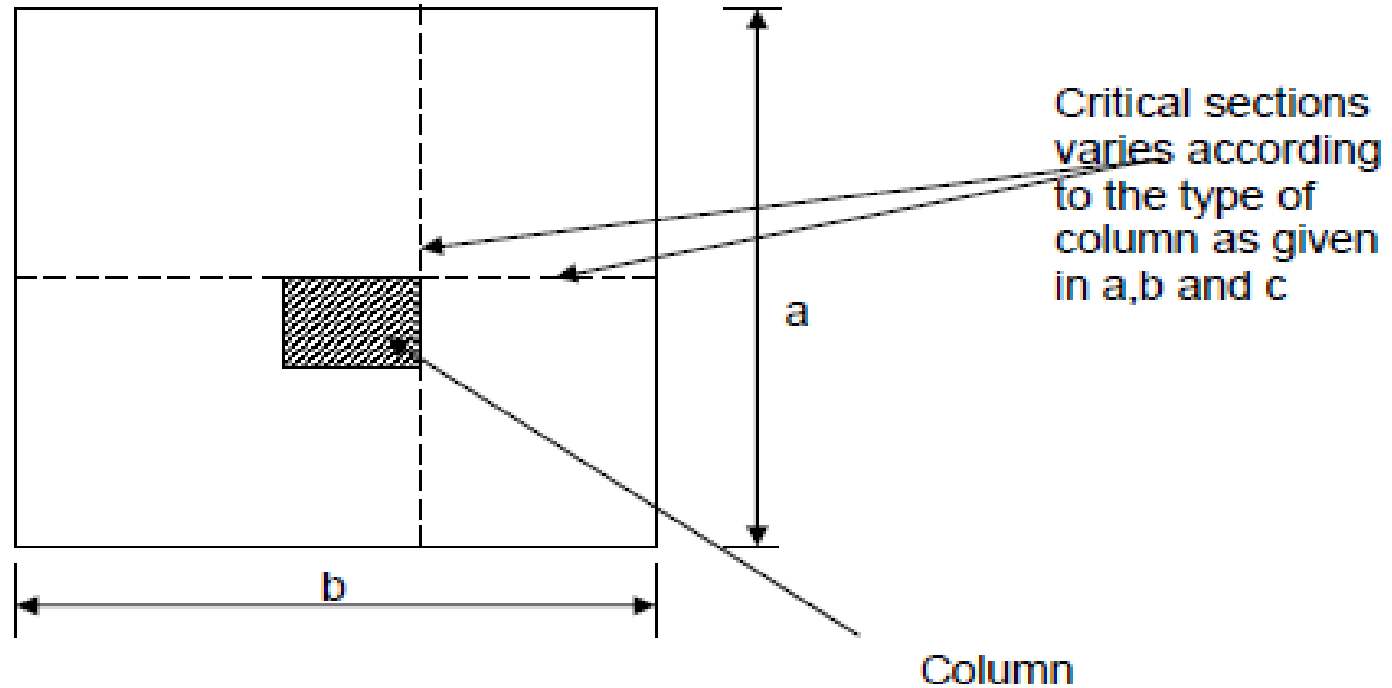
calculated actual shear stresses should be compared with diagonal shear resistance.

iii) Bending Moment

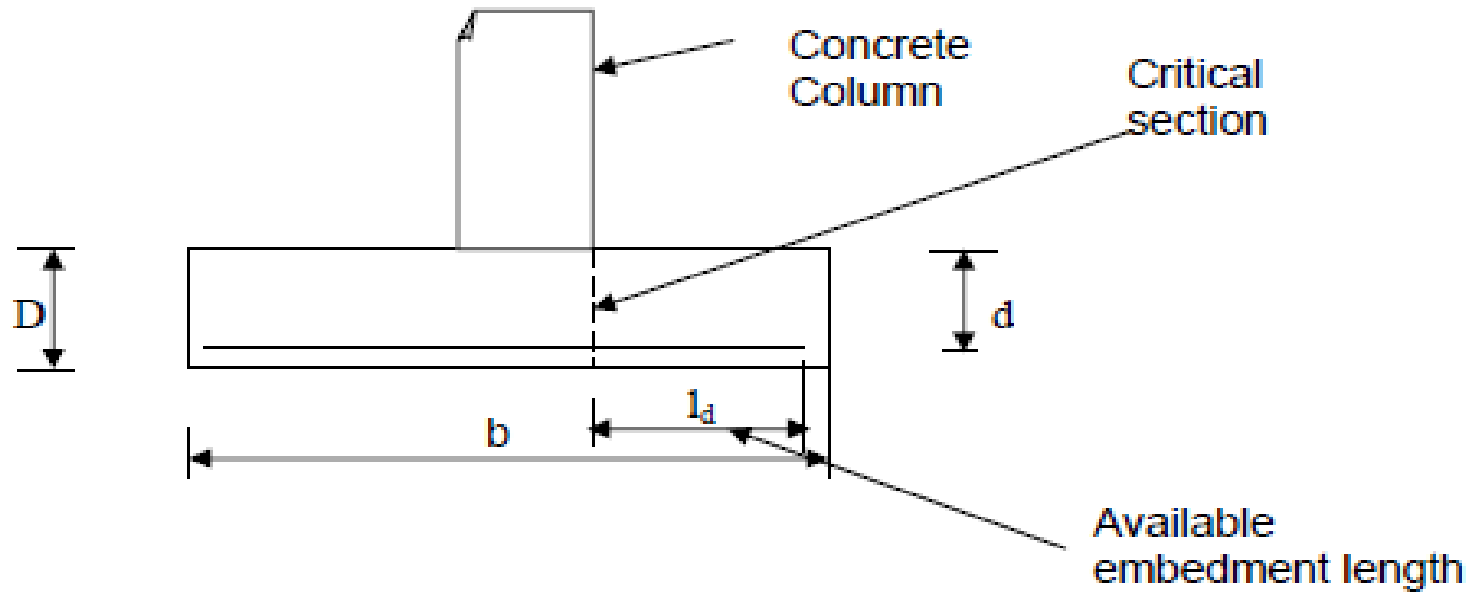
The external moment on any section of a footing shall be determined by passing a vertical plane through the footing, and computing the moment of the forces acting over the entire area of the footing on one side of that vertical plane. The critical sections for the bending moment vary according to the type of columns.

According to EBCS 2-1995, the critical section for moment shall be taken as follows:

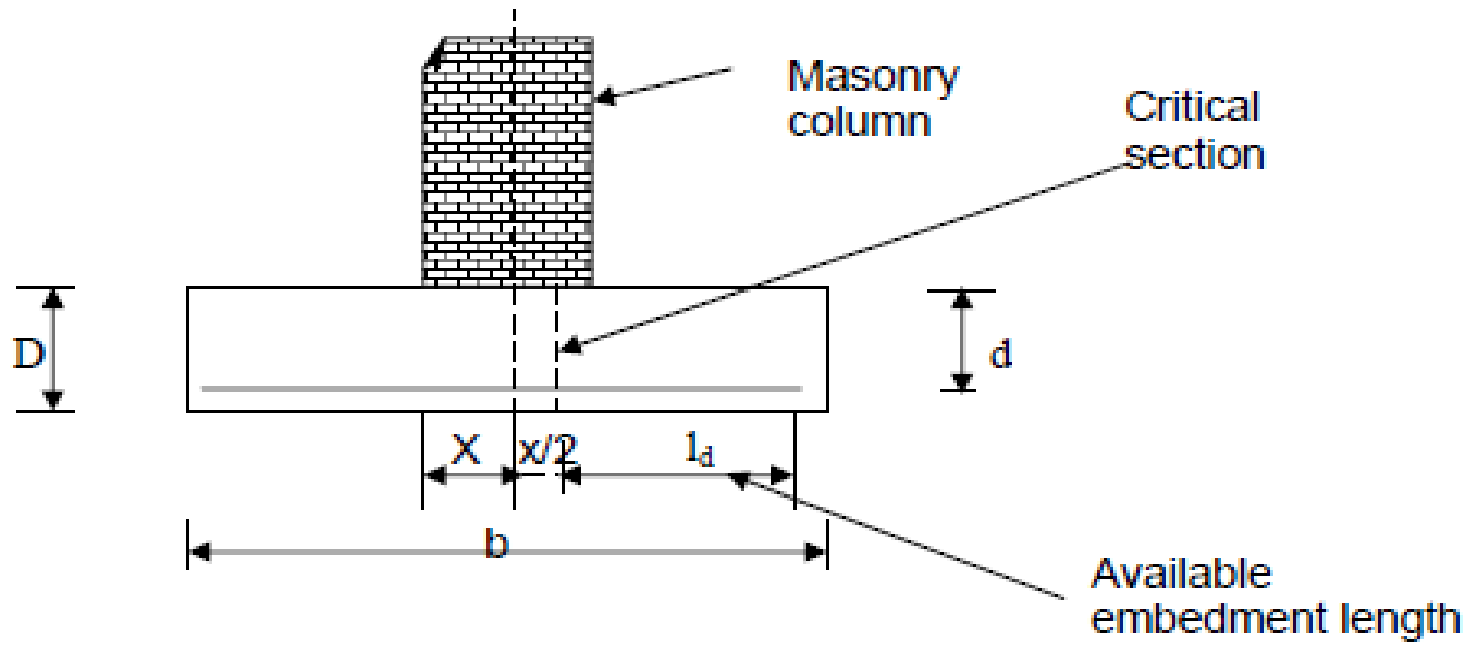
- a) At the face of column, pedestal or wall for footings supporting a concrete pedestal or wall
- b) Halfway between middle and edge of wall, for footings supporting a masonry wall
- c) Halfway between face of column and edge of steel base for footings supporting a column with base plates.



a)



b)



c)

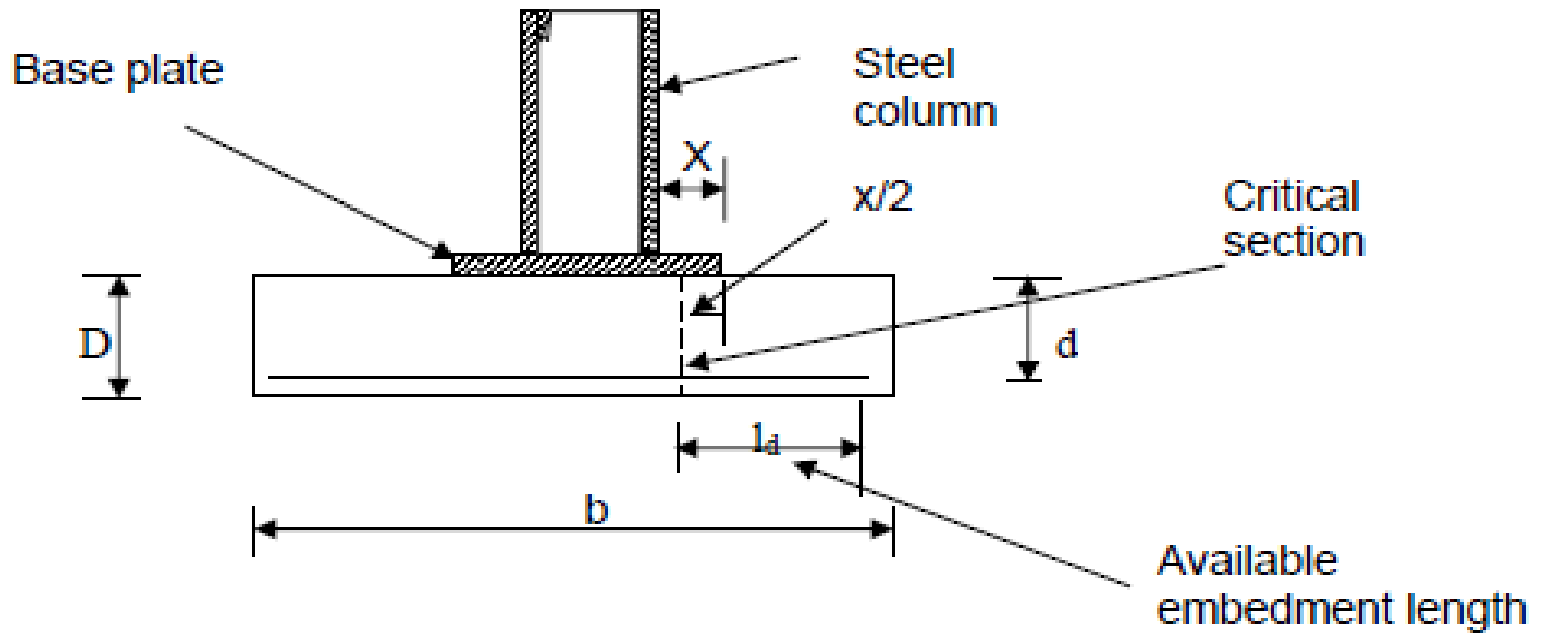


Fig. Critical sections for moments

Flexural Reinforcement

1. Distribution: In one-way footings and two-way square footings, reinforcement shall be distributed uniformly across the entire width of footing.
2. In two-way rectangular footings, reinforcement shall be distributed as follows:
 - a) Reinforcement in long direction shall be distributed uniformly across the entire width of footing
 - b) For reinforcement in the short direction, a portion of the total reinforcement given by Eqn. shall be distributed uniformly over a band width (centered on center line of column or pedestal) equal to the length of the short side of footing. The remainder of the reinforcement required in the short direction shall be distributed uniformly out side the center band width of the footing.

$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

Where β is the ratio of long side to short side of footing (a/b).

IV. Development length

The reinforcement bars must extend a sufficient distance into the concrete to develop proper anchorage. This distance is called the development length. The necessary development length may be calculated using the following equation.

$$l_d = \frac{\phi f_{yd}}{4f_{bd}}$$

Minimum Footing cover (According to EBCS2-1995)

The thickness of footing above bottom reinforcement shall not be less than 150mm for footing on soil, or 300mm for footing on piles.

Concrete cover to reinforcement (According to EBCS2-1995)

- Concrete cast directly against the earth, the minimum cover should be greater than 75mm
- Concrete cast against prepared ground (including blinding) the minimum cover should be greater than 40mm.

Spacing of reinforcement

The clear horizontal and vertical distance between bars shall be at least equal to the largest of the following values: (EBCS2-1995)

- a) 20mm
- b) the diameter of the largest bar
- c) the maximum size of the aggregate plus 5mm

The spacing between main bars for slabs shall not exceed the smaller of $2h$ or 350mm

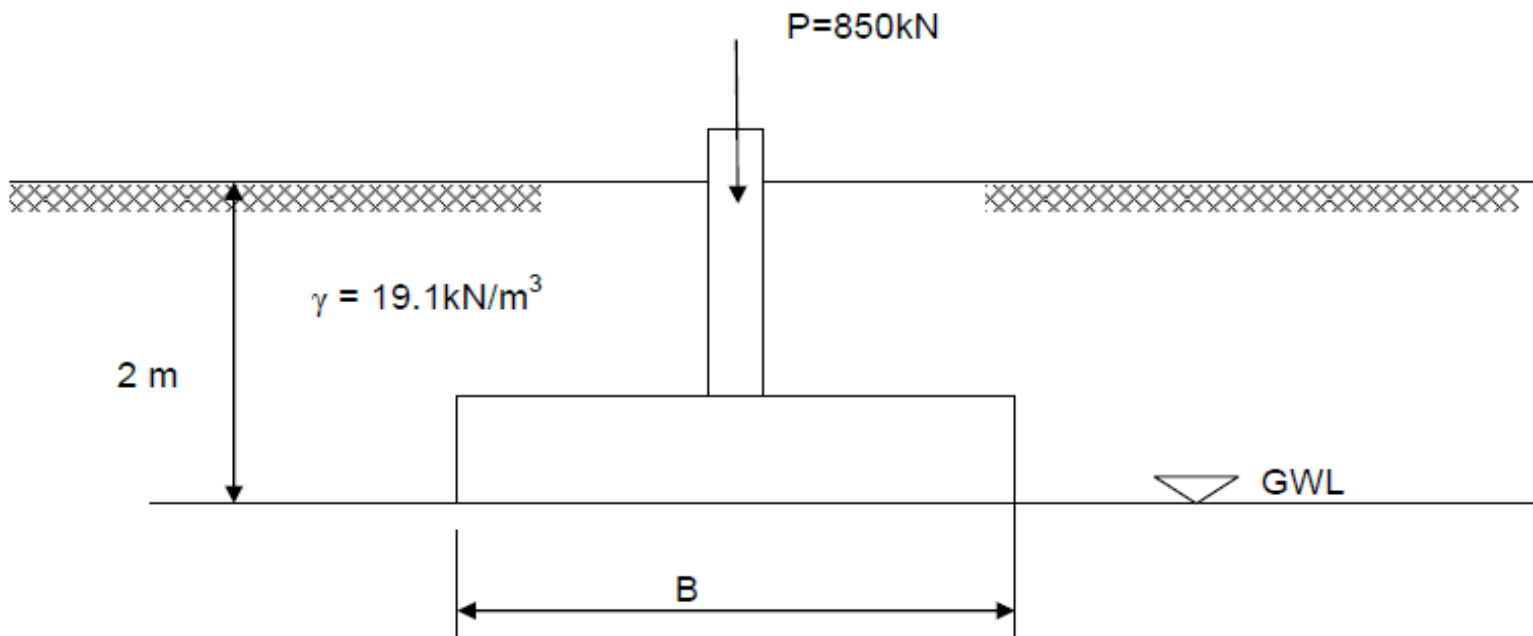
The spacing between secondary bars shall not exceed 400mm

Examples

1. Determine the dimensions of a square footing necessary to sustain an axial column load of 850kN as shown in Fig. below, if

a) An allowable presumptive bearing pressure of 150kN/m² is used.

b) $C_u = 40 \text{ kN/m}^2$; $C = 7.5 \text{ kN/m}^2$; $\phi' = 22.50$



Solution

a) Using presumptive value

$$A = \frac{P}{\sigma_{as}} = \frac{850}{150} = 5.67m^2 = B^2$$

b) Using the bearing capacity formula

i) Initial loading condition

$$\sigma_f = 5.14 C_u S_c d_c i_c + q S_q d_q i_q$$

Shape factors

$$S_c = 1.2, S_q = 1$$

Depth factors

$$d_c = (1 + 0.4(2/B)) , d_q = 1$$

Load inclination factors

$$i_c = 1 , i_q = 1$$

Hence

$$\sigma_{ult} = 5.14 \cdot 40 \cdot 1.2 \cdot (1 + 0.8/B) \cdot 1 + 19.1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = (244.8 + 195.84/B + 38.2)$$

$$A \sigma_{ult} = P F_s$$

$$A = \frac{P \cdot F_s}{\sigma_{ult}} = \frac{850 \cdot 2}{253 + 195.84/B} = B^2$$

$$253 B^2 + 195.84B - 1700 = 0$$

The dimension of the footing would be 2.25m X 2.25m

ii) Final or long term loading condition

$$\sigma_{ult} = CN_c S_c d_c i_c + \frac{1}{2} B' \gamma N_\gamma S_\gamma d_\gamma i_\gamma + q N_q S_q d_q i_q$$

Bearing capacity factors

$$N_c = 17.45, N_\gamma = 6.82, N_q = 8.23$$

Shape factors

$$S_c = 1 + (N_q / N_c) = 1.47, S_\gamma = 0.6, S_q = 1 + \tan \phi = 1.41$$

Depth factors

$$d_c = 1 + 0.4 (2 / B) = 1 + 0.8/B, d_\gamma = 1, d_q = 1 + 2 \tan 22.5 (1 - \sin 22.5)^2 (D_f / B) = 1 + 0.63/B$$

Load inclination factors

$$i_c = 1, i_\gamma = 1, i_q = 1$$

Hence

$$\sigma_{ult} = 7.5 * 17.45 * 1.47 * (1 + 0.8/B) * 1 + \frac{1}{2} B' * 9.1 * 6.82 * 0.6 * 1 * 1 + 19.1 * 2 * 8.23 * 1.41 * (1 + 0.63/B) * 1 = 192.39 + 153.91/B + 18.62B + 443.28 + 279.27/B$$

$$A * \sigma_{ult} = P * F_s$$

$$B^2 = \frac{P * F_s}{\sigma_{ult}} = \frac{850 * 2}{635.67 + \left(433.18 / B\right) + 18.62B}$$

