TEXTILE ECONOMICS AND COSTING

CHAPTER 1: TIME VALUE OF MONEY

1.1 INTRODUCTION: BASIC CONCEPTS & DEFINITIONS

- ***Engineering economics*** deals with the methods that enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations
- ***Time Value Of Money:*** the capacity of money to enlarge itself with the passage of time

  E.g. Interest, rent, profit

- ***Interest:*** money earned by the original sum of money regardless of whether the earned money is referred to as **"interests," "profits," or "rent"** in ordinary commercial parlance

Cont.

❖ ***Interest Rate:*** the time rate at which a sum of money earns interest expressed in percentage form

❖ ***Payment-*** any exchange of money regardless of whether the firm **received or expended** the stipulated sum of money

- ***receipt***: money that enters the firm
- ***disbursement or expenditure***: money that leaves the firm.

❖ ***Investment:*** the productive use of money to earn interest

❖ ***Capital:*** the money that earns interest

1.2 CASH FLOW AND CASH-FLOW DIAGRAMS

❖***Cash Flow:*** the set of payments associated with an investment

❖***Cash-flow Diagram:*** a diagram that depicts payments

- Time & payments are represented by **horizontal axis & vertical bar** respectively
- If the set of payments consists **exclusively** of receipts or disbursements, the bars may all be placed above the horizontal axis.
- If **both** types of payments are present, the bars representing **receipts** will be placed **above** the horizontal axis and those representing **disbursements** will be placed **below** it.

E.G. As an illustration, assume that a project has the following cash flow: a disbursement of $20,000 now, a receipt of $5,000 three years hence, a receipt of $12,000 five years hence, and a receipt of $14,000 eight years hence. The cash-flow diagram appears in Fig. 1.1, where the unit of time is 1 year.

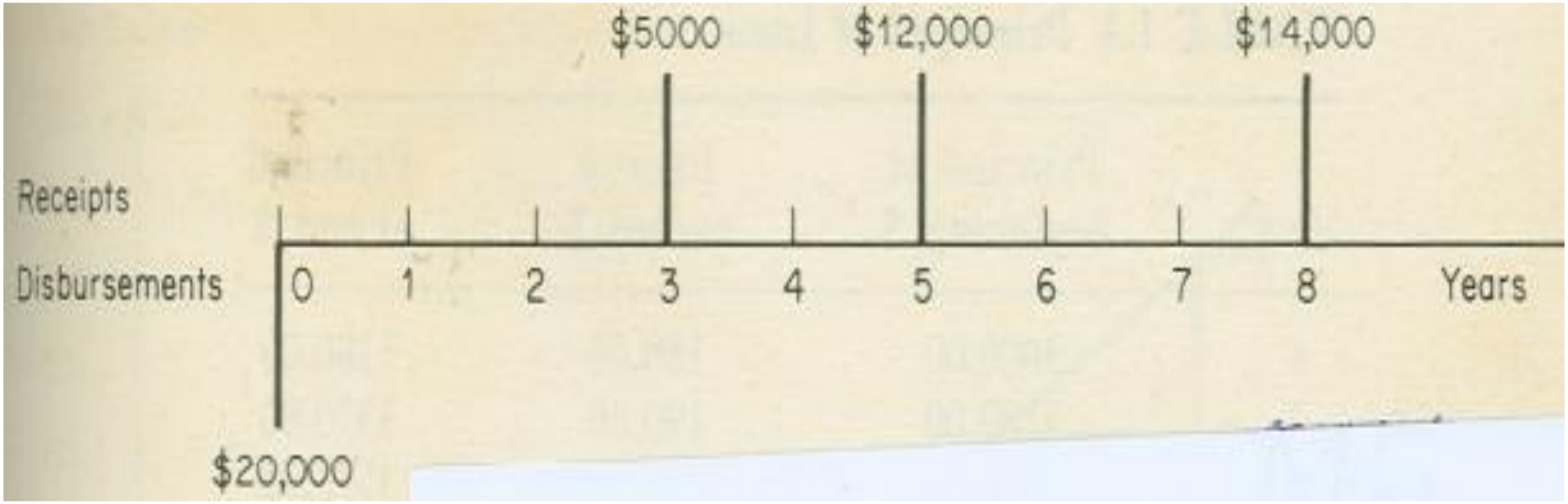
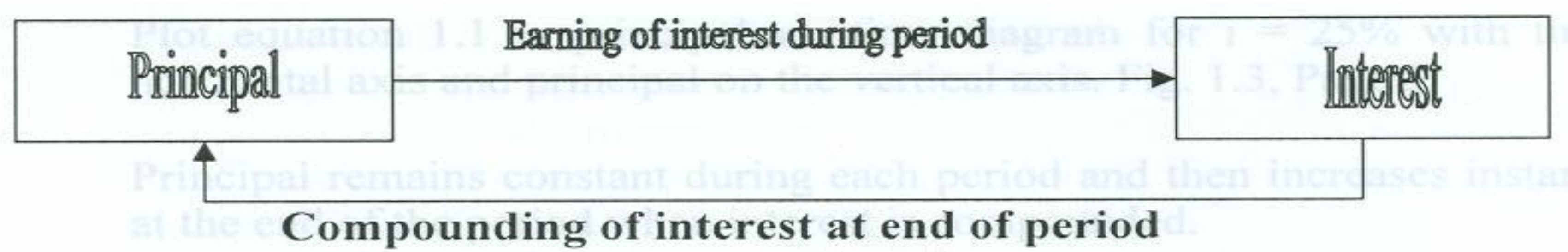


