Chapter Two

Transformer

2.1 Introduction

Transformer is a static electromagnetic machine (i.e., it has no moving parts) that transfers electrical energy from one electrical circuit to another electrical circuit through the medium of magnetic field and without a change in the frequency.

The electric circuit which receives energy from the supply mains is called *primary winding* and the other circuit which delivers electrical energy to the load is called *secondary winding*.

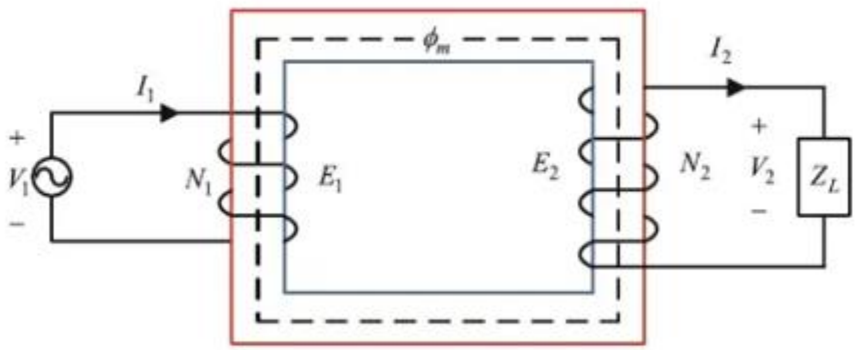


Fig 2.1 single phase transformer

If the secondary winding has more turns than the primary winding, then the secondary voltage is higher than the primary voltage and the transformer is called a *step-up* transformer. When the secondary winding has less turns than the primary windings, then the secondary voltage is lower than the primary voltage and the transformer is called *step down* transformer.

Transformers are commonly used for

- Changing the voltage and current levels in a given electrical system,
- Establishing electrical isolation,
- Impedance matching, and
- Measuring instruments.

Isolating transformers are used to electrically isolate electric circuits from each other or to block dc signals while maintaining ac continuity between the circuits, and to eliminate electromagnetic noise in many types of circuits.

Transformers are used extensively only in ac power systems. AC electrical power can be generated at one central location, its voltage stepped up for transmission over long distances at very low losses and its voltage stepped down again for final use.

If we take the same arrangement and apply an alternating voltage to one coil, it will induce an alternating e.m.f. in the other coil; this is called the transformer effect. The coil or winding to which the supply is connected is called the primary and the winding from which the induced voltage is taken is called the secondary.

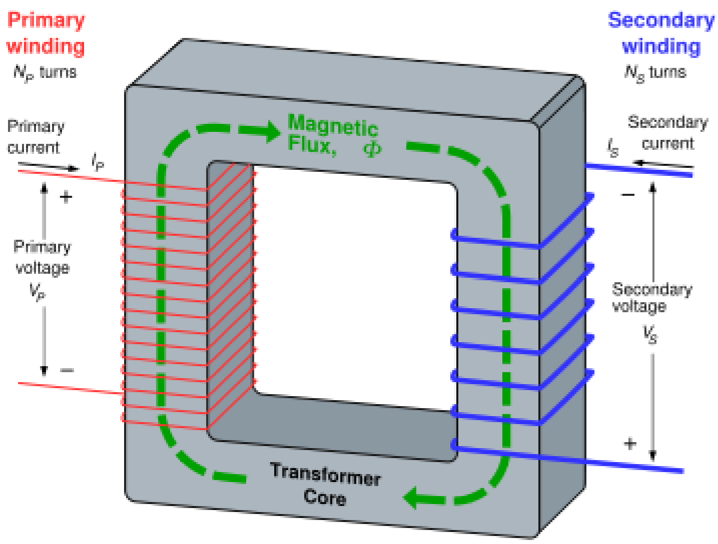


Fig.2.2 Transformer windings

The relationship between the voltage, current and number of turns for each winding is as follows:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{I_S}{I_P}$$

Where: V_P= primary voltage; I_P= primary current; N_P= primary turns; V_S= secondary voltage; I_S= secondary current, and N_S= secondary turns.

Example 2.1

A single-phase transformer has 796 turns on the primary and 365 turns on the secondary winding. If the primary voltage and the primary current are 240V and 10A respectively. Calculate the current of the secondary winding and the turns ratio of the transformer and give a conclusion about the transformer type.