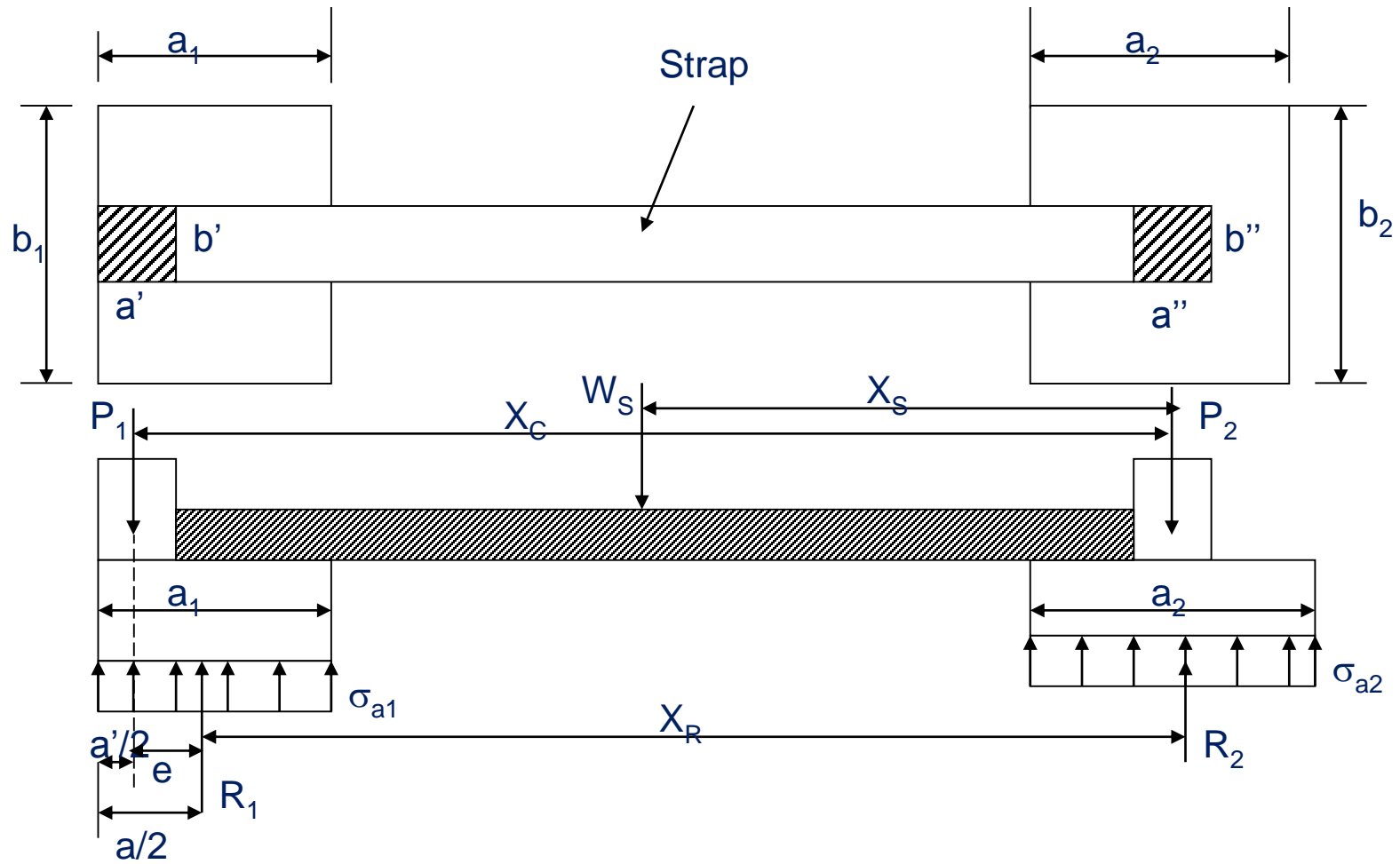
$$18.62 * B^3 + 635.67 * B^2 + 433.18 * B = 1700$$

From the above the dimension of the footing would be 1.35m X1.35m

Design of Strap or Cantilever Footings

A strap footing is used in the following two cases

1. When $X' < L/3$
 2. When the distance between the two columns is so large that a combined footing becomes excessively long and narrow.
- Essentially a strap footing consists of a rigid beam connecting two pads (footings) to transmit unbalanced shear and moment from the statically unbalanced footing to the second footing.
 - Design Assumptions
 - strap is infinitely rigid
 - strap is a pure flexural member and does not take soil reaction. (To confirm with this, strap is constructed slightly above soil or soil under strap is loosened).



1. a) Assume a_1 and establish the eccentricity, e of the soil reaction force R_1 .

$$e = \frac{a_1 - a'}{2}$$

$$e = X_C - X_R$$

- b) Determine the magnitude of the soil reaction force by taking moments about R_2 .

$$R_1 = P_1 \frac{X_c}{X_R} + W_s \frac{X_s}{X_R}$$

- In this equation the weight of the strap, W_s , may be neglected if the strap is relatively short.

- c) Determine the reaction R_2 from equilibrium consideration

$$R_2 = P_1 + P_2 + W_s - R_1$$

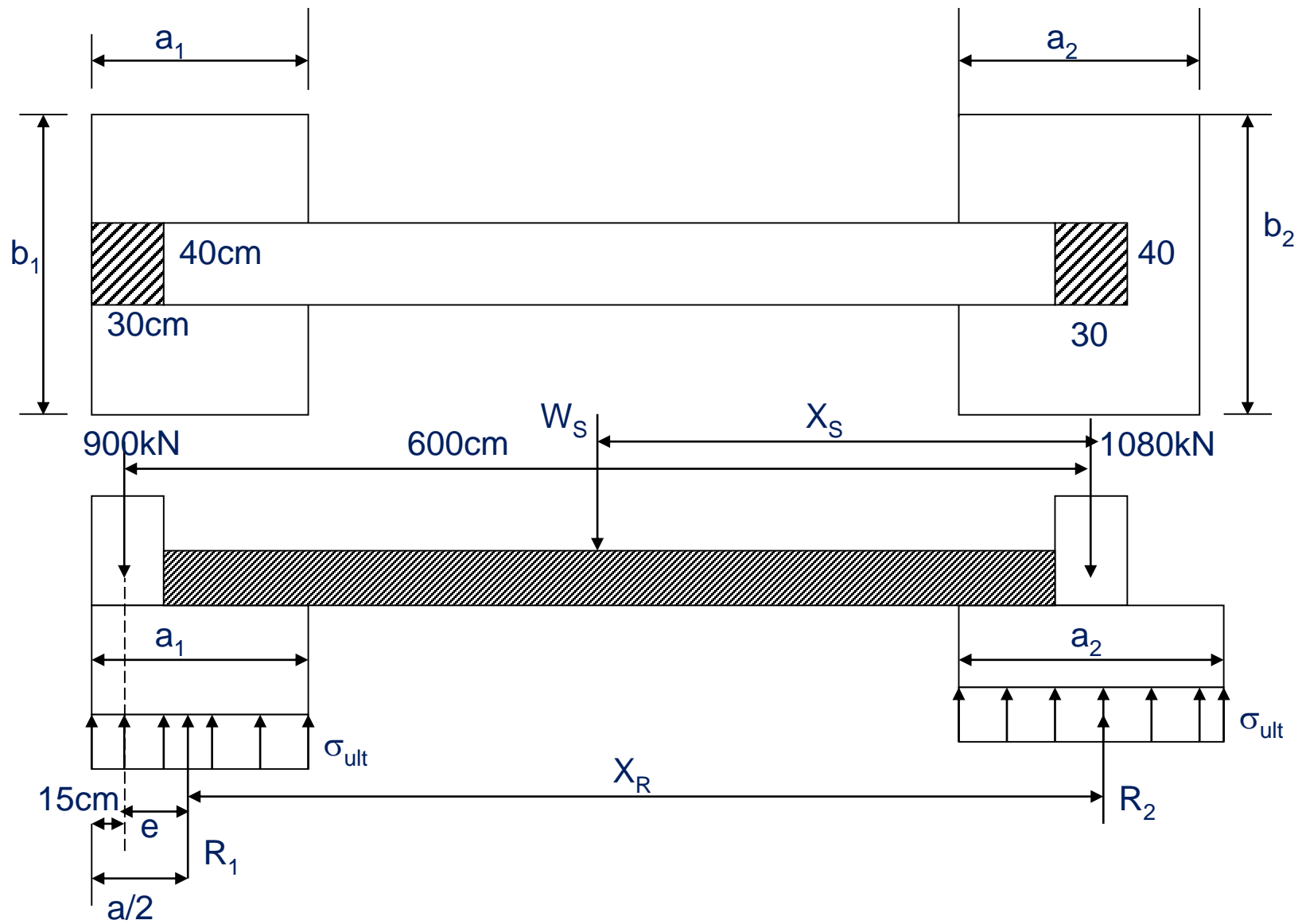
2. Determine sizes of footings using known values of R_1 , R_2 and σ_{ult}

$$b_1 = \frac{R_1}{\sigma_{ult1} * a_1}$$

$$b_2 = \frac{R_2}{\sigma_{ult2} * a_2}$$

- For square footing $b_2 = a_2 = \sqrt{\frac{R_2}{\sigma_{a2}}}$
- For rectangular footing assume some value of a_2 and determine b_2 .

- It should be noted that the actual bearing pressures under the footings should not vary different from each other in order to minimize differential settlement.
3. Determine and draw shear force and bending moment diagrams along the length of the footings.
 4. Select depths of footings for shear requirement.
 5. Select steel reinforcement for bending requirement.
 6. In short direction, the footings analyzed as spread footing subject to uniform soil pressure.
 7. Design strap as flexural member for the shear and moment obtained above.



SOLUTION

- Proportioning of footing
- Trial 1
- Assume $a_1 = 3.00\text{m}$
- Then $e = (3.00/2) - 0.15 = 1.35\text{m}$
- $X_R = 6.00 - e = 6.00 - 1.35 = 4.65\text{m}$
- Neglecting the weight of the strap, the soil reaction R_1 will be determined by taking moment about R_2 .
- $4.65 * R_1 = 6.00 * 900 \Rightarrow R_1 = 1161.30\text{kN}$
- $b_1 = R_1 / (a_1 \sigma_{ult}) = 1161.30 / (3 * 100) = 3.87$, use $b_1 = 3.90\text{m}$
- Hence dimension of footing 1 $3.00 \times 3.90\text{m}$

- Now for the second footing
- $R_2 = P_1 + P_2 - R_1 = 900 + 1080 - 1161.30 = 818.70 \text{ kN}$
- Area of footing = $R_2 / \sigma_{ult} = 818.70 / 100 = 8.19 \text{ m}^2$
- Take $a_1 = a_2 = 3.00 \text{ m}$
- $b_2 = 8.19 / 3 = 2.73$, use $b_2 = 2.75 \text{ m}$

- Dimension of footing 2 $3.00 \times 2.75 \text{ m}$

- $q_1 = R_1 / a_1 = 1161.30 / 3.00 = 387.1 \text{ kN/m}$
- $q_2 = R_2 / a_2 = 818.7 / 3.00 = 272.90 \text{ kN/m}$

- Since the distribution per meter run under the pad is different for each pad, a second trial should be made

- Trial 2

- Take $a_1 = 2.50\text{m}$

- Then $e = (2.50/2) - 0.15 = 1.10\text{m}$

- $X_R = 6.00 - e = 6.00 - 1.1 = 4.90\text{m}$

- $4.90 * R_1 = 6.00 * 900 \Rightarrow R_1 = 1102.04\text{kN}$

- $b_1 = R_1 / (a_1 \sigma_{ult}) = 1102.04 / (2.50 * 100) = 4.41$, use $b_1 = 4.45\text{m}$

- Dimension of footing 1 $2.50 \times 4.45\text{m}$

- $R_2 = P_1 + P_2 - R_1 = 900 + 1080 - 1102.04 = 877.96\text{kN}$

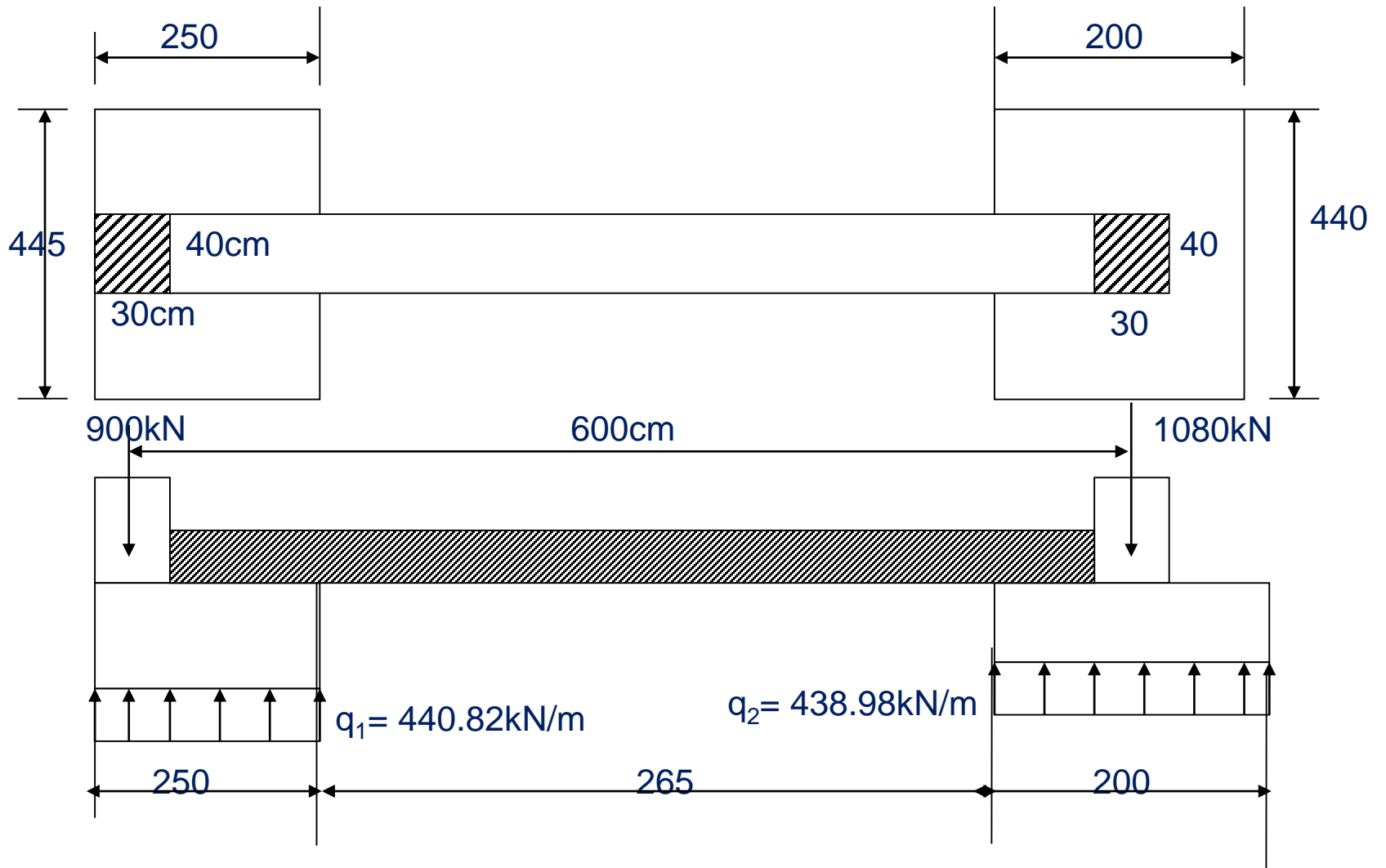
- Area of footing $= R_2 / \sigma_{ult} = 877.96 / 100 = 8.78\text{m}^2$

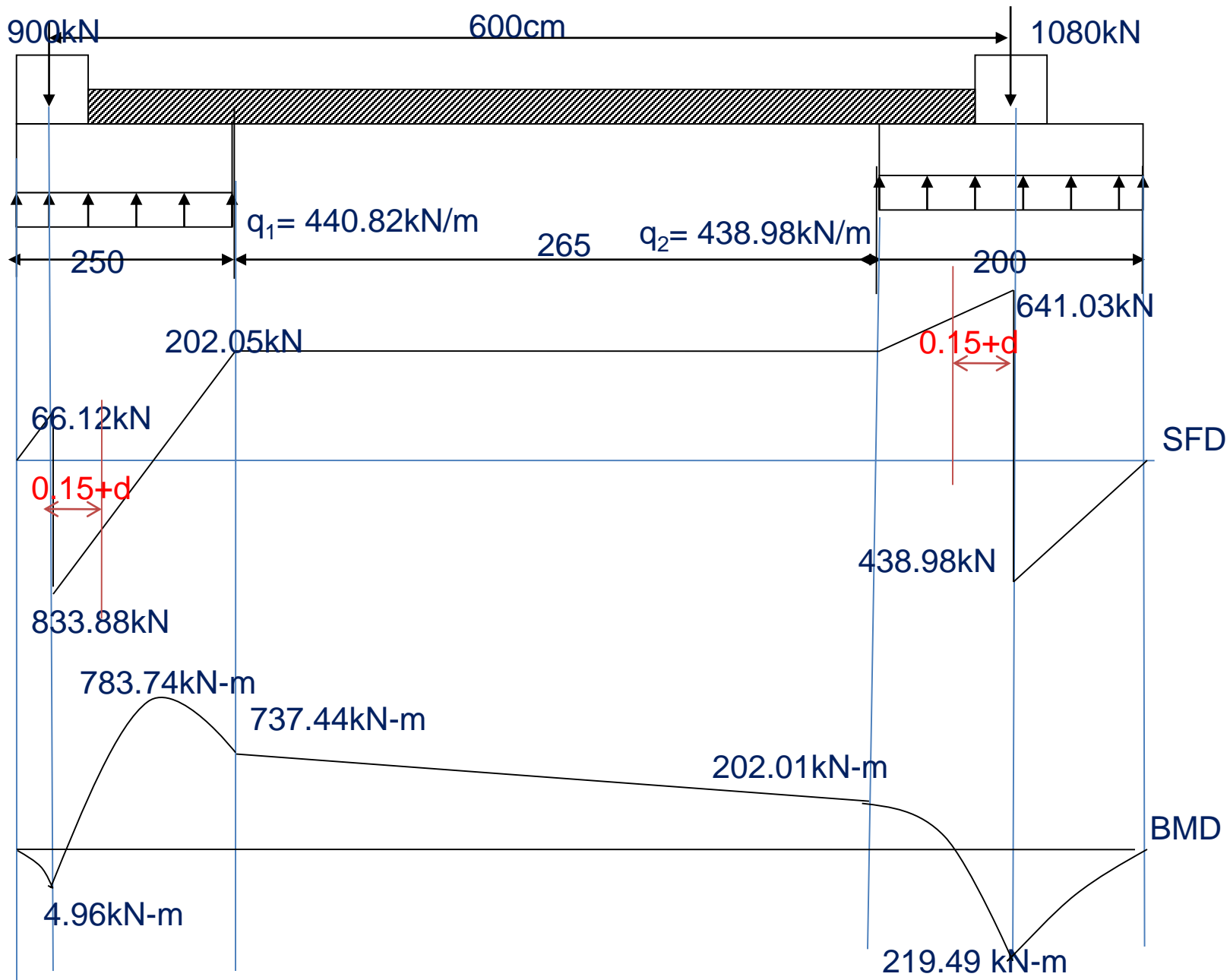
- Take $a_2 = 2.00\text{m}$ (or use $R_1/a_1 = R_2/a_2$ to determine a_2)

- $b_2 = 8.78 / 2.00 = 4.39$, use $b_2 = 4.40\text{m}$

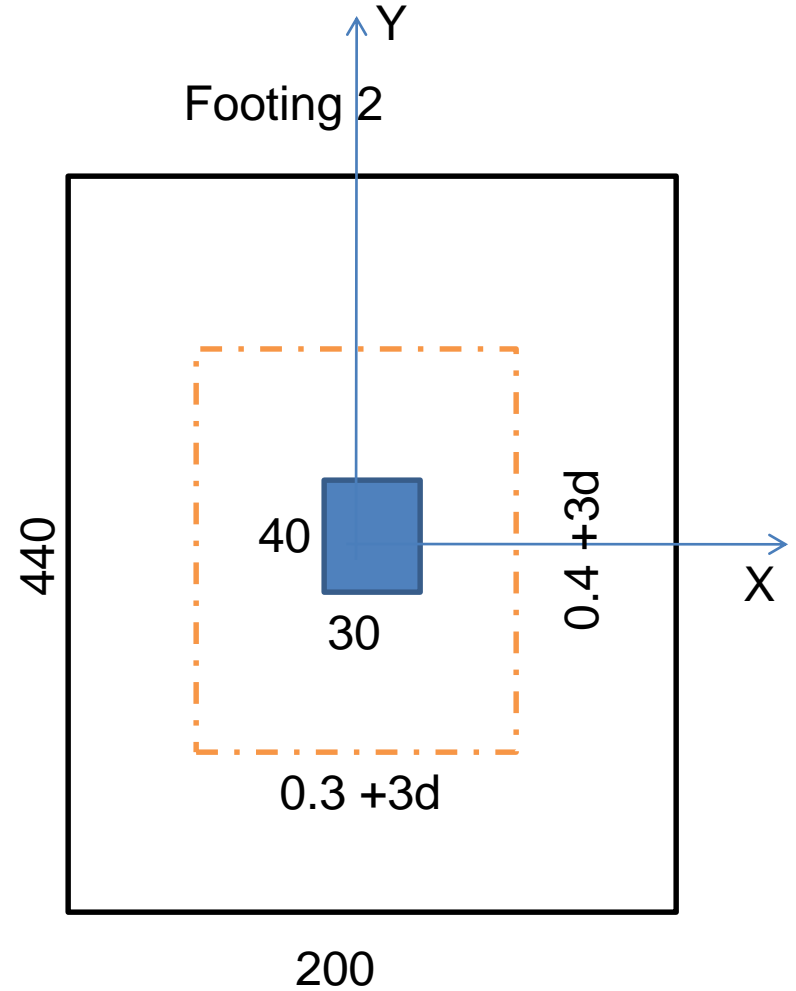
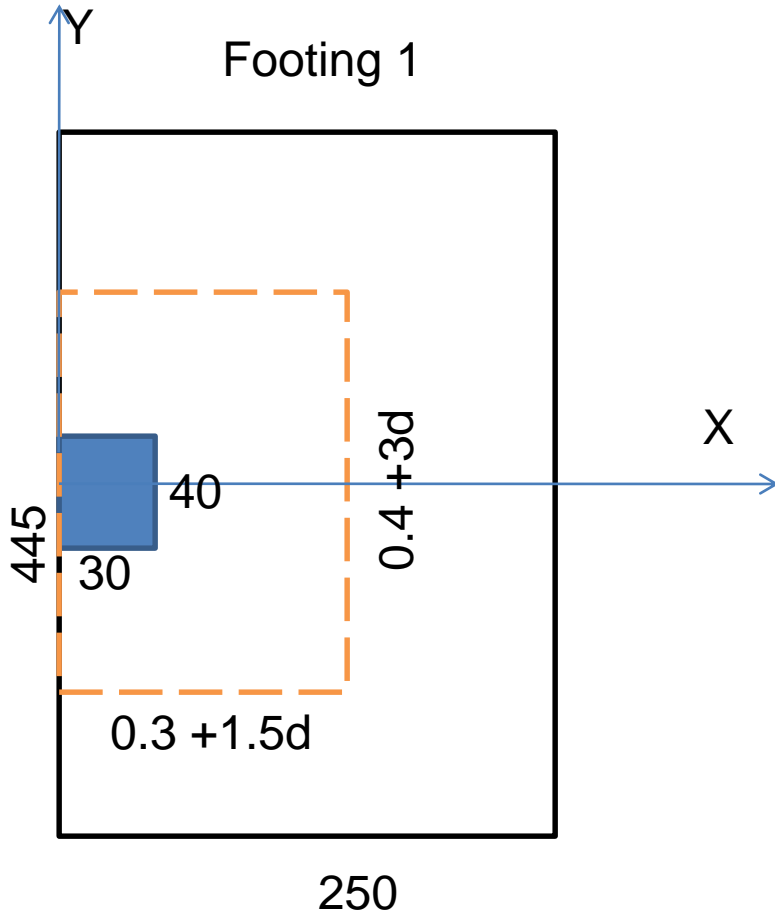
- Dimension of footing 2 $2.00 \times 4.40\text{m}$

- $q_1 = R_1/a_1 = 1102.04/2.50 = 440.82\text{kN/m}$
- $q_2 = R_2/a_2 = 877.96 / 2.00 = 438.98\text{kN/m}$
- The difference between stress distribution is a small amount. Hence the final proportions of the footings as shown below would be acceptable.