Fig. Cash-flow diagram

1.3 Basic Relationship b/n Money & Time

❖Since money expands by reproducing itself, the functional relationship between money and time must be formulated.



- **Principal:** The sum of money that is earning interest at a given instant
- **Interest Period:** a time interval at which a given sum earns interest, the *interest* that has been earned up to that date is converted to *principal*,

Cont.

- **Compounding of Interest:** process of converting the interest to principal,

Table 1.1 Principal of Loan

Year	Principal at the beginning of the year ($)	Interest earned at the end of the year at 6% per annum ($)	Principal at the end of the year ($)
1	3000.00	180.00	3180.00
2	3180.00	190.80	3370.80
3	3370.80	202.25	3573.05
4	3573.05	214.38	3787.43
5	3787.43	227.25	4014.68

How does it Come?

Cont.

- *Present value(p):* sum (principal) deposited in savings account at beginning of an interest period
- *Future value* = principal in account at expiration of *n* interest periods
- ***i***=*Interest rate (in %),* ***n***=*number of interest period*
- The principal at the end of the first period is P + Pi = P(1+i). Thus, the principal is multiplied by the factor (1+i) during each period.
- the principal at the end of n^{th} period is: $\mathbf{F = P(1+i)^n}$ **(1.1)**

E.g. Given: p=3000 Birr, i= 6%, principal at the end of 5^{th} interest period is: $F = P(1+i)^5 = 3000(1.06)^5 =$ <u>$4014.68</u>.

1.4 Significance of Time Value of Money

❖In analyzing an investment or comparing alternative investments, it is necessary to consider the *timing and the amount* of each payment. Analyze the following:

- If you have a choice of having 1000 Birr today or $1000 Birr a year later, what would you like to choose?

now

❖Is it preferable to expend 1000 Birr now or one year hence?

A year hence

❖The sooner the money is received, the better; the longer the expenditure of money delayed the better.

1.5 Notation for Compound-Interest Factors

- Each compound interest factor will be represented symbolically in the following general format: $(A/B,n,i)$

  Where,

  - A and B denote two sums of money
  - n = the number of interest periods
  - i = the interest rate

1.6 Calculation of Future Worth

❖In equation $F = P(1+i)^n$, *P* and *F* refer to the *present worth* and *future worth* respectively of the given sum of money

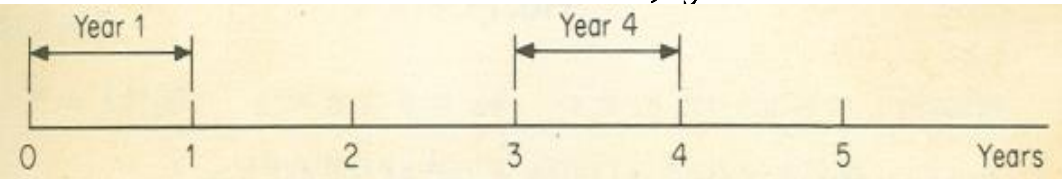
❖The factor $(1+i)^n$ is termed the ***single-payment future-worth*** **factor**.