- Thickness of the footings



- Punching shear
- Footing 1

Take $d = 0.40\text{m}$ and $\rho = \rho_{\min} = 0.50/f_{yk} = 0.50 / 300 = 0.0017$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.4 = 1.20$$

$$u_1 = P_{r1} = 0.90 + 0.90 + 1.60 = 3.40\text{m}$$

- Net shear force developed under column 1

$$V_d = 900 - \sigma * (0.90 * 1.60), \quad \sigma = R_1 / a_1 * b_1 = 99.06\text{kPa}$$

$$V_d = 900 - 99.06 * (0.90 * 1.60) = 757.35\text{kN}$$

- Punching shear resistance under column 1

$$V_{up} = 0.25 f_{ctd} k_1 k_2 u d \quad (\text{MN})$$

$$V_{up} = 0.25 * 1000 * 1.085 * 1.20 * 3.40 * 0.40$$

$$= 442.68\text{kN} < V_d \text{ .. Not OK!}$$

- Since the developed shear force is greater than the punching shear resistance, one may increase the depth

Take $d = 0.55\text{m}$ and $\rho = \rho_{\min} = 0.50/f_{yk} = 0.50/300 = 0.0017$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.55 = 1.05$$

$$u_1 = P_{r1} = 1.125 + 1.125 + 2.05 = 4.30\text{m}$$

- Net shear force developed under column 1

$$V_d = 900 - 99.06 * (1.125 * 2.05) = 671.54\text{kN}$$

- Punching shear resistance under column 1

$$V_{up} = 0.25 * 1000 * 1.085 * 1.05 * 4.3 * 0.55 = 673.58\text{kN} > V_d \dots \text{OK!}$$

- Wide beam shear

- X- direction

- The magnitude of the wide beam shear is read from the shear force diagram at a distance of d from the face of column 1.

- $V_d = 440.82 (0.30 + d) - 900 = 440.82 (0.30 + 0.55) - 900$
 $= 525.30 \text{ kN}$

- Wide beam shear resistance

$$V_{ud} = 0.25 f_{ctd} k_1 k_2 b_w d \quad (\text{MN})$$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.60 - d = 1.60 - 0.55 = 1.05$$

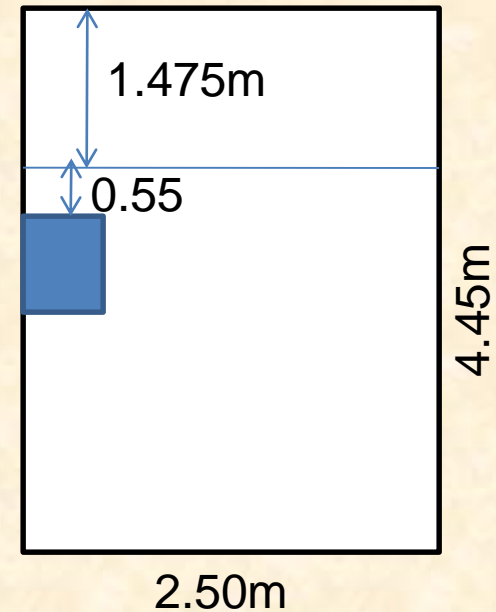
$$b_w = 4.45 \text{ m}$$

- $V_{ud} = 0.25 * 1000 * 1.085 * 1.05 * 4.45 * 0.55 = 697.08 \text{ kN} > V_d \text{ ..OK!}$

- **Y- direction**

- The developed shear force in direction Y may be estimated as for the case of an isolated footing

- Developed shear
- $V_d = 99.06(1.475 * 2.50) = 365.28 \text{ kN}$
- Wide beam shear resistance
- $V_{ud} = 0.25 * 1000 * 1.085 * 1.05 * 2.50 * 0.53$
 $= 377.38 \text{ kN} > V_d \dots\dots\dots \text{OK!}$



- Footing 2
- Punching shear

Take $d = 0.40 \text{ m}$ and $\rho = \rho_{\min} = 0.50 / f_{yk} = 0.50 / 300 = 0.0017$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.60 - d = 1.60 - 0.40 = 1.20$$

$$u_2 = P_{r2} = 1.50 + 1.50 + 1.60 + 1.60 = 6.20 \text{ m}$$

-

- Net shear force developed under column 2

$$V_d = 1080 - \sigma * (1.50 * 1.60), \quad \sigma = R_2 / a_2 * b_2 = 99.77 \text{ kPa}$$

$$V_d = 1080 - 99.77 * (1.50 * 1.60) = 840.55 \text{ kN}$$

- Punching shear resistance under column 2

$$V_{up} = 0.25 f_{ctd} k_1 k_2 u d \quad (\text{MN})$$

- $V_{up} = 0.25 * 1000 * 1.085 * 1.20 * 6.20 * 0.40$
 $= 807.24 \text{ kN} < V_d \text{ .. Not OK! , increase the depth.}$

- Take $d = 0.45 \text{ m}$ and $\rho = \rho_{min} = 0.50 / f_{yk} = 0.50 / 300 = 0.0017$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.60 - d = 1.60 - 0.45 = 1.15$$

$$u_2 = P_{r2} = 1.65 + 1.65 + 1.75 + 1.75 = 6.80 \text{ m}$$

- Net shear force developed under column 2

$$V_d = 1080 - 99.77 * (1.65 * 1.75) = 791.91 \text{ kN}$$

- Punching shear resistance under column 2

$$V_{up} = 0.25 * 1000 * 1.085 * 1.15 * 6.80 * 0.45 = 954.53 \text{ kN} > V_d \dots \text{OK!}$$

- Wide beam shear

- X- direction

- The wide beam shear is read from the shear force diagram at a distance of d from the face of column 2.

- $V_d = 438.98 (1.00 + 0.20 + 0.45) - 1080 = 355.68 \text{ kN}$

- Wide beam shear resistance

$$V_{ud} = 0.25 f_{ctd} k_1 k_2 b_w d \quad (\text{MN})$$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.60 - d = 1.60 - 0.45 = 1.15$$

$$b_w = 4.40\text{m}$$

- $V_{ud} = 0.25 * 1000 * 1.085 * 1.15 * 4.40 * 0.45 = 617.64 > V_d \dots \text{OK!}$

- Y- direction**

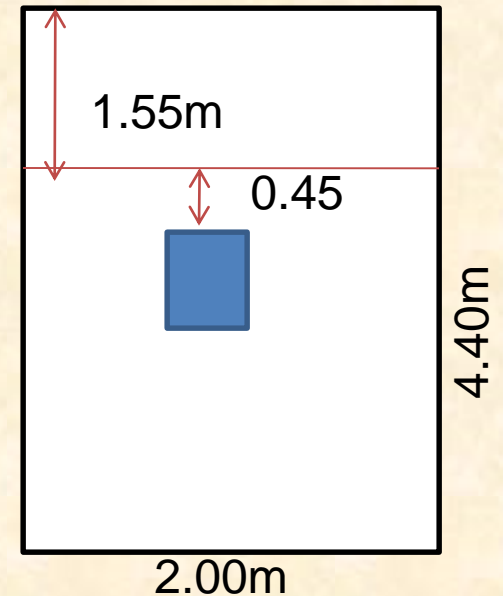
- Developed shear

- $V_d = 99.77(1.55 * 2.00) = 309.29\text{kN}$

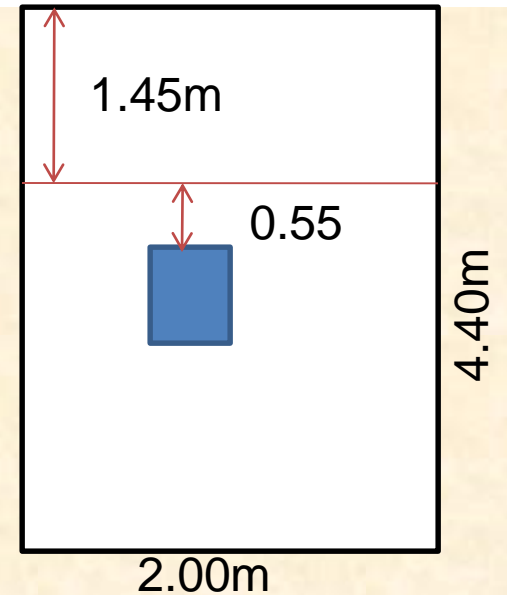
- Wide beam shear resistance

- $V_{ud} = 0.25 * 1000 * 1.085 * 1.15 * 2.00 * 0.43$
 $= 268.27\text{kN} < V_d \dots \dots \dots \text{NOT OK!}$

- Increase the effective depth d to 0.55m



- Developed shear
- $V_d = 99.77(1.45 * 2.00) = 289.33 \text{ kN}$
- Wide beam shear resistance
- $V_{ud} = 0.25 * 1000 * 1.085 * 1.05 * 2.00 * 0.53$
 $= 301.90 \text{ kN} > V_d \dots\dots\dots \text{OK!}$



- Moment capacity of concrete
- Footing 1

– X – direction , Capacity of concrete

$$M = 0.32 * f_{cd} * b d^2$$

$$= 0.32 * 11.33 \times 10^3 * 4.45 * (0.55)^2$$

$$= 4880.51 \text{ kN} - \text{m} \gg M_{\text{max}} = 783.74 \text{ kN} - \text{m} \dots \text{Ok!}$$

- Y- direction
- Developed moment

$$M_d = 99.06 * \frac{(2.025)^2}{2} * 2.5 = 507.76kN - m$$

- Capacity of concrete

$$\begin{aligned} M &= 0.32 * f_{cd} * b d^2 \\ &= 0.32 * 11.33 \times 10^3 * 2.50 * (0.53)^2 \\ &= 2546.08kN - m \gg M_d \dots Ok! \end{aligned}$$

- Footing 2

– X – direction, Capacity of concrete

$$M = 0.32 * f_{cd} * b d^2$$

$$= 0.32 * 11.33 \times 10^3 * 4.40 * (0.55)^2$$

$$= 4825.67 kN - m \gg M_{max} = 219.49 kN - m \dots \text{Ok!}$$

– Y-direction

$$M_d = 99.77 * \frac{(2.00)^2}{2} * 2.0 = 399.08 kN - m$$

– Capacity of concrete

$$M = 0.32 * 11.33 \times 10^3 * 2.0 * (0.53)^2$$

$$= 2036.86 kN - m \gg M_d \dots \dots \dots \text{OK!}$$

- Calculation of Reinforcements

- Footing 1

- X-direction

- From BMD , M = 783.74kN-m

$$\rho = \frac{f_{cd}}{f_{yd}} \left[1 - \sqrt{1 - \frac{2M}{f_{cd} b d^2}} \right]$$
$$= \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 783.74}{11.33 \times 10^3 * 4.45 * (0.55)^2}} \right] = 0.0023 > \rho_{\min} \dots \text{ok!}$$

$$A_s = \rho b d = 0.0023 * 445 * 55 = 56.29 \text{cm}^2$$

- Use $\phi = 20$, $a_s = 3.14 \text{cm}^2$

- No. of bars , $n = 56.29 / 3.14 = 17.9$, use 18 bars

- Spacing = $(445 - 2 * 5) / (n - 1) = 25.6 \text{cm}$

Use 18 ϕ 20 c/c 255mm (The reinforcement should be placed at the top)

- Provide also minimum bottom reinforcement in the X-direction
- $A_{s_{\min}} = \rho_{\min} bd = 0.0017 * 445 * 55 = 41.61\text{cm}^2$
- Use $\phi = 14$, $a_s = 1.54\text{cm}^2$
 No. of bars , $n = 41.61/1.54 = 27$
 Spacing = $(445 - 2*5) / (n - 1) = 16.73\text{cm}$
- Use 27 $\phi 14$ c/c 165mm
- Y-direction
- $M_d = 507.76\text{kN-m}$

$$\rho = \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 507.76}{11.33 \times 10^3 * 2.50 * (0.53)^2}} \right] = 0.0029 > \rho_{\min} \dots \text{ok!}$$

$$A_s = \rho b d = 0.0029 * 250 * 53 = 38.425\text{cm}^2$$

- Use $\phi = 20$, $a_s = 3.14\text{cm}^2$
 No. of bars , $n = 38.425/3.14 = 12.2$, use 13 bars
 Spacing = $(250 - 2 \cdot 5) / (n - 1) = 20\text{cm}$
 Use 13 ϕ 20 c/c 200mm (bottom reinforcement)
- Provide also minimum top reinforcement in the Y-direction
- $A_{s_{\min}} = \rho_{\min} bd = 0.0017 * 250 * 53 = 22.53\text{cm}^2$
- Use $\phi = 14$, $a_s = 1.54\text{cm}^2$
 No. of bars , $n = 22.53/1.54 = 14.6$, use 15 bars
 Spacing = $(250 - 2 \cdot 5) / (n - 1) = 17.1\text{cm}$
 Use 15 ϕ 14 c/c 170mm

– Footing 2

– X-direction

– From BMD , $M = 219.49$ kN-m

$$\rho = \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 219.49}{11.33 \times 10^3 * 4.40 * (0.55)^2}} \right] = 0.0006 < \rho_{\min}$$

$$A_s = \rho_{\min} b d = 0.0017 * 440 * 55 = 41.14 \text{ cm}^2$$

- Use $\phi = 20$, $a_s = 3.14 \text{ cm}^2$

No. of bars , $n = 41.14 / 3.14 = 13.20$, use 14 bars

Spacing = $(440 - 2 * 5) / (n - 1) = 33.08 \text{ cm}$

Use 14 ϕ 20 c/c 330mm (The reinforcement should be placed at the bottom)

- Y-direction
- $M_d = 399.08 \text{ kN-m}$

$$\rho = \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 399.08}{11.33 \times 10^3 * 2.00 * (0.53)^2}} \right] = 0.0028 > \rho_{\min} \dots \text{ok!}$$

$$A_s = \rho b d = 0.0028 * 200 * 53 = 29.68 \text{ cm}^2$$

- Use $\phi = 20$, $a_s = 3.14 \text{ cm}^2$
 No. of bars , $n = 29.68 / 3.14 = 9.5$, use 10 bars
 Spacing = $(250 - 2 * 5) / (n - 1) = 26.67 \text{ cm}$
 Use 10 ϕ 20 c/c 265mm (bottom reinforcement)

- Development length
- Footing 1
 - X-direction

$$l_d = \frac{\phi f_{yd}}{4 f_{bd}} = \frac{2 * 260.87}{4 * 1} = 130.44cm$$

- Available development length, $l_a = 220 - 5 = 215cm > l_d$ OK!

- Y-direction

$$l_d = \frac{\phi f_{yd}}{4 f_{bd}} = \frac{2 * 260.87}{4 * 1} = 130.44cm$$

- Available development length, $l_a = 202.5 - 5 = 197.5cm > l_d$ OK!

- Footing 2

- X-direction

$$l_d = \frac{\phi f_{yd}}{4 f_{bd}} = \frac{2 * 260.87}{4 * 1} = 130.44cm$$

- Available development length, $l_a = 80 - 5 = 75cm < l_d \dots\dots$ NOT OK!
- Bend the bars upward with a minimum length of $\dots\dots cm$

- Y-direction

$$l_d = \frac{\phi f_{yd}}{4 f_{bd}} = \frac{2 * 260.87}{4 * 1} = 130.44cm$$

- Available development length, $l_a = 200 - 5 = 195cm > l_d \dots\dots$ OK!

Design of Mat/Raft Foundation

- Mat or raft foundation is a large concrete slab supporting several columns in two or more rows.
- It is used where the supporting soil has low bearing capacity.
- The bearing capacity increased by combining all individual footings in to one mat –since bearing capacity is proportional to width and depth of foundations.
- In addition to increasing the bearing capacity, mat foundations tend to bridge over irregularities of the soil and the average settlement does not approach the extreme values of isolated footings.
- Thus mat foundations are often used for supporting structures that are sensitive to differential settlement.

- **Design of uniform mat**
- Design Assumptions
 - mat is infinitely rigid
 - planner soil pressure distribution under mat
- **Design Procedure**
 - I. Determine the line of action of the resultant of all the loads acting on the mat
 - II. Determine the contact pressure distribution as under
 - If the resultant passes through the center of gravity of the mat, the contact pressure is given by

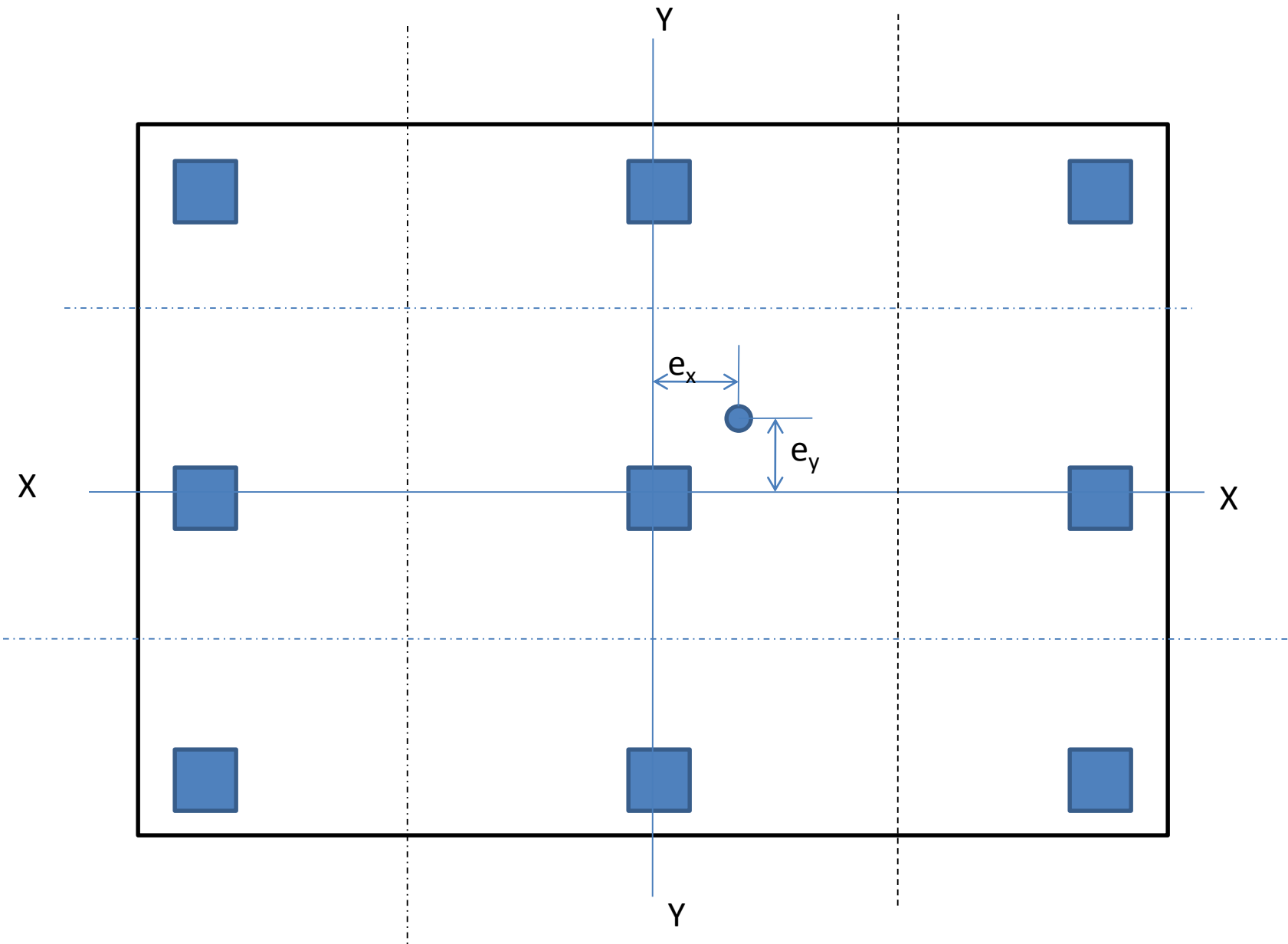
$$\sigma = \frac{Q}{A}$$

- If the resultant has an eccentricity of e_x and e_y in the x and y direction

$$\sigma_{\max/\min} = \frac{Q}{A} \pm \frac{Qe_x}{I_{yy}} x \pm \frac{Qe_y}{I_{xx}} y$$

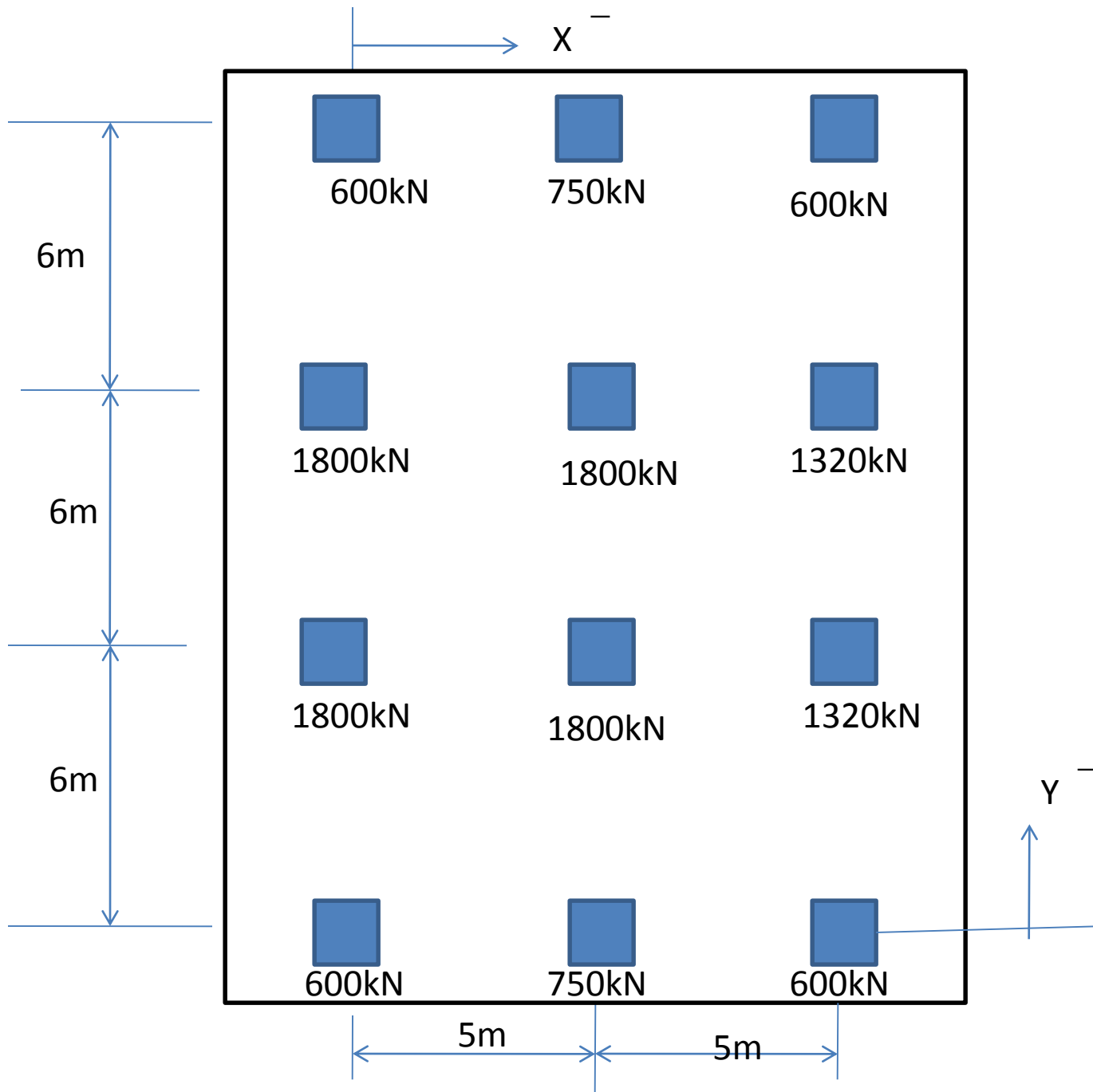
The maximum contact pressure should be less than the allowable soil pressure

- Divide the slab mat into strips in x and y directions. **Each strip is assumed to act as independent beam subjected to the contact pressure and the columns loads.**
- Determine the modified column loads
- Draw the shear force and bending moment diagrams for each strip.
- Select depth of mat for shear requirement
- Select steel reinforcement for moment requirement



Example

- A mat foundation is to be design by the conventional method (rigid method) for the loadings shown in Fig. below.
 - All columns are 40X40cm
 - Ultimate soil bearing pressure , $q_{ult} = 100\text{kPa}$
 - $f_{yk} = 300\text{MPa} \Rightarrow f_{yd} = 300/1.15 = 260.87 \text{ Mpa}$
 - C25 $\Rightarrow f_{ck} = 20\text{MPa} \Rightarrow f_{ctk} = 1.5 \text{ MPa}$,



- **Location of c.g. of loads**

- $\sum P = (600 + 750 + 600) * 2 + (1800 + 1800 + 1320) * 2 = 13740 \text{ kN}$

- $13740 \bar{X} = (750 + 1800 + 1800 + 750) * 5 + (600 + 1320 + 1320 + 600) * 10$

$$\bar{X} = 4.65 \text{ m}$$

$$e_x = 5 - 4.65 = 0.35$$

$$X' = 5 + 0.35 = 5.35 \text{ m}$$

- $B_{\min} = 2 * (5.35 + 0.20 + 0.15) = 11.40 \text{ m}$

- $13740 \bar{Y} = (600 + 750 + 600) * 18 + (1800 + 1800 + 1320) * 12 + (1800 + 1800 + 1320) * 6$

$$\bar{Y} = 9 \text{ m}$$

$$e_y = 6 + 6/2 - 9 = 0$$

- $L_{\min} = 2 * (9 + 0.20 + 0.15) = 18.70 \text{ m}$

- **Dimension of Mat 11.40 X 18.70m**

-

- Actual contact pressure

$$\sigma = \Sigma P / (BL) = 13740 / (11.40 * 18.70) = 64.45 \text{ kPa} < \sigma_{ult} = 100 \text{ kPa}$$

- Thickness of the mat

- Punching shear

- Punching shear under 1800kN load

Take $d = 0.70 \text{ m}$ and $\rho = \rho_{min} = 0.50 / f_{yk} = 0.50 / 30$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.70 = 0.90, \text{ Take } K_2 = 1$$

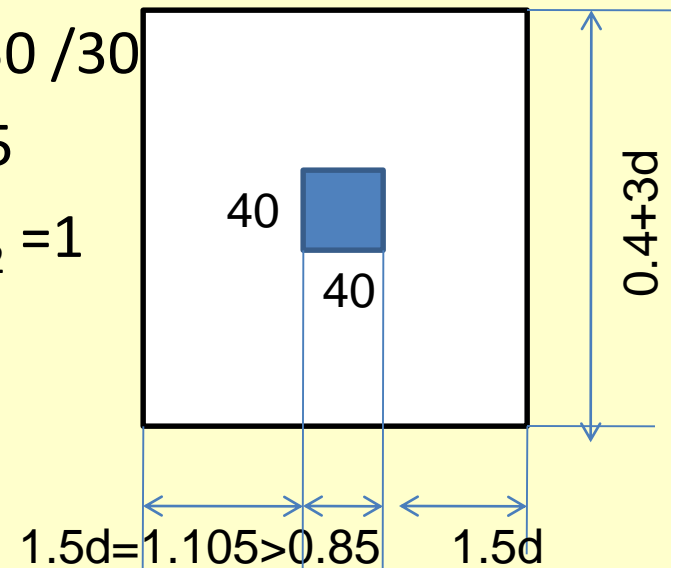
$$P_r = (0.85 + 0.4 + 1.105)2 + (0.4 + 3(0.70))$$

$$= 7.21 \text{ m}$$

- Net shear force developed

- $V_d = 1800 - \sigma * (2.355 * 2.50), \sigma = 64.45 \text{ kPa}$

- $V_d = 1800 - 64.45 * (2.355 * 2.50) = 1420.55 \text{ kN}$



- Punching shear resistance

$$V_{up} = 0.25f_{ctd} k_1 k_2 u d \quad (\text{MN})$$

- $V_{up} = 0.25 * 1000 * 1.085 * 1.00 * 7.21 * 0.70$
 $= 1369.00 \text{ kN} < V_d \dots$ **NOT OK! Increase the depth**

Take $d = 0.75 \text{ m}$ and $\rho = \rho_{min} = 0.50 / f_{yk} = 0.50 / 300 = 0.0017$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.75 = 0.85, \text{ Take } K_2 = 1$$

$$P_r = (0.85 + 0.4 + 1.125)2 + (0.4 + 3(0.75))$$

$$= 7.40 \text{ m}$$

- Net shear force developed
- $V_d = 1800 - \sigma * (2.375 * 2.65), \sigma = 64.45 \text{ kPa}$
- $V_d = 1800 - 64.45 * (2.375 * 2.65) = 1394.37 \text{ kN}$

- Punching shear resistance

$$V_{up} = 0.25f_{ctd} k_1 k_2 u d \quad (\text{MN})$$

- $V_{up} = 0.25 * 1000 * 1.085 * 1.00 * 7.40 * 0.75$
 $= 1505.44 \text{ kN} > V_d \dots \text{OK!}$

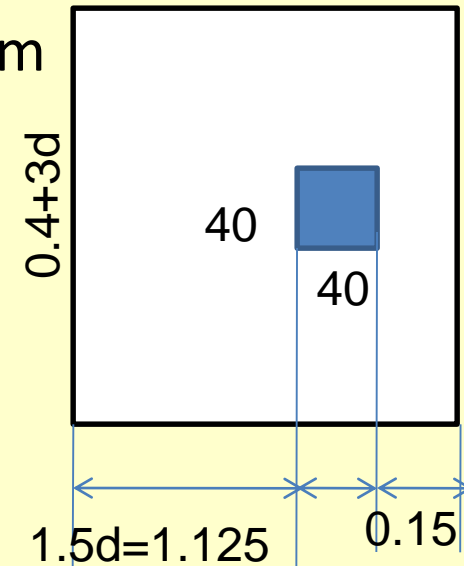
- Check punching shear under 1320kN

$$P_r = (1.125 + 0.15 + 0.4)2 + (0.4 + 3(0.75)) = 6.00 \text{ m}$$

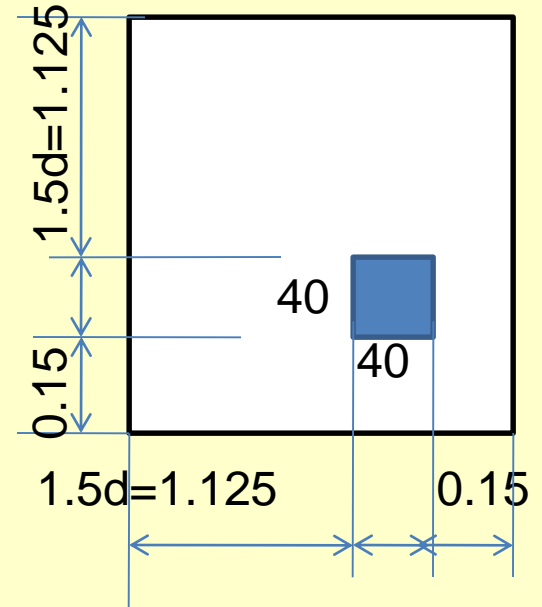
- Net shear force developed
- $V_d = 1320 - 64.45 * (1.675 * 2.65) = 1033.92 \text{ kN}$
- Punching shear resistance

$$V_{up} = 0.25f_{ctd} k_1 k_2 u d \quad (\text{MN})$$

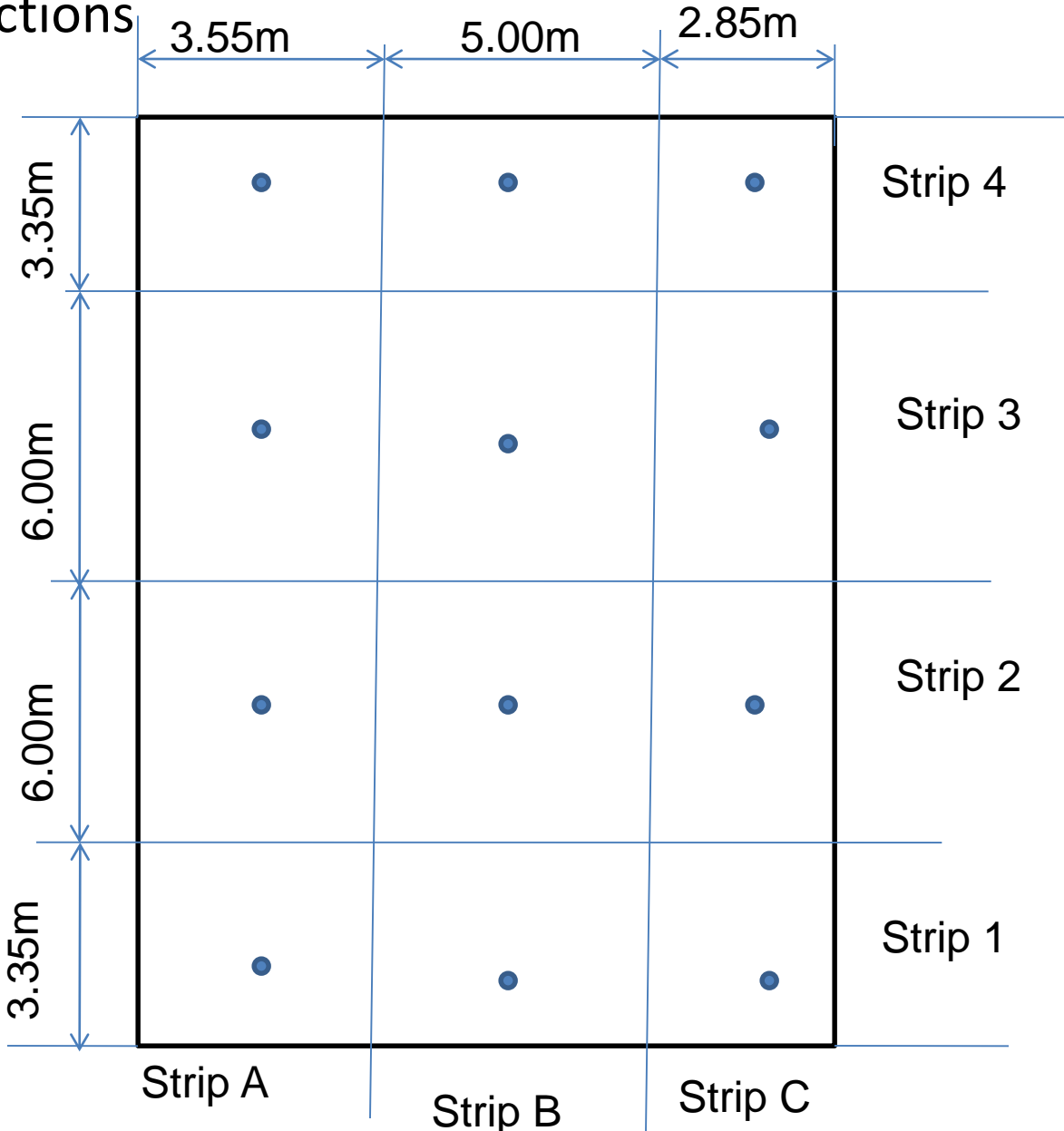
- $V_{up} = 0.25 * 1000 * 1.085 * 1.00 * 6.00 * 0.75$
 $= 1220.63 \text{ kN} > V_d \dots \text{OK!}$



- Check punching shear under 600kN
- $P_r = (1.125 + 0.15 + 0.4) + (1.125 + 0.15 + 0.4)$
 $= 3.35\text{m}$
- Net shear force developed
- $V_d = 600 - 64.45 * (1.675 * 1.675) = 419.18\text{kN}$
- Punching shear resistance
 $V_{up} = 0.25 f_{ctd} k_1 k_2 u d \quad (\text{MN})$
- $V_{up} = 0.25 * 1000 * 1.085 * 1.00 * 3.35 * 0.75$
 $= 681.52\text{kN} > V_d \dots \text{OK!}$

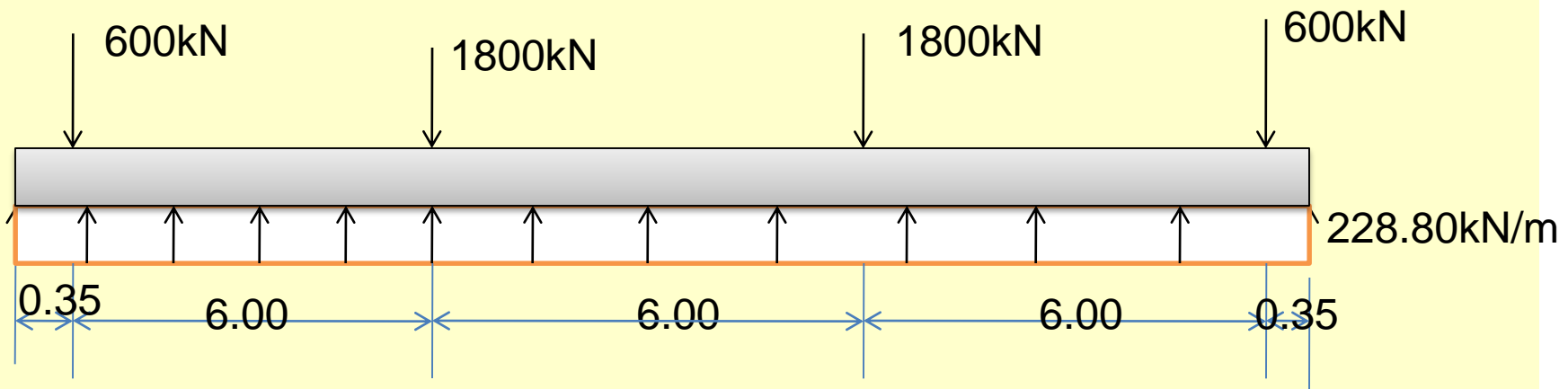


- **Soil reaction analysis:-** Divide the slab mat into strips in x and y directions