- According to the above convention, $(F/P,n,i) = (1+i)^n$ ……(1.1a)

❖Equation (1.1) may now be rewritten as:

- F = *P*(*F/P,n,i*)……..(1.1b), *where (F/P,n,i) can be taken from table of compound interest factors*.

❖To specify the timing of payment, select a particular date as zero time and number the units of time from that date, year *n* starts at *n-l* years after zero time.



Q. Show the beginning & end of payment?

Cont.

Eg.1.1 If \$2360 is invested at an interest rate of 7 percent per annum, what will be the value of this sum of money at the beginning of year 7 after the investment?

Solution $F = 2360(F/P,6,7\%) = 2360(1.50073) =$ <u>\$3542</u>

Eg. 1.2 If \$4500 is deposited in an account earning interest at 8 percent per annum **compounded quarterly**, what will be the principal at the end of 6 years?

Solution $P = \$4500$ $n = 6 \times 4 = 24$ $i = 8\%/4 = 2\%$

$F = 4500(F/P,24,2\%) = 4500(1.60844) =$ <u>\$7238</u>

1.7 Calculation of Present Worth

❖ Equation (1.1) converts present sum P to a future sum F. however, it is necessary to perform the inverse operation, which is to convert a future sum to a present sum. $P = \frac{F}{(1+I)^n} = F(1+i)^{-n}$ ……….(1.2)

❖ The factor $(1+i)^{-n}$ is termed the ***single-payment present-worth*** **factor**. It can be written as: $(P/F,n,i) = (1+i)^{-n}$ (1.2a)

❖ equation (1.2) may be rewritten as: $P = F(P/F,n,i)$ (1.2b)

❖ When a future sum of money is converted to its present worth, it is said to be ***discounted***.

Eg.1.3 What sum of money deposited in a savings account at the present date will amount to $3800 four years hence? If the interest rate of the fund is 5 percent per annum.

Cont.

Solution: $P = 3800(F/P,4,5\%) = 3800(0.82270) =$ <u>$3126</u>

Eg.1.4 A savings account earns interest at the rate of 6 percent per annum **compounded quarterly**. What sum of money must be deposited in this account in the present date if it is to amount to $5000 seven years hence?

Solution

$F = \$5000$ $n = 7 \text{ X } 4 = 28$ $i = 1.5\%$

$P = 5000(P/F,28,1.5\%) = 5000(0.65910) =$ <u>$3296</u>

1.8 Calculation of Interest Rate

❖the value of the present worth *P* and the future worth *F* of a given sum of money are known and it is required to find out the interest rate by which they are related. Solving equation (1.1) gives:

$$i = \left(\frac{F}{P}\right)^{1/n} - 1 \quad \text{.........................(1.3)}$$

Eg.1.5 An individual borrowed \$3000 and discharged the debt by a payment of \$4500 four years later. What annual interest rate did this individual pay, to the nearest tenth of a percent?

Solution *P* = \$3000 *F* = \$4500 *n* = 4

By above equation, $i = \left(\frac{4500}{3000}\right)^{1/4} - 1$ = 1.107 – 1 = 0.107 = <u>10.7%</u>

1.9 Calculation of Required Investment Duration

❖It is often necessary to find how long it will take a given sum of money to increase to some specified value when invested at a known interest rate.

$$n = \frac{\log(F/P)}{\log(1+i)} \text{..........................} (1.4)$$

Eg. 1.6

If a given sum of money is invested at 13 percent, in how many years will it treble in value?