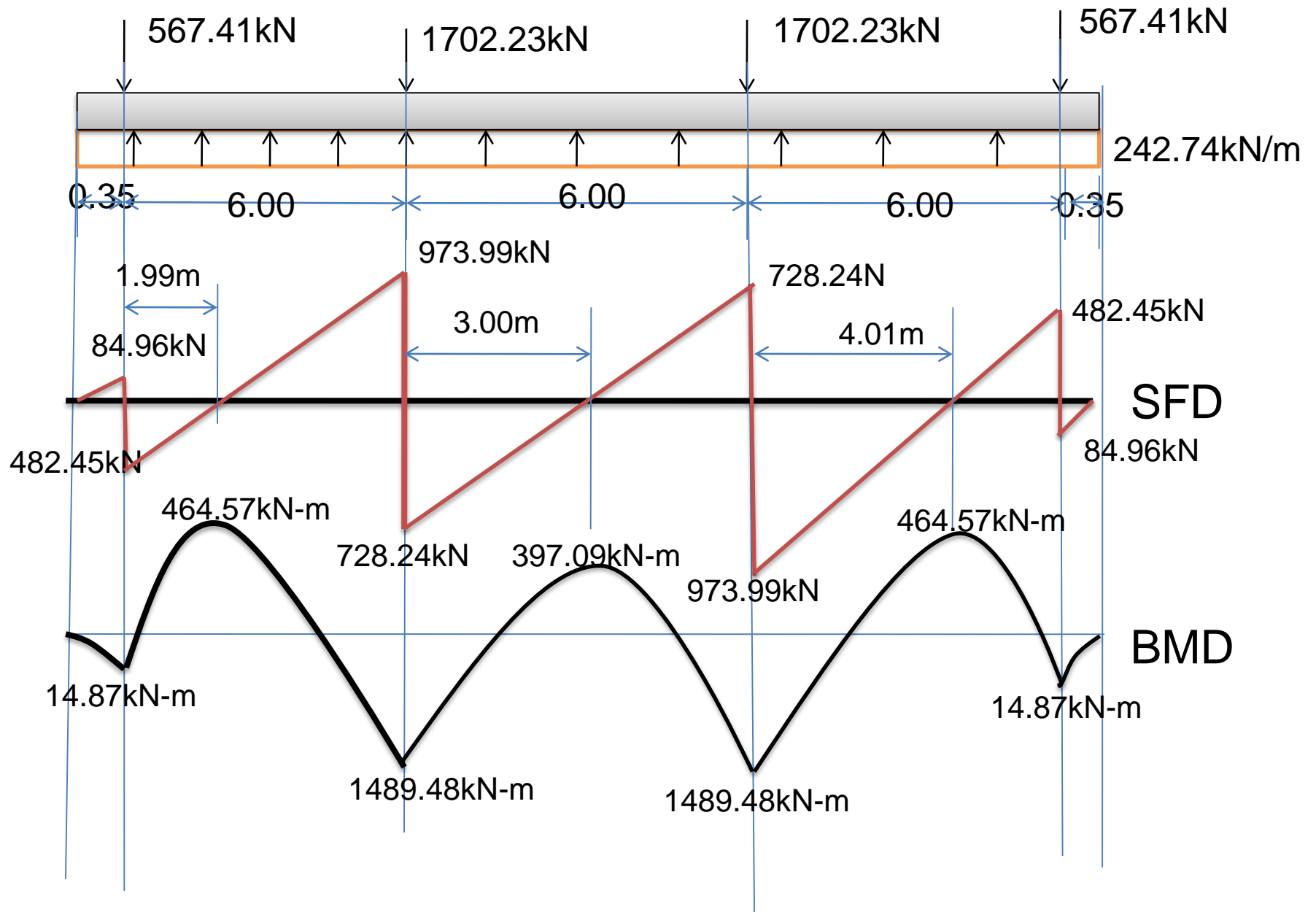
- Strip A, $(64.45) * 3.55 = 228.80\text{kN/m}$
- Strip B, $(64.45) * 5.00 = 322.25\text{kN/m}$
- Strip C, $(64.45) * 2.85 = 183.68\text{kN/m}$
- Strip 1 & Strip 4, $(64.45) * 3.35 = 215.91\text{kN/m}$
- Strip 2 & Strip 3 $(64.45) * 6.00 = 386.70\text{kN/m}$

- Shear force and Bending moment diagrams for each strip
- **Strip A**

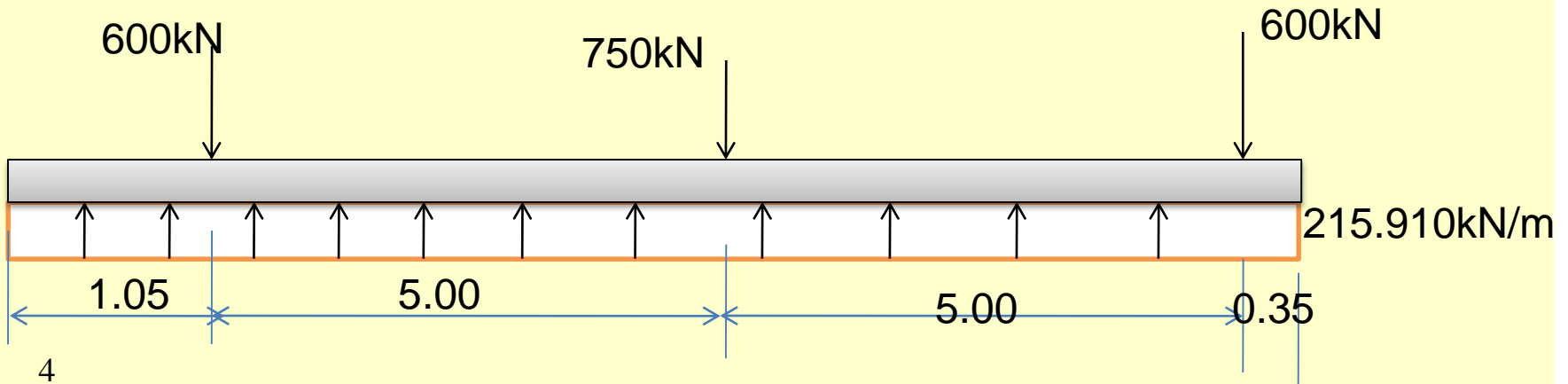


$$\sum_{i=1}^4 P_i = 600 + 1800 + 1800 + 600 = 4800 \text{ kN}$$

- $\Sigma R = 228.80 * 18.70 = 4278.56 \text{ kN}$
- $\Sigma V = \Sigma P - \Sigma R = 4800 - 4278.56 = 521.44 \neq 0$
- Hence take average of ΣP and ΣR
- I.e., $(4800 + 4278.56) / 2 = 4539.28 \text{ kN}$
- $\sigma_{\text{avg}} = (4539.28) / 18.70 = 242.74 \text{ kN/m}$
- $P_{1\text{avg}} = P_{4\text{avg}} = (4539.28 / 4800) * 600 = 567.41 \text{ kN}$
- $P_{2\text{avg}} = P_{3\text{avg}} = (4539.28 / 4800) * 1800 = 1702.23 \text{ kN}$

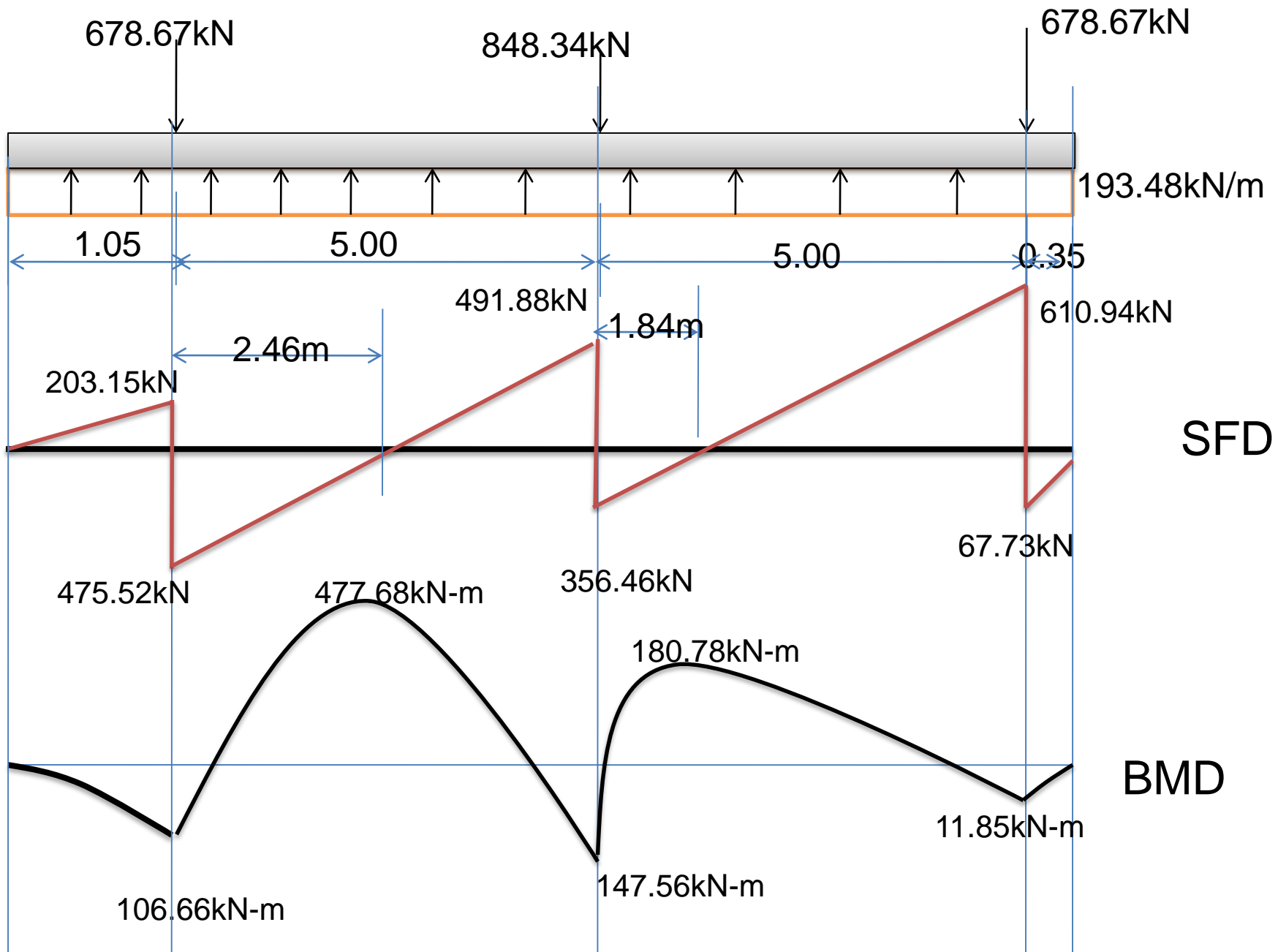


- Strip 1 & Strip 4, $(64.45) * 3.35 = 215.91 \text{ kN/m}$



$$\sum_{i=1}^4 P_i = 600 + 750 + 600 = 1950 \text{ kN}$$

- $\Sigma R = 215.91 * 11.40 = 2461.37 \text{ kN}$
- $\Sigma V = \Sigma P - \Sigma R = 1950 - 2461.37 = -511.37 \neq 0$
- Hence take average of ΣP and ΣR
- i.e., $(1950 + 2461.37) / 2 = 2205.69 \text{ kN}$
- $\sigma_{\text{avg}} = (2205.69) / 11.40 = 193.48 \text{ kN/m}$
- $P_{1\text{avg}} = P_{3\text{avg}} = (2205.69 / 1950) * 600 = 678.67 \text{ kN}$
- $P_{2\text{avg}} = (2205.69 / 1950) * 750 = 848.34 \text{ kN}$



Design of rectangular combined footing

- Example

- Given Column 1 size 30x 30

Reinf. 4 ϕ 22

- Column 2 size 40x 40cm

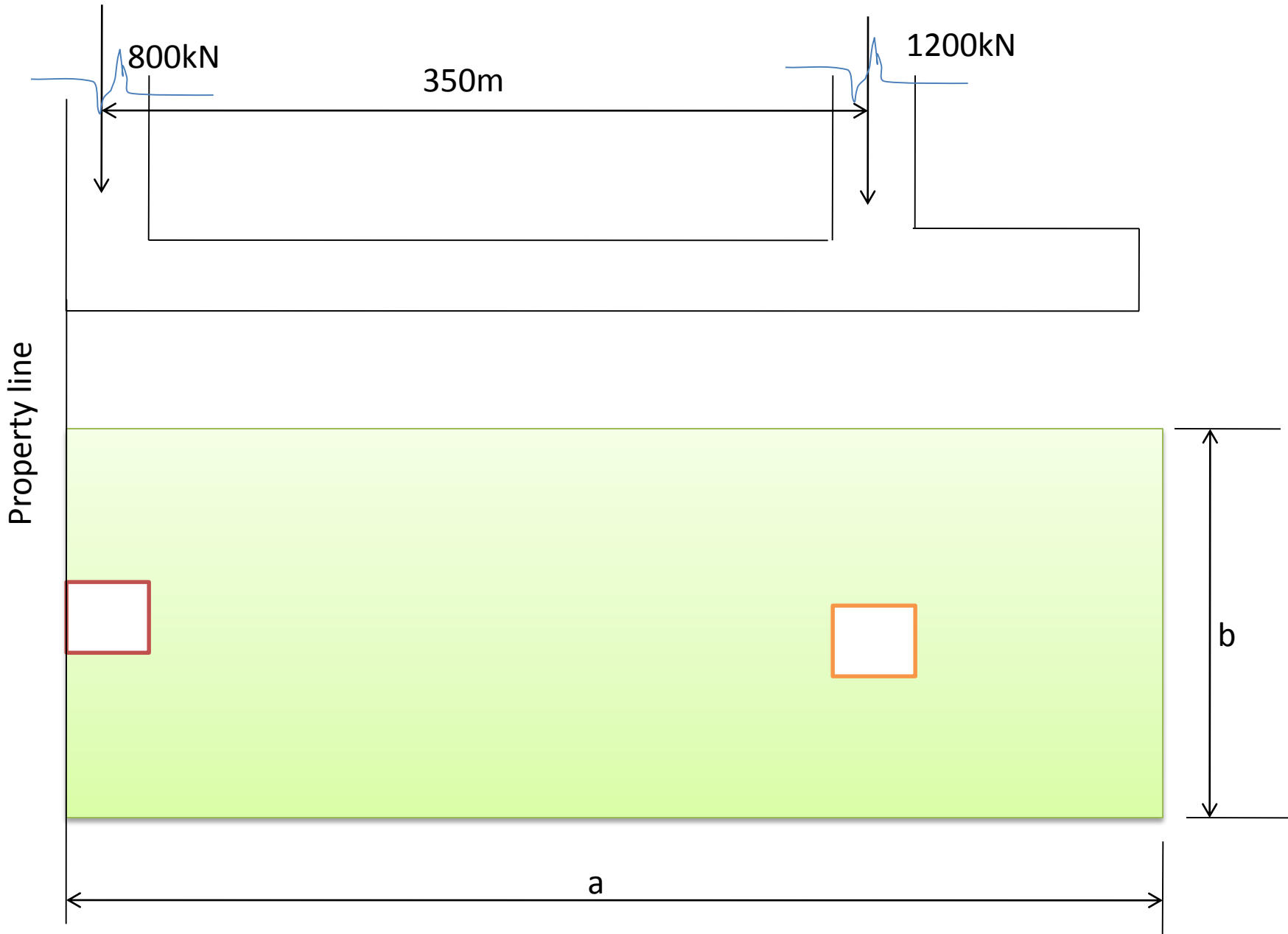
Reinf. 4 ϕ 24

Ultimate soil bearing pressure , $q_{ult} = 150\text{kPa}$

- $f_{yk} = 300\text{MPa} \Rightarrow f_{yd} = 300/1.15 = 260.87 \text{ MPa}$

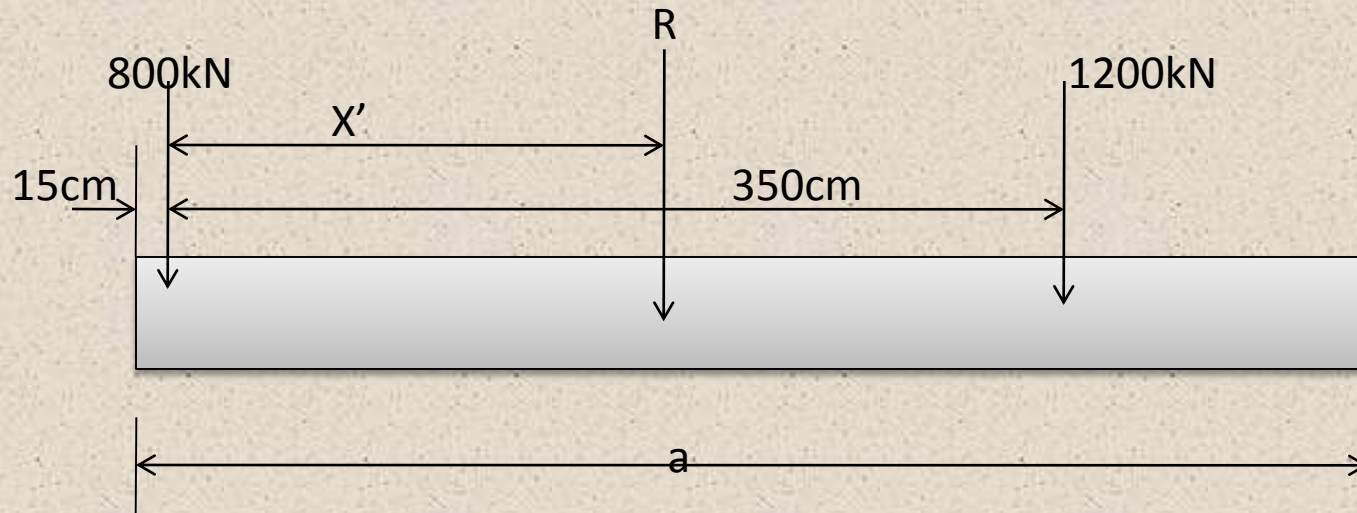
- C25 $\Rightarrow f_{ck} = 20\text{MPa} \Rightarrow f_{ctk} = 1.5 \text{ MPa},$

- Required:- Design of rectangular combined footing



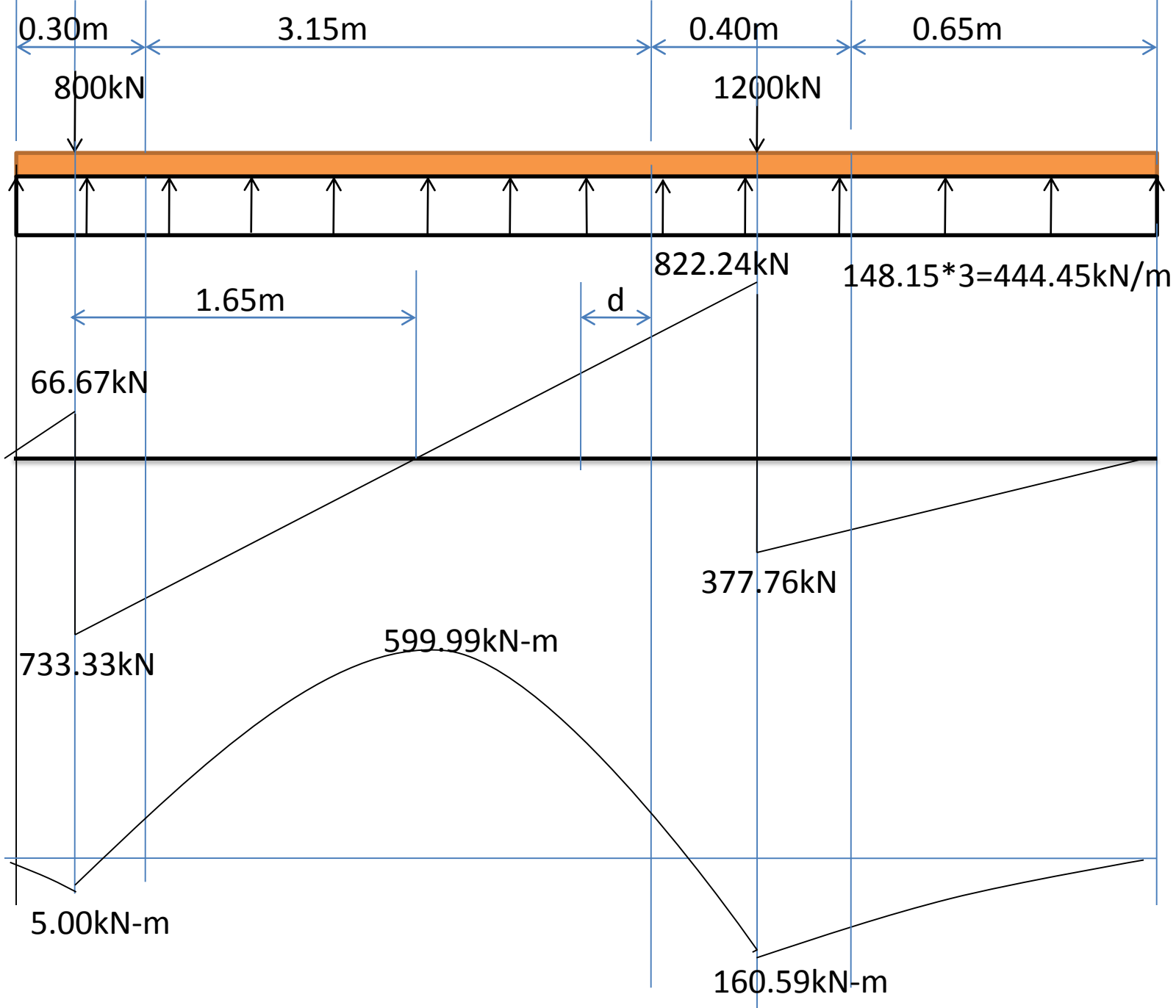
SOLUTION

- Proportioning of footing



- $R = 800 + 1200 = 2000\text{kN}$
- $RX' = 1200 * 350 \Rightarrow X' = (1200 * 350) / 2000 = 210\text{cm}$
- $a = 2 (X' + 15) = 450 \text{ cm}$