Solution $F = 3P$

$$n = \frac{\log(F/P)}{\log(1+i)} \qquad \frac{\log(3P/P)}{\log(1+0.13)} = \frac{\log 3}{\log 1.13} = \frac{0.47712}{0.05308} = \underline{9}$$

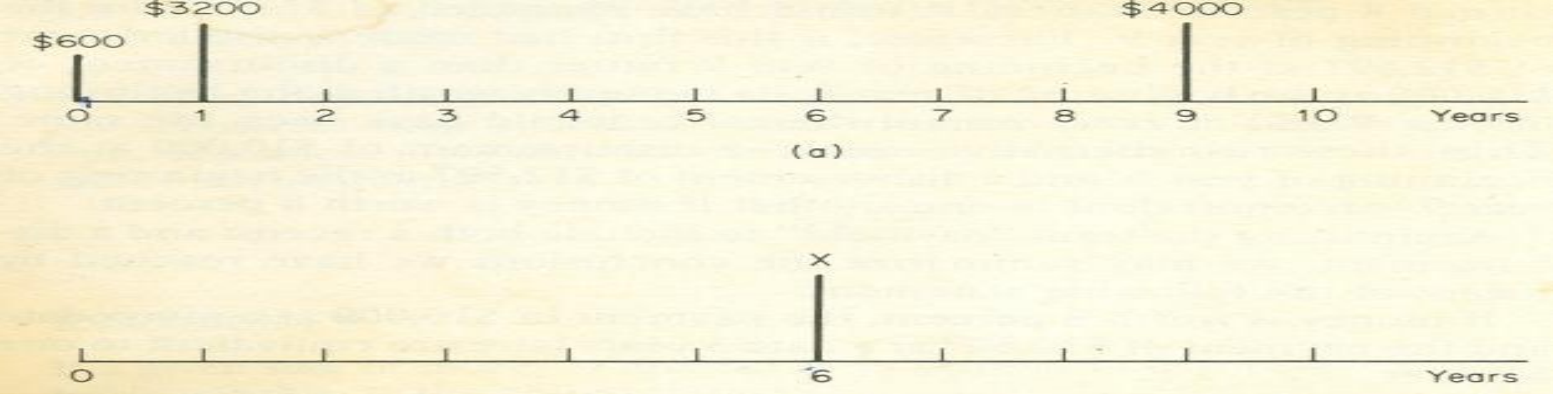
1.10 Meaning of Equivalence

❖two alternative payments are *equivalent* to one another if the ***monetary worth of the firm will eventually be the same regardless of which payment is made.***

Technique of Equivalence

- Determine single equivalent value at a point in time for plan 1.
- Determine single equivalent value at a point in time for plan 2.
- *Both at the same interest rate and at the same time point.*
- Judge the **relative attractiveness** of the two alternatives from the comparable equivalent values

Eg.1.7 If money is worth 10 percent, with single payment made at the beginning of year 7 is equivalent to the following set of payments: \$600 at the beginning of year 1, \$3200 at the beginning of year 2, and \$4000 at the beginning of year 10?



Equivalence of single payment (b) & set of payments *(a)*

Solution: Equivalent payment= $600(F/P,6) + 3200(F/P,5) + 4000(P/F,3)$

$= 600(1.77156) + 3200(1.61051) + 4000(0.75131)$

$= \underline{\$9222}$

- If a single payment is equivalent to a given set of payments, the amount of the single payment is called ***the value of the set of payments at the specified date***. When all payments in a set of payments are transformed to their equivalent payments at a common date, the specific date is called ***the valuation date***.

1.11 comparison of Sets of Payments

- ❖Two sets of payments may be compared by finding the value of each set at a common valuation date.

- ❖**E.g. 1.8** A business firm contemplating the installation of labor-saving machinery has a choice of two models. Model A will cost \$36,500 and model B will cost \$36,300. The major repairs required under each model are estimated to be the following: model A, \$1500 at the end of the fifth year and \$2000 at the end of the tenth year; model B, \$3800 at the end of the ninth year. The models are alike in all other respects. If this firm is earning 7 percent on its capital, which model is more economical?

Solution

- Select the purchase date as the valuation date. The value of the specified expenditures is as follows:

Model A

$36{,}500 + 1500(P/F,5) + 2000(P/F,10) = 36{,}500 + 1500(0.71299) + 2000(0.50835)$

$= \underline{\$38{,}586}$

Model B:

$36{,}300 + 3800(P/F,9) = 36{,}300 + 3800(0.54393) = \underline{\$38{,}367}$

- Therefore, Model B is more economical.