- $q_{ult} = R/A = 2000 / (4.50 * b)$
- $150 = 2000 / (4.50 * b) \Rightarrow b = 2.96\text{m}$
- Take $b = 3.00\text{m}$
- **Actual contact pressure**
- $\sigma = R/ab = 2000 / (4.50 * 3.00) = 148.15\text{kPa} < q_{ult} \dots\dots\dots \text{ok!}$
- **Shear force and bending moment diagrams**



- Thickness of the footing
- Wide beam shear
- The magnitude of the wide beam shear is read off from the shear force diagram at a distance of d from the face of the column.
- $V = 444.45 (3.45 - d) - 800 = 733.35 - 444.45d$
- Take $d = 0.60\text{m}$ and $\rho = \rho_{\min} = 0.5/f_{yk} = 0.5 / 300 = 0.0017$
- Then, $V = 466.68\text{kN}$
- The wide beam shear resistance according to EBCS-2 is given by

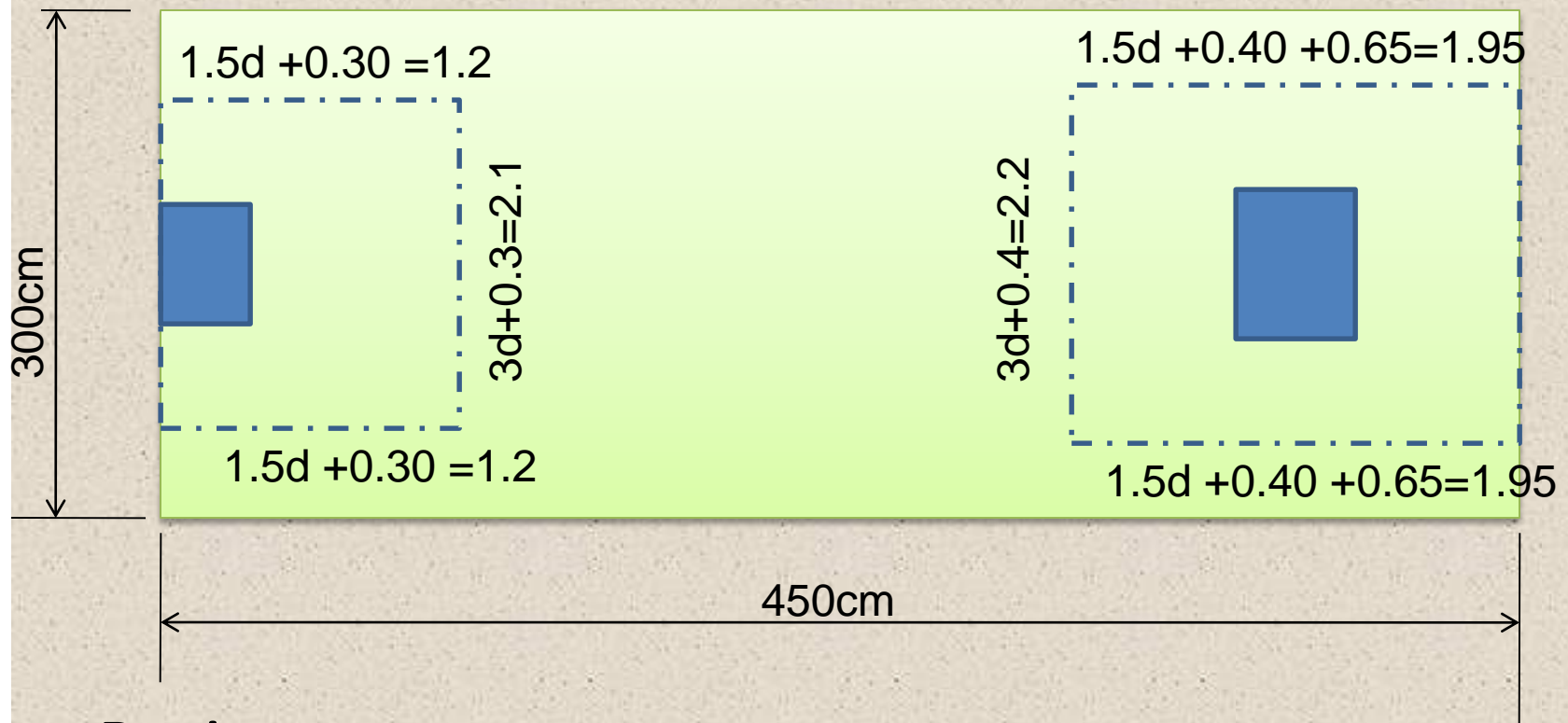
$$V_{ud} = 0.25f_{ctd} k_1 k_2 b_w d \quad (\text{MN})$$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.6 = 1$$

$$V_{ud} = 0.25 * 1 * 1.085 * 1 * 3.0 * 0.60 = 0.488\text{MN} = 488\text{kN} > V \dots \text{OK!}$$

- Punching shear



- Perimeters $P_{r1} = 1.2 + 1.2 + 2.1 = 4.5\text{m}$
 $P_{r2} = 1.95 + 1.95 + 2.2 = 6.1\text{m}$

- Net shear force developed under column 1

$$V_1 = 800 - 148.15 * (1.2 * 2.1) = 426.66 \text{ kN}$$

- Net shear force developed under column 2

$$V_2 = 1200 - 148.15 * (1.95 * 2.2) = 564.43 \text{ kN}$$

- Punching shear resistance

$$V_{up} = 0.25 f_{ctd} k_1 k_2 u d \quad (\text{MN})$$

$$d = 0.60 \text{ m and } \rho = \rho_{\min} = 0.5 / f_{yk} = 0.5 / 300 = 0.0017$$

$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.6 = 1$$

$$u_1 = P_{r1} = 4.5 \text{ m}, u_2 = P_{r1} = 6.1 \text{ m}$$

- Punching shear resistance under column 1

- $V_{up} = 0.25 * 1000 * 1.085 * 1 * 4.5 * 0.6 = 732.38 \text{ kN} > V_1 \dots \text{ok!}$

- Punching shear resistance under column 2
- $V_{up} = 0.25 * 1000 * 1.085 * 1 * 6.1 * 0.6 = 992.78 \text{ kN} > V_2 \dots \text{ok!}$
- **Moment capacity of concrete**

$$\begin{aligned}
 M &= 0.32 * f_{cd} * b d^2 \\
 &= 0.32 * 11.33 \times 10^3 * 3.00 * (0.6)^2 \\
 &= 3915.65 \text{ kN-m} \gg M_{\max} \dots \text{Ok!}
 \end{aligned}$$

- **Calculation of reinforcement**
 - Long direction

The reinforcement shall be calculated for the maximum moment

$$M_{\max} = 599.99 \text{ kN-m}$$

$$\rho = \frac{f_{cd}}{f_{yd}} \left[1 - \sqrt{1 - \frac{2M}{f_{cd} b d^2}} \right]$$

$$= \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 599.99}{11.33 \times 10^3 * 3.0 * (0.6)^2}} \right] = 0.0022 > \rho_{\min}$$

$$A_s = \rho b d = 0.0022 * 300 * 60 = 39.6 \text{ cm}^2$$

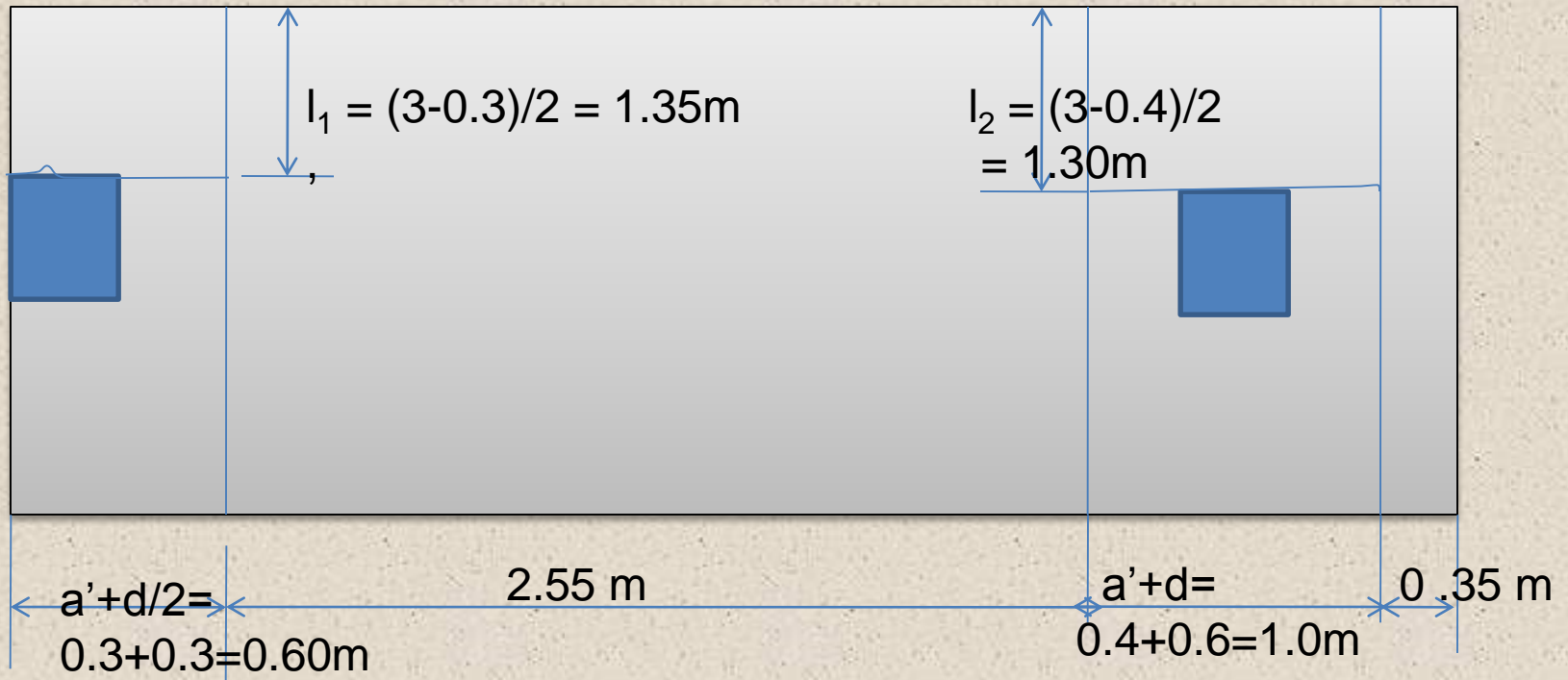
Use $\phi = 20$, $a_s = 3.14 \text{ cm}^2$

No. of bars, $n = 39.6 / 3.14 = 12.6$, use 13 bars

Spacing = $(300 - 2 * 5) / (n - 1) = 24.2 \text{ cm}$

Use 13 ϕ 20 c/c 240mm

– Short direction



Effective width at exterior and interior columns being $a+d/2$ and $a+d$, respectively

- Contact pressure under columns 1 and 2

$$\sigma_1 = \frac{800}{3.00 \times 0.60} = 444.44 \text{ kN} / \text{m}^2$$

$$\sigma_2 = \frac{1200}{3.00 \times 1.00} = 400 \text{ kN} / \text{m}^2$$

- Bending moment
- $M_1 = 444.44 * 1.35 * 0.60 * 1.35 / 2 = 243 \text{ kN-m}$
- $M_2 = 400 * 1.30 * 1.00 * 1.30 / 2 = 338 \text{ kN-m}$

- Calculation of Reinforcements

- Under column 1

Moment capacity of concrete

$$M = 0.32 * f_{cd} * b d^2$$

$$= 0.32 * 11.33 \times 10^3 * 0.60 * (0.58)^2$$

$$= 731.79 \text{ kN} - \text{m} > M_1 \dots \text{Ok!}$$

$$\rho = \frac{f_{cd}}{f_{yd}} \left[1 - \sqrt{1 - \frac{2M}{f_{cd} b d^2}} \right]$$

$$= \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 243}{11.33 \times 10^3 * 0.60 * (0.58)^2}} \right] = 0.0049 > \rho_{\min} \dots \text{ok!}$$

$$A_s = \rho b d = 0.0049 * 60 * 58 = 17.05 \text{ cm}^2$$

Use $\phi = 20$, as = 3.14cm

No. of bars , n= 17.05/3.14 =5.4 , use 6 bars

Spacing = (60-5)/ (n-1) =9.2cm

Use 6 ϕ 20 c/c 90mm

– Under column 2

Moment capacity of concrete

$$\begin{aligned} M &= 0.32 * f_{cd} * b d^2 \\ &= 0.32 * 11.33 \times 10^3 * 1.00 * (0.58)^2 \\ &= 1219.65 kN - m > M_2 \dots \text{Ok!} \end{aligned}$$

$$\rho = \frac{f_{cd}}{f_{yd}} \left[1 - \sqrt{1 - \frac{2M}{f_{cd} b d^2}} \right]$$

$$= \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 338}{11.33 \times 10^3 * 1 * (0.58)^2}} \right] = 0.0040 > \rho_{\min} \dots \text{ok!}$$

$$A_s = \rho b d = 0.0040 * 100 * 58 = 23.20 \text{ cm}^2$$

Use $\phi = 20$, as = 3.14cm

No. of bars , $n = 23.20 / 3.14 = 7.4$, use 8 bars

Spacing = $(100) / (n-1) = 14.3 \text{ cm}$

Use 8 ϕ 20 c/c 140mm

- The reinforcement between the two strips will be nominal reinforcement to prevent shrinkage cracks
- **Short direction**
- $A_{s_{\min}} = \rho_{\min} b d = 0.0017 * 255 * 58 = 25.14 \text{ cm}^2$

Use $\phi = 20$, $a_s = 3.14\text{cm}$

No. of bars , $n = 25.14/3.14 = 8$, use 8 bars

Spacing = $(245)/(7)$

$= 35\text{cm} < 400\text{mm}$ (s_{\max} for secondary bars)..... ok

Use 8 ϕ 20 c/c 350mm

• Long direction

$A_{s_{\min}} = \rho_{\min} bd = 0.0017 * 300 * 60 = 30.60\text{cm}^2$

No. of bars , $n = 30.16/3.14 = 9.7$, use 10 bars

Spacing = $(300-10)/(9)$

$= 32.2\text{cm} < 400\text{mm}$ (s_{\max} for secondary bars)..... ok

Use 10 ϕ 20 c/c 320mm

•Cantilever portion

–Bottom reinforcement , long direction

–Critical moment from bending moment diagram is $M = 160.59\text{kN-m}$

$$\rho = \frac{f_{cd}}{f_{yd}} \left[1 - \sqrt{1 - \frac{2M}{f_{cd} b d^2}} \right]$$
$$= \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 160.59}{11.33 \times 10^3 * 3 * (0.60)^2}} \right] = 0.00057 < \rho_{\min}$$

$$A_s = \rho_{\min} b d = 0.0017 * 300 * 60 = 30.60 \text{cm}^2$$

Use $\phi = 20$, $a_s = 3.14\text{cm}$

No. of bars , $n = 30.6/3.14 = 9.7$, use 10 bars

Spacing = $(300 - 2 * 5) / 9 = 32.22\text{cm}$

Use 10 ϕ 20 c/c 320mm

Short direction

Provide minimum reinforcement

- $A_{s_{\min}} = \rho_{\min} bd = 0.0017 * 35 * 58 = 3.45 \text{cm}^2$
- No. of bars , $n = 3.45/3.14 = 1.1$, use 2 bars
Spacing = $(25)/ 2 = 12.5 \text{cm}$
Use 2 ϕ 20 c/c 125mm

- Development length

- Short direction

- Under column 1

$$l_d = \frac{\phi f_{yd}}{4 f_{bd}} = \frac{2 * 260.87}{4 * 1} = 130.44 \text{cm}$$

- Available development length, $l_a = 135 - 5 = 130\text{cm} < l_d$

Bend the bars upward with a minimum length of 10cm

- Under column 2

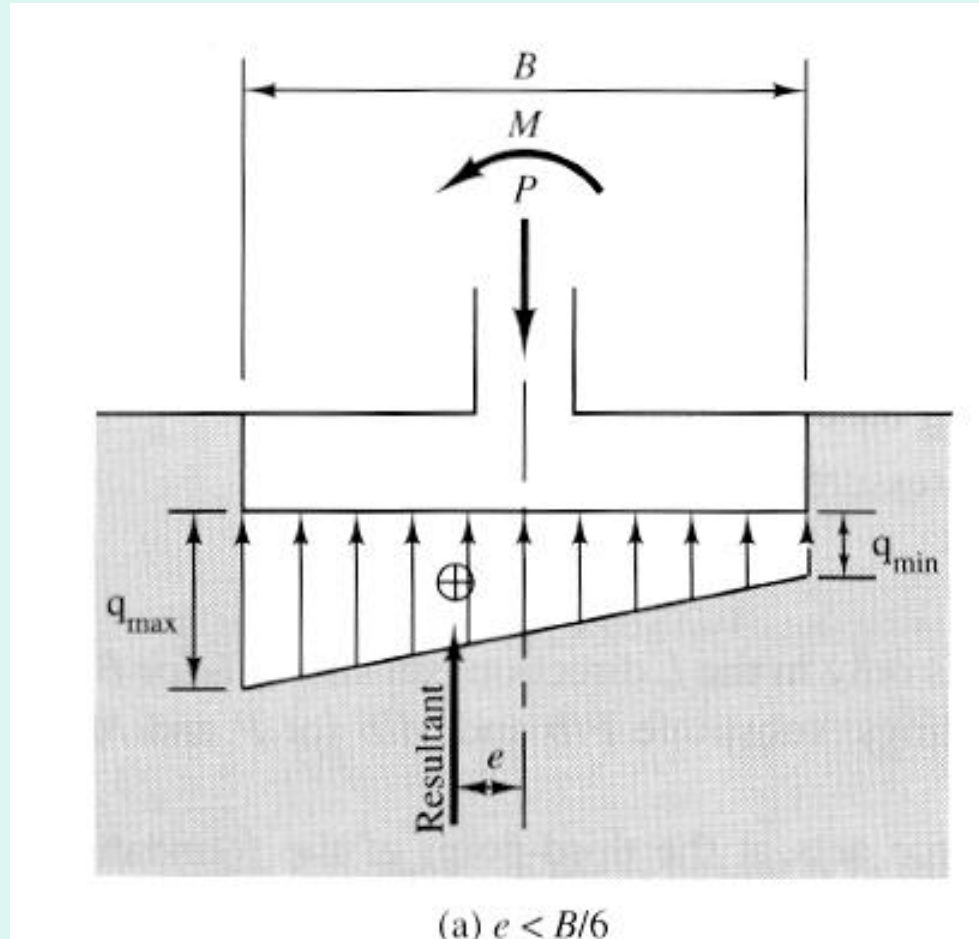
$$l_d = \frac{\phi f_{yd}}{4 f_{bd}} = \frac{2 * 260.87}{4 * 1} = 130.44\text{cm}$$

- Available development length, $l_a = 130 - 5 = 125\text{cm} < l_d$

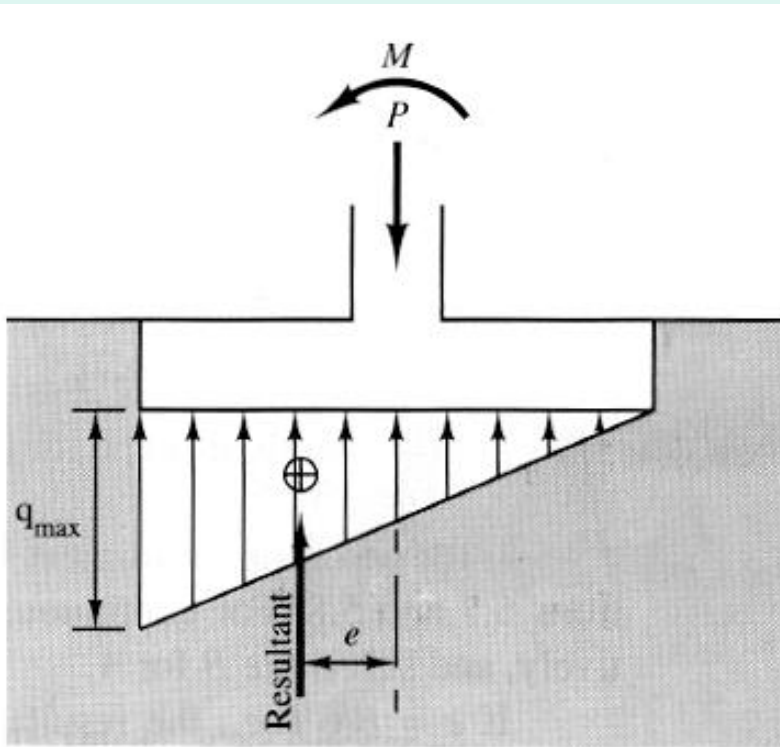
Bend the bars upward with a minimum length of 10cm