1.12 Change of Interest Rate

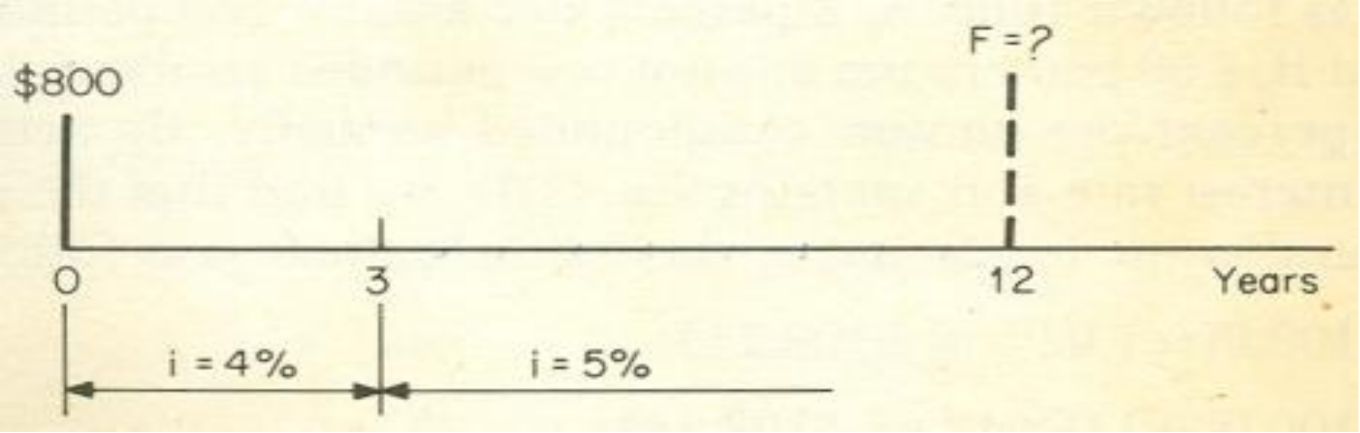
❖If the interest rate changes at a particular date, this date serves to **divide time into two intervals**, each characterized by a specific interest rate. If a given sum of money is to be carried from one interval to the other, it is necessary to **find its value at the boundary point.**

E.g.1. 9 The sum of $800 was deposited in a fund that earned 4 percent per annum for the first 3 years and 5 percent per annum thereafter. What was the principal in the fund 12 years after the date of deposit?

Solution

- At the end of the third year, the principal was 800(F/P,3,4%). The principal then grew at the rate of 5 percent per annum for the remaining 9 years, and it then amounted to:
  - 800(F/P,3,4%)(F/P,9,5%)

= 800(1.12486)(1.55133) = $1396



1.13 Equivalent & Effective Interest Rates

❖Consider the sum of $**100,000** is deposited in funds **A**, **B** & **C** at the beginning of a given year. The interest rates are: 8% per annum compounded quarterly for fund A; **8.08% per annum compounded semiannually for fund B**; 8.243% per annum compounded annually for fund C. The principal of each fund one year later is as follows:

- Fund A: $100{,}000(1 + 0.08/4)^4 = \$108{,}243$
- Fund B: $100{,}000(1 + 0.0808/2)^2 = \$108{,}243$
- Fund C: $100{,}000(1 + 0.08243) = \$108{,}243$

❖the three funds yield an *identical principal* after 1 year, their interest rates are *equivalent* to one another

Cont.

Effective Interest Rates

❖If a given interest rate applies to a period **less than a year**, its equivalent rate for an annual period is known as *its effective rate*.

In general,

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

- Where, i_e = effective interest rate
- r = nominal annual interest rate
- m = number of interest periods contained in 1 year

❖The concept of effective interest rates **enables** comparison of interest rates that apply to unequal intervals of time.

Cont.

E.g.1.10 What amount must be deposited today in an account paying 15% per year, compounded monthly in order to have $2,000 in the account at the end of 5 years?

a) calculate using nominal interest rate

b) calculate using annual effective interest rate

Solution a) i=15%/12=1.25%   F=2000   n=5*12=60

P=2000(P/F,60,1.25%)= 949.1352

b) $i_a=(1+\frac{r}{m})^m-1=(1+0.0125)^{12}-1=0.16075$, ***16.075%***

P=2000(P/F,5,16.075)= 949.1537

Cont.