Design of eccentrically loaded foundation

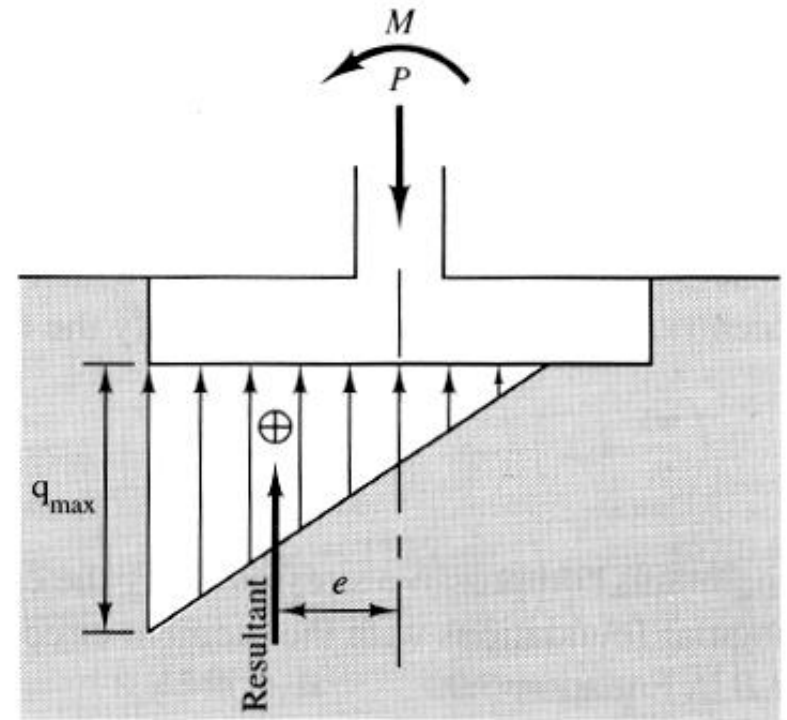
Eccentric Loads or Moments



Eccentric Loads or Moments

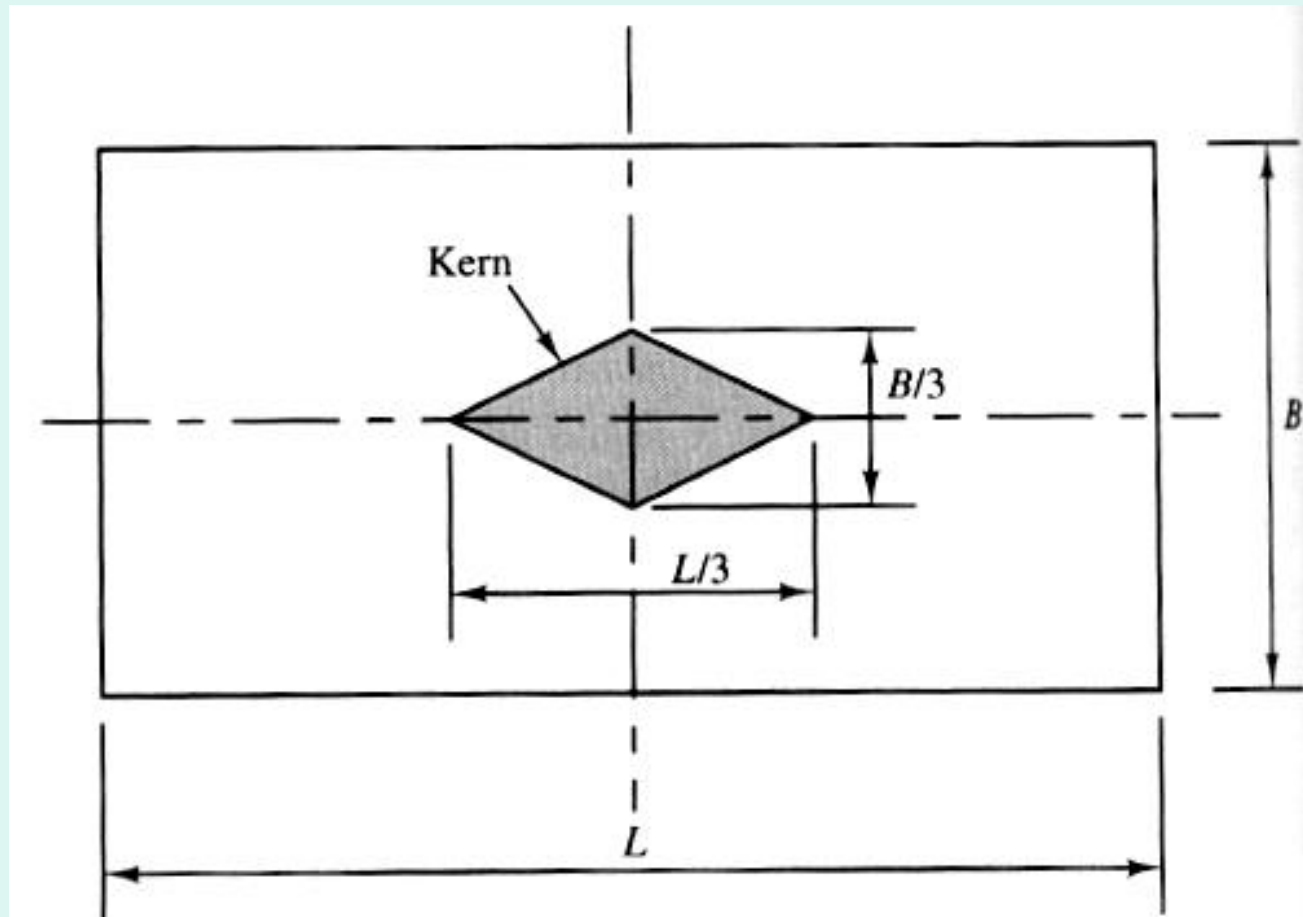


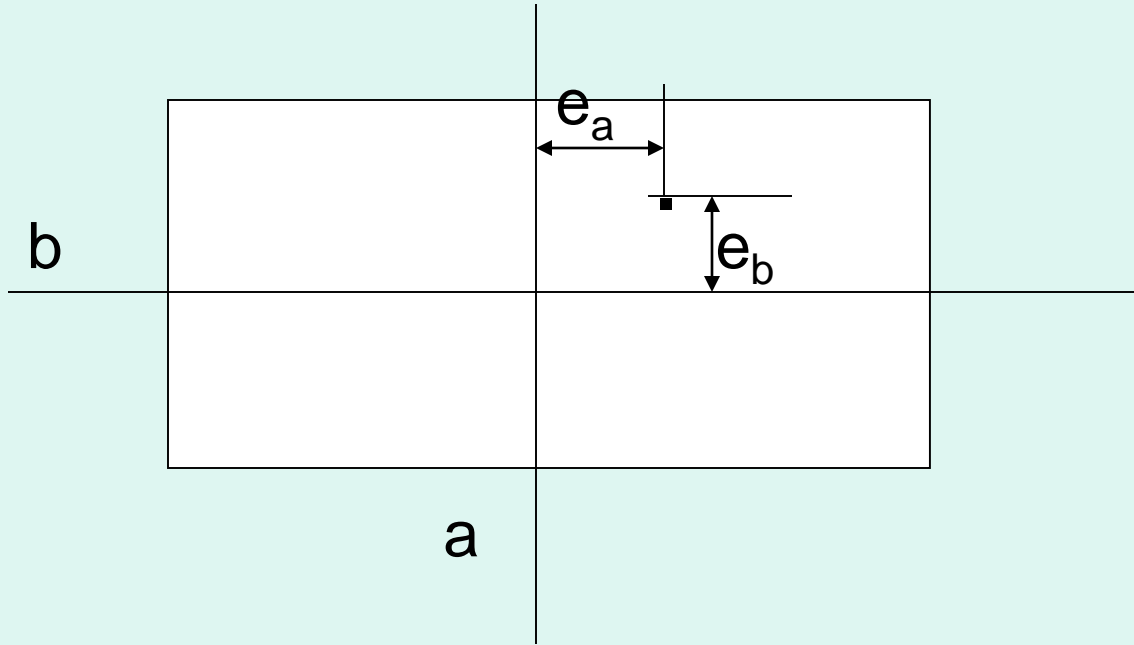
(b) $e = B/6$



(c) $e > B/6$

Two-way Eccentric Loads or Moments





$$\left. \begin{array}{l} \sigma_{\max} \\ \sigma_{\min} \end{array} \right\} = P/ab (1 \pm 6e_b/b \pm 6e_a/a)$$

For contact pressure to remain (+) ve everywhere,

$$\frac{6e_B}{B} + \frac{6e_L}{L} \leq 1.0$$

Examples

Given R.C. column size 30X50 cm with 4 ϕ 22.

$$P = 1500\text{kN}$$

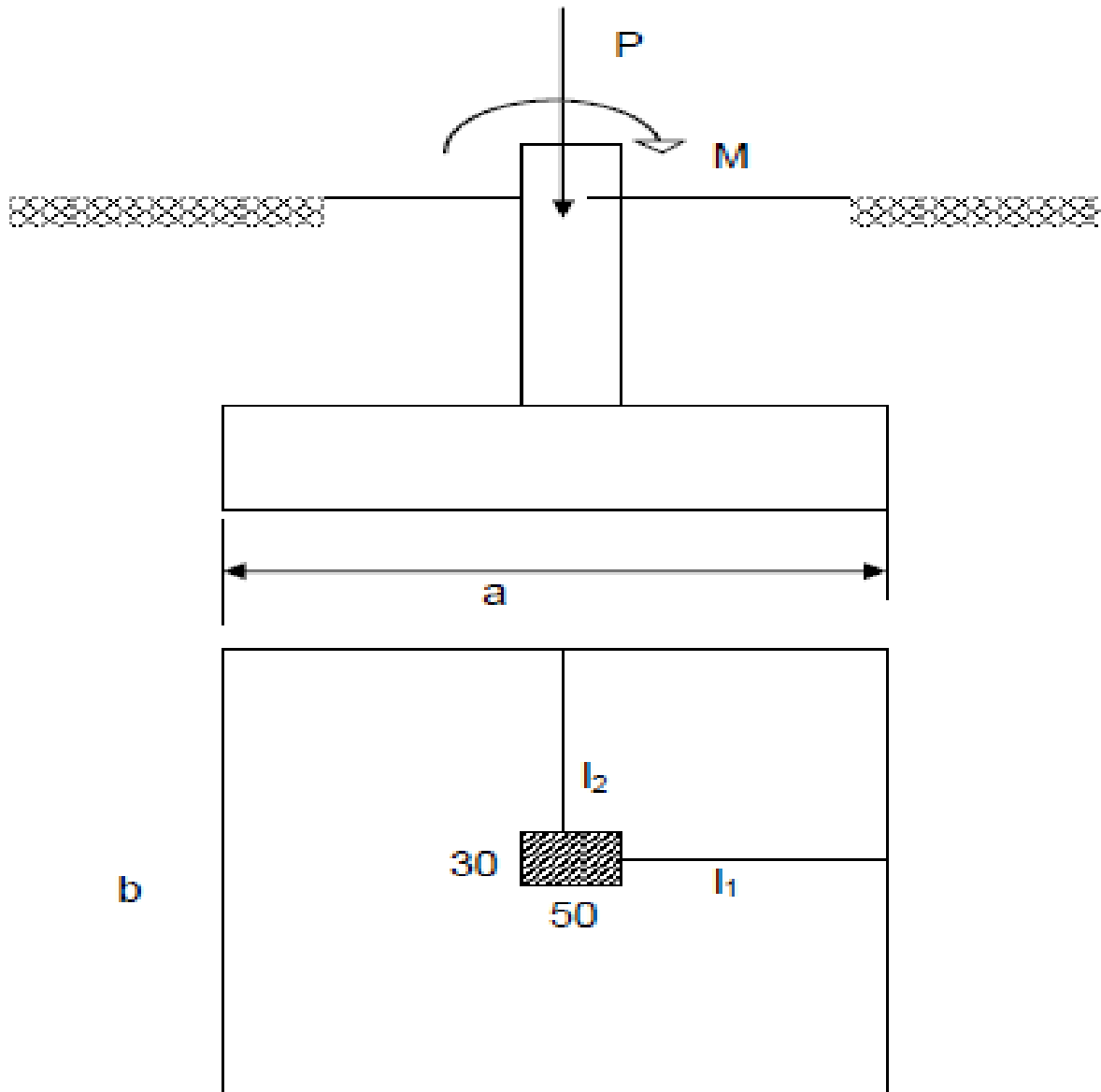
$$M = 375 \text{ kN-m}$$

Ultimate soil bearing pressure = 400kPa

$$f_{yk} = 300\text{MPa} \Rightarrow f_{yd} = 300/1.15 = 260.87 \text{ MPa}$$

$$\text{C25} \Rightarrow f_{ck} = 20\text{MPa} \Rightarrow f_{ctk} = 1.5 \text{ MPa},$$

Required:- Design of rectangular R.C. footing



Solution

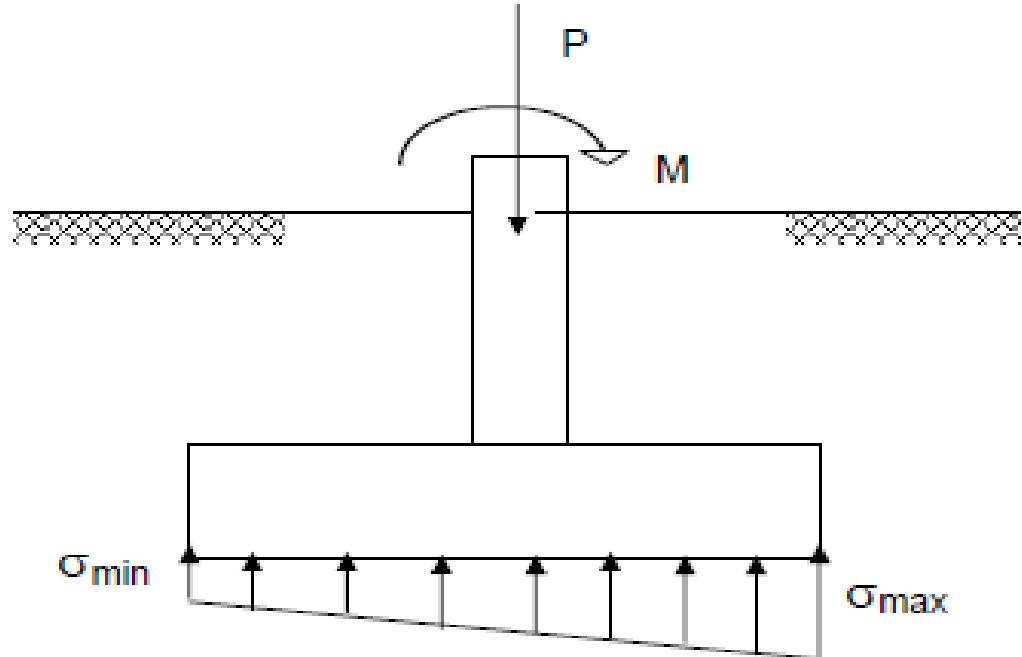
Size of footing

Let $l_1 = l_2$

$$\text{Then } \frac{a - 50}{2} = \frac{b - 30}{2} \Rightarrow a - b = 50 - 30 = 20\text{cm} = 0.2\text{m}$$

$$\text{Eccentricity, } e_a = \frac{M}{P} = \frac{375}{1500} = 0.25\text{m}$$

Contact pressure



$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e_a}{a} \right) = \frac{P}{ab} \left(1 + \frac{6e_a}{a} \right)$$

$$400 = \frac{1500}{(0.2 + b)b} \left(1 + \frac{6 * 0.25}{(0.2 + b)} \right)$$

$$400(0.20b + b^2) = 1500 + \frac{2550}{(0.2 + b)}$$

$$400b^3 + 160b^2 - 1484b - 2550 = 0$$

by trial and error $b = 2.345$ m

Take $b = 2.4$ m

Then $a = b + 0.20$ m = 2.60 m

Actual contact pressure

$$\sigma_{\max} = \frac{1500}{(2.6)2.4} \left(1 + \frac{6 * 0.25}{(2.6)} \right) = 379.07 \text{ kN} / \text{m}^2 < \sigma_{\text{uit}} \quad \text{ok}$$

$$\sigma_{\min} = \frac{1500}{(2.6)(2.4)} \left(1 - \frac{6 * 0.25}{(2.6)} \right) = 101.70 \text{ kN} / \text{m}^2 > 0 \quad \text{ok}$$

Thickness of the footing

i, Punching shear

The Punching shear resistance according to EBCS-2 is given by

$$V_{\text{up}} = 0.25 f_{\text{ctd}} k_1 k_2 u d \quad (\text{MN})$$

Take $d = 0.40\text{m}$ and $\rho = \rho_{\min} = 0.5 / f_{\text{yk}} = 0.5 / 300 = 0.0017$

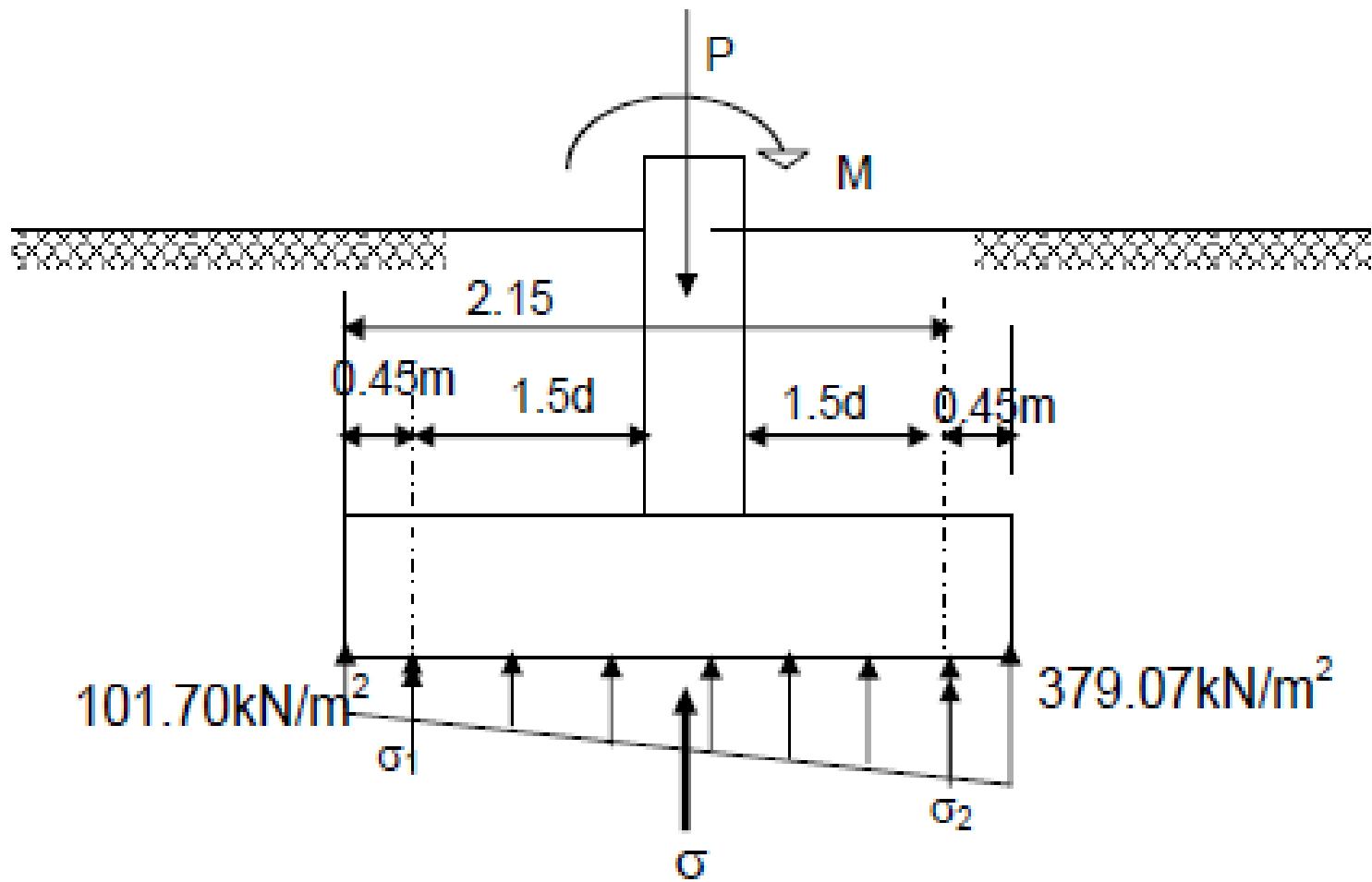
$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.4 = 1.2$$

$$u = 2(3d + b') + 2(3d + a') = 12d + 2b' + 2a' = 12 * 0.4 + 2 * 0.5 + 2 * 0.3 = 6.4$$

Then

$$V_{\text{up}} = 0.25 * 1 * 1.085 * 1.2 * 6.4 * 0.4 = 0.83328 \text{ MN} = 833.28 \text{ kN}$$



$$\sigma_1 = 101.7 + \frac{0.45 * (379.07 - 101.70)}{2.60} = 149.71 \text{ kN} / \text{m}^2$$

$$\sigma_2 = 101.7 + \frac{2.15 * (379.07 - 101.7)}{2.60} = 331.06 \text{ kN} / \text{m}^2$$

$$\sigma = \frac{\sigma_1 + \sigma_2}{2} * 1.7 = \frac{331.06 + 149.71}{2} * 1.7 = 408.65 \text{ kN} / \text{m}$$

$$V = 408.65 * 1.5 = 612.98 \text{ kN}$$

Net shear force developed = 1500 – 612.98 = 887.02 kN > V_{up} not ok !