E.g.1.11 Make a ratio comparison (to four significant figures) of the following interest rates: rate 1, 6% per annum compounded monthly; rate 2, 4 % per annum compounded semiannually.

Solution

❖Let the second subscripts 1 and 2 refer to rates 1 and 2 respectively. Using Equation (on slide 25), the effective rates are calculated as follows:

$$i_{e,1} = (1.005)^{12} - 1 = 6.168\%$$

$$i_{e,2} = (1.02)^{2} - 1 = 4.040\%$$

Then:

$$\frac{i_{e,1}}{i_{e,2}} = \frac{6.168}{4.040} = \underline{1.527}$$

Effect of Taxes on Investment Rate

❖The income received by a firm is referred as the ***original income***, and that part of this income that remains after the payment of taxes will be called the ***residual income***.

❖The investment rates as calculated on the basis of the original income and the residual income are knows as *the* ***before-tax*** and ***after-tax rates*** respectively.

Let C = amount invested

I = annual income from investment

i_b = before-tax investment rate

i_a = after-tax investment rate

t = tax rate

Cont.

Hence, Original income = I and $i_b = \dfrac{I}{C}$

Residual income = I(1-t) and $i_a = \dfrac{I(1-t)}{C}$

Therefore, $i_a = i_b(1-t)$

E.g.1.12 A corporation wishes to earn 7 percent on its capital after payment of taxes. The income from a prospective investment will be subject to federal, state, and municipal taxes and the total tax rate will be 52 percent. What must be the minimum rate of return of the prospective investment before payment of taxes?

Solution: $$i_b = \frac{i_a}{1-t} = \frac{0.07}{1-0.52} = 0.146 = \underline{14.6\%}$$

1.15 Opportunity Costs; Sunk Costs

- Consider that an organization has a choice of two alternative investments, **A** and **B**. If it **undertakes A**, it forfeits the income that would accrue under B. Therefore, the income associated with investment B is referred to as an *opportunity cost of investment A.* For example, a firm can dispose of an existing machine for $2000 or keep it. This prospective income is an opportunity cost of retaining the machine.
- A *sunk cost* is an expenditure that was made in the past and that exerts no direct influence on future cash flows. Therefore, it is irrelevant in an economy analysis. Eg. Manufacturing cost of a merchandise manufactured several years before and stored simply b/c of little demand it has now.

1.16 Inflation

- costs in general tend to increase with the passage of time. This general increase in costs is termed *inflation*.

- A loss in the purchasing power of money over time

For a given commodity, let

C_o = cost of commodity at beginning of first year

C_r = cost of commodity at end of r^{th} year

q = (effective) rate of inflation for r^{th} year

- The rate of inflation for a given year is the ratio of the **increase in cost** of the commodity during that year to the cost at the **beginning** of the year, that is: $q = \dfrac{C_r - C_{r-1}}{C_{r-1}}$

Cont.

- If the annual rate of inflation q remains constant for n years, the cost of the commodity at the end of that period is:

$$C_n = C_o(1+q)^n$$

E.g.1.13 A commodity cost \$30 per unit at the beginning of the first year. The annual rate of inflation was 2 percent for the first 3 years, 5 percent for the fourth year and 8 percent for the fifth year. What did the commodity cost the end of the fifth year?

Solution

- $C_3 = 30(1.02)^3$ $C_4 = C_3(1.05)$ $C_5 = C_4(1.08)$ Then, $C_5 = 30(1.02)^3(1.05)(1.08) = \underline{\$36.10/\text{unit}}$

Cont.

E.g. 1.14 The inflation rate was 5.6 percent for the first year, 4.9 percent for the second year and 8.7 percent for the third year. Find the value of q_e (equivalent uniform inflation rate) for this three-year period, to three significant figures.

Solution

$$(1 + q_e)^3 = (1.056)(1.049)(1.087)$$

$$(1 + q_e) = \{(1.056)(1.049)(1.087)\}^{1/3} = 1.0639$$

$$q_e = 1.0639 - 1 = 0.0639$$

Then, $q_e = \underline{6.39\%}$