Since the developed shear force is greater than the punching shear resistance, one may increase the depth.

Take $d = 0.45 \text{ m}$ and $\rho = \rho_{min} = 0.5 / f_{yk} = 0.5 / 300 = 0.0017$

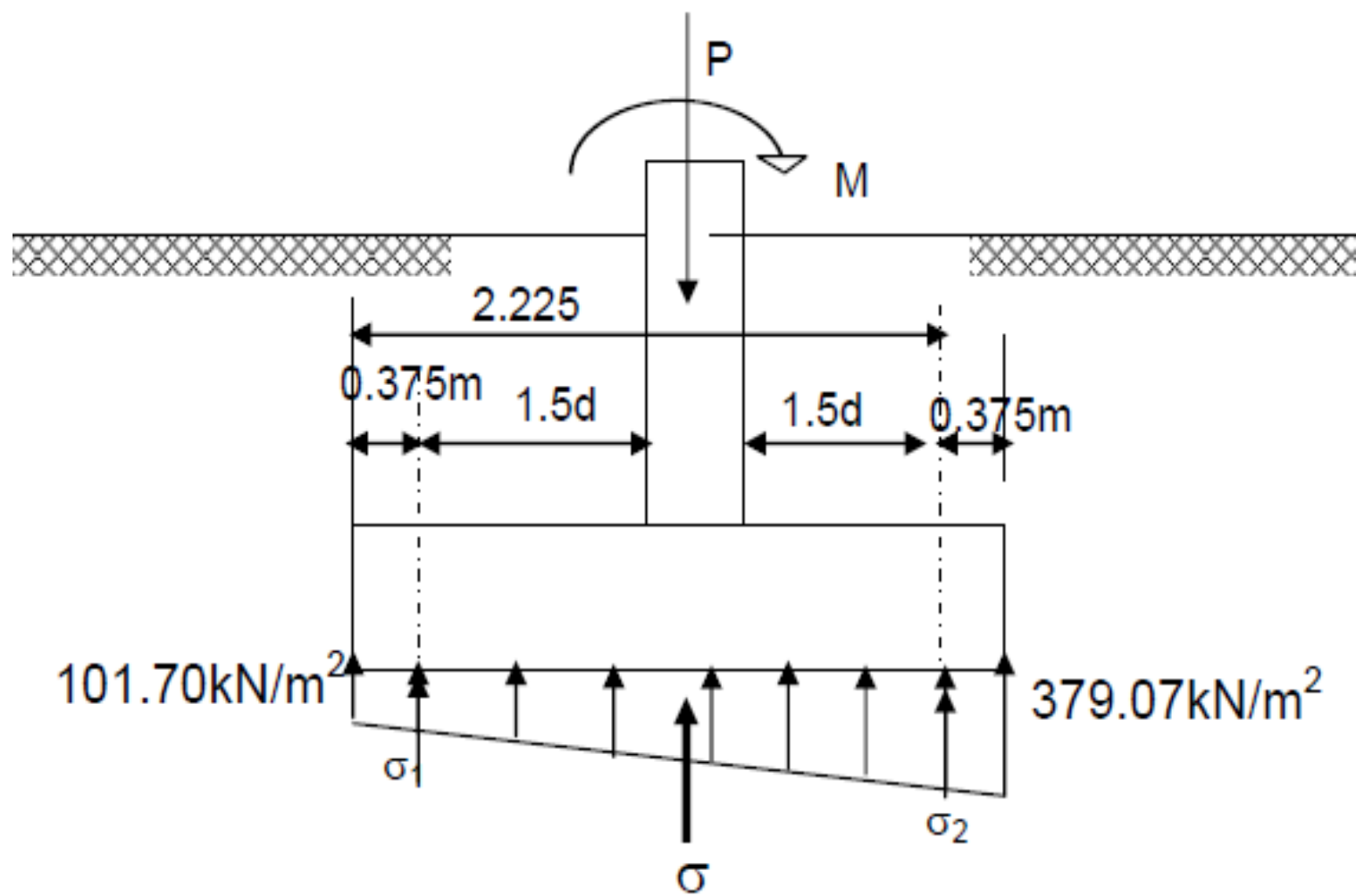
$$k_1 = (1 + 50\rho) = (1 + 50 * 0.0017) = 1.085$$

$$k_2 = 1.6 - d = 1.6 - 0.45 = 1.15$$

$$u = 2(3d + b') + 2(3d + a') = 12d + 2b' + 2a' = 12 * 0.45 + 2 * 0.5 + 2 * 0.3 = 7$$

Then

$$V_{up} = 0.25 * 1 * 1.085 * 1.15 * 7 * 0.45 = 0.98260 \text{ MN} = 982.60 \text{ kN}$$



$$\sigma_1 = 101.7 + \frac{0.375 * (379.07 - 101.70)}{2.60} = 141.71 \text{ kN} / \text{m}^2$$

$$\sigma_2 = 101.7 + \frac{2.225 * (379.07 - 101.7)}{2.60} = 339.07 \text{ kN} / \text{m}^2$$

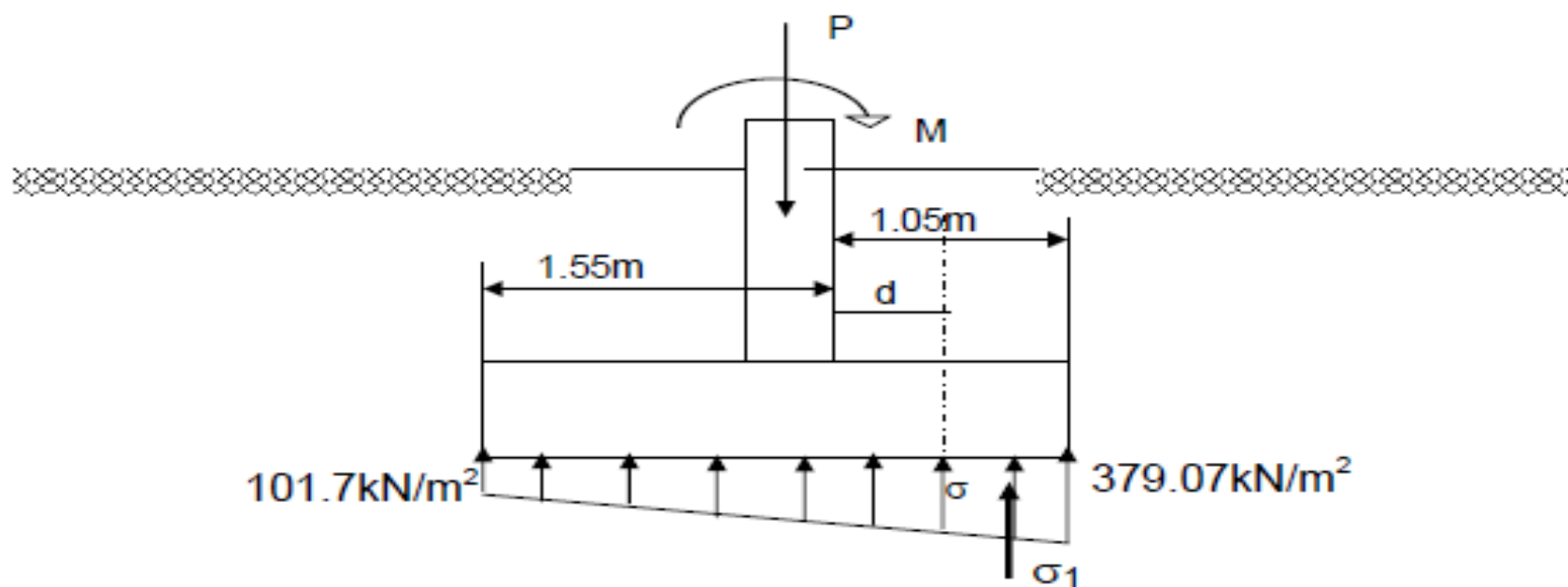
$$\sigma = \frac{\sigma_1 + \sigma_2}{2} * 1.85 = \frac{339.07 + 141.71}{2} * 1.85 = 444.72 \text{ kN} / \text{m}$$

$$V = 444.72 * 1.65 = 733.79 \text{ kN}$$

Net shear force developed = 1500 – 733.79 = 766.21 kN < V_{up} ok !

The depth satisfies the punching shear requirement for the assumed ρ_{\min} .

ii, Wide beam shear



Contact stress at distance d from the face of the column, σ

$$\sigma = 101.7 + \frac{(379.07 - 101.7)(1.55 + 0.45)}{2.60} = 315.06 \text{ kN/m}^2$$

$$\sigma_1 = \left(\frac{\sigma_{\max} + \sigma}{2} \right) (1.05 - d) = \left(\frac{379.07 + 315.06}{2} \right) 0.6 = 208.24 \text{ kN/m}$$

Developed wide beam shear

$$V_d = 208.24 * 2.4 = 499.78 \text{ kN}$$

The wide beam shear resistance according to EBCS-2 is given by

$$\begin{aligned} V_{ud} &= 0.25 f_{ctd} k_1 k_2 b_w d \quad (\text{MN}) \\ &= 0.25 * 1 * 1.085 * 1.15 * 2.4 * 0.45 = 0.33689 \text{ MN} = 336.89 \text{ kN} < V_d \text{ not ok !} \end{aligned}$$

Since the developed shear force is greater than the wide beam shear resistance, one may increase the depth

Take $d = 0.60\text{m}$

Contact stress at distance d from the face of the column, σ

$$\sigma = 101.7 + \frac{(379.07 - 101.7)(1.55 + 0.60)}{2.60}$$

$$\sigma = 331.06 \text{ kN} / \text{m}^2$$

$$\begin{aligned}\sigma_1 &= \left(\frac{\sigma_{\max} + \sigma}{2} \right) (1.05 - d) = \left(\frac{379.07 + 331.06}{2} \right) 0.45 \\ &= 159.78 \text{ kN} / \text{m}\end{aligned}$$

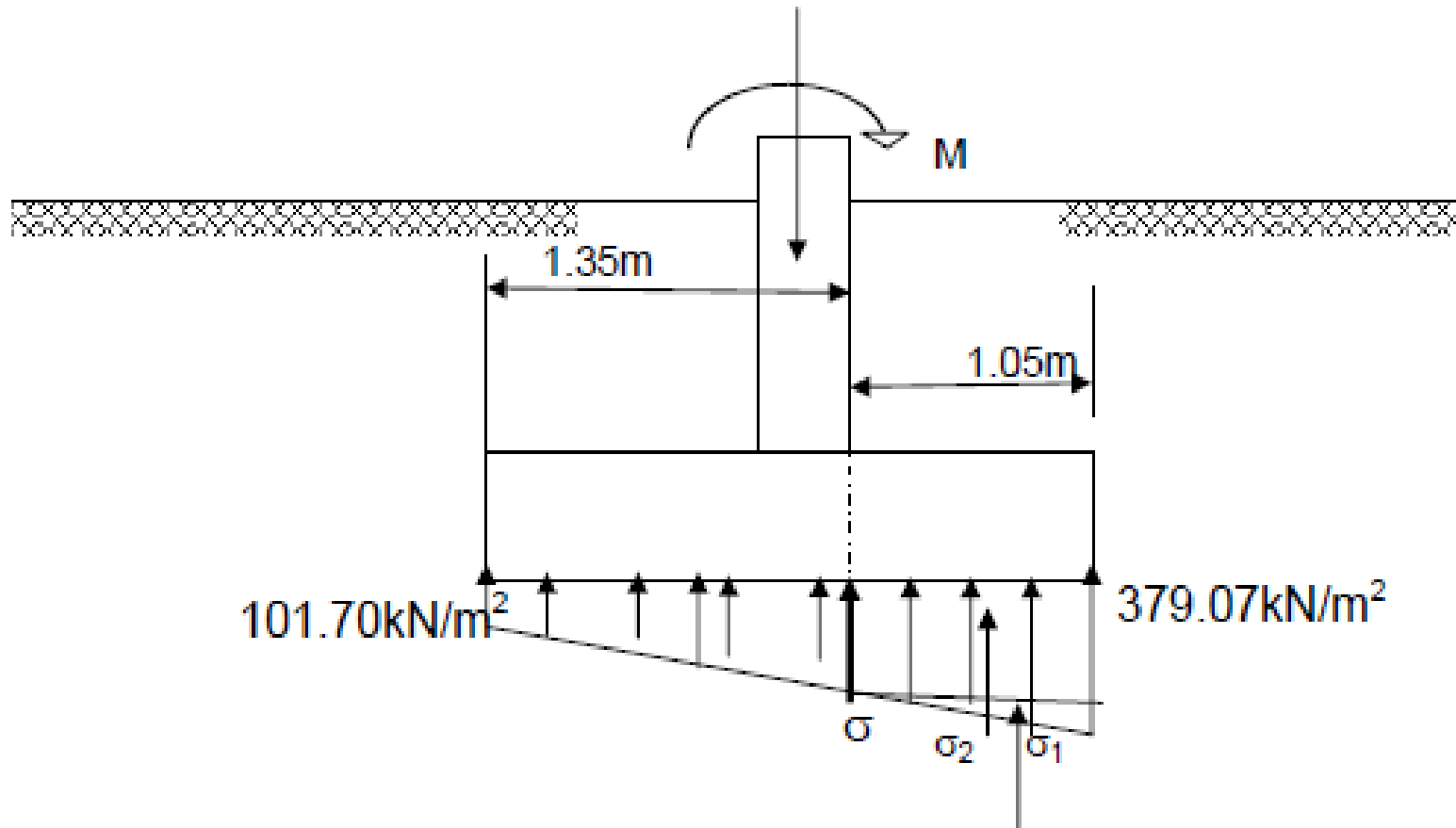
Developed wide beam shear

$$V_d = 159.78 * 2.4 = 383.47 \text{ kN}$$

Wide beam shear resistance

$$\begin{aligned}V_{ud} &= 0.25 f_{ctd} k_1 k_2 b_w d \quad (\text{MN}) \\ &= 0.25 * 1 * 1.085 * 1 * 2.4 * 0.60 = 0.3906 \text{ MN} = 390.60 \text{ kN} > V_d \quad \text{ok!}\end{aligned}$$

Bending Moment



$$\sigma = 101.70 + \frac{1.55 * (379.07 - 101.7)}{2.60} = 267.06 \text{ N / m}^2$$

$$\sigma_1 = \frac{1}{2}(1.05)(379.07 - 267.06) = 58.81 \text{ kN / m}$$

$$\sigma_2 = (1.05)(267.06) = 280.41 \text{ kN / m}$$

$$M = \left[\sigma_1 \frac{2}{3}(1.05) + \sigma_2 \left(\frac{1.05}{2} \right) \right] b$$

$$M = \left[(58.81) \frac{2}{3}(1.05) + (280.41) \left(\frac{1.05}{2} \right) \right] 1 = 188.38 \text{ kN - m / m}$$

Moment capacity of concrete

$$M = 0.32 * f_{cd} * b d^2$$

$$= 0.32 * 11.33 \times 10^3 * 1.0 * (0.6)^2 = 1305.22 \text{ kN - m / m}$$

Calculation of reinforcement

Long direction

$$\rho = \frac{f_{cd}}{f_{yd}} \left[1 - \sqrt{1 - \frac{2M}{f_{cd} b d^2}} \right]$$
$$= \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 188.38}{11.33 \times 10^3 * 1.0 * (0.6)^2}} \right] = 0.0021 > \rho_{\min}$$

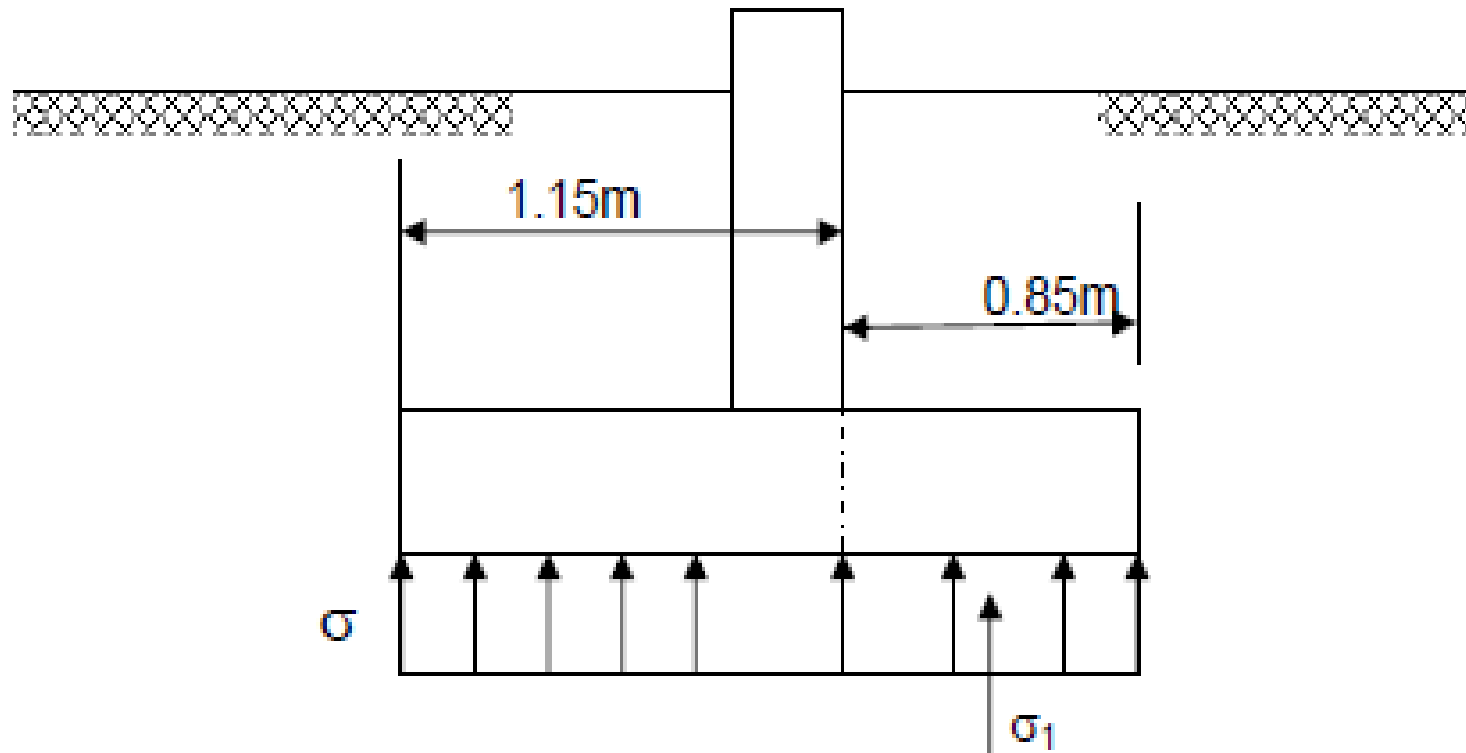
$$A_s = \rho b d = 0.0021 * 100 * 60 = 12.6 \text{ cm}^2 / \text{m}$$

use $\phi 16$

$$\text{spacing} = \frac{a_s * 100}{A_s} = \frac{2.01 * 100}{12.6} = 16 \text{ cm}$$

Use $\phi 16 \text{ c/c } 16 \text{ cm}$

Short direction



Average contact pressure, σ

$$\sigma_{avg} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{avg} = \frac{379.07 + 101.7}{2} = 240.39 \text{ kN} / \text{m}^2$$

$$M = \left[\sigma_1 \left(\frac{1.05}{2} \right) \right] a$$

$$M = \left[240.39 \left(\frac{1.05}{2} \right) \right] 1 = 126.21 \text{ kN} - \text{m} / \text{m}$$

$$\rho = \frac{f_{cd}}{f_{yd}} \left[1 - \sqrt{1 - \frac{2M}{f_{cd} b d^2}} \right]$$

$$= \frac{11.33}{260.87} \left[1 - \sqrt{1 - \frac{2 * 126.21}{11.33 * 10^3 * 1.0 * (0.584)^2}} \right] = 0.0014 < \rho_{min}$$

$$A_s = \rho_{min} b d = 0.0017 * 100 * 58.4 = 9.928 \text{ cm}^2 / \text{m}$$

$$\text{spacing} = \frac{a_s * 100}{A_s} = \frac{2.01 * 100}{9.98} = 20.2 \text{ cm}$$

Use $\phi 16\text{c}/\text{c}20\text{cm}$

Since there is no much difference between a and b, distribute these reinforcement uniformly.

Development length

$$l_d = \frac{\phi f_{yd}}{4f_{bd}}$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = 260.87 \text{ MPa} \quad ; \quad f_{bd} = f_{ctd}$$

$$f_{ctd} = \frac{0.35 \sqrt{f_{ck}}}{\gamma_c} = \frac{0.35 \sqrt{20}}{1.5} = 1 \text{ MPa}$$

$$l_d = \frac{\phi f_{yd}}{4f_{bd}} = \frac{1.6 * 260.87}{4 * 1} = 104.35 \text{ cm}$$

$l_{\text{davailable}} = 100 \text{ cm} < l_d$, bend the bars upward with a minimum length of 10cm