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

$$V_S = \frac{V_P * N_S}{N_P} = \frac{240 * 365}{796} = 110\ V$$

Also $\frac{V_P}{V_S} = \frac{I_S}{I_P}$

$$I_S = \frac{V_P * I_P}{V_S} = \frac{10 * 240}{110} = 21.81\ A$$

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{I_S}{I_P} = a$$

$$a = \frac{796}{365} = \frac{21.82A}{10A} = 2.18$$

Note the larger secondary current. The secondary winding would need to have a larger conductor size than the primary winding to carry this current. If the transformer were of the step-up type the secondary current would be smaller.

2.2. Transformer principle

A transformer normally consists of a pair of windings the set of windings connected to the source side of the transformer are called primary windings, and those connected to the load are named secondary windings. They are linked by a magnetic circuit or core. When an alternating voltage is applied to the primary windings, a current will flow which sets up an alternating magneto-motive force (m.m.f). Hence an alternating flux is developed in the core, which in turn induces electromagnetic force (e.m.f) in both windings. In the secondary winding the induced e.m.f. is the secondary open-circuit voltage. If a load is connected to the secondary winding which permits the flow of secondary current.

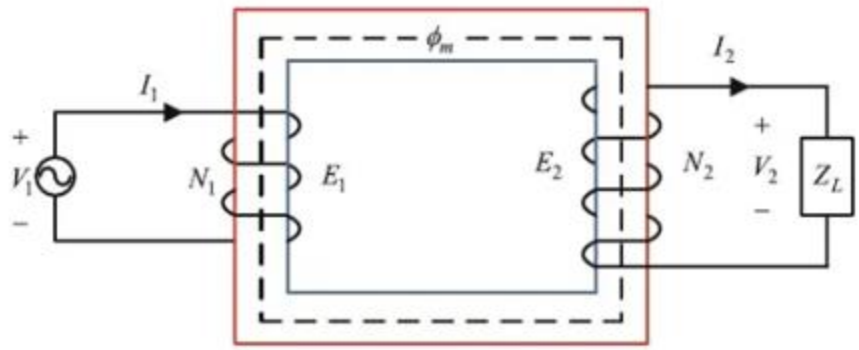


Fig 2.3: Schematic diagram of a transformer winding

- The product of number of turns (N) of the coil and the magnetic flux (φ) linking the coil is called flux linkages i.e. Flux linkages $= N\Phi$.

- The primary winding P is connected to an alternating voltage source, therefore, an alternating current I_m starts flowing through N_1 turns.
- The alternating mmf sets up an alternating flux ϕ which is confined to the high permeability iron path.
- The alternating flux induces voltage E_1 in the primary P and E_2 in secondary S.
- If a load is connected across the secondary, load current starts flowing.

Principle of electromagnetic induction states that:

- *When the primary coil is connected to an alternating voltage (Vp) source, Alternating current (Ip) starts to flow through the primary winding (Np). The resulting MMf = NpIp, produces an alternating flux (ϕ) in the core. This alternating flux induces emf in the primary (Ep) and in the secondary (Es) windings.*

The following points may be noted carefully:

- The transformer action is based on the laws of electromagnetic induction.
- There is no electrical connection between the primary and secondary. The a.c. power is transferred from primary to secondary through magnetic flux.
- There is no change in frequency i.e., output power has the same frequency as the input power.

- The losses that occur in a transformer are:

(a) Core losses—eddy current and hysteresis losses

(b) Copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

2.3. EMF equation

In electrical transformer, ones alternating electrical source is applied to the primary winding and due to this, magnetizing current flowing through the primary winding which produces alternating flux in the core of transformer. This flux links with both primary and secondary windings. As this flux is alternating in nature, there must be a rate of change of flux. According to Faraday's law of electromagnetic induction if any coil or conductor links with any changing flux, there must be an induced emf in it.

Faraday's and lenz's laws of induction

According to Faraday's law of induction, whenever a flux passes through a turn of a coil, a voltage (i.e., an electromotive force [e.m.f]) is induced, in each turn of that coil, that is directly proportional to the rate of change in the flux with respect to time. Therefore, induced voltage can be found from

$$e_{ind} = \frac{d\Phi}{dt}$$

Where Φ is the flux that passes through the turn. If such a coil has N turns and the same flux passes through all of them, the resulting induced voltage between the two terminals of the coil becomes

$$e_{ind} = N\frac{d\Phi}{dt}$$

However, according to Lenz's law of induction, if the coil ends were connected together, the voltage built-up would produce a current that would create a new flux opposing the original flux change.

$$e_{ind} = -\frac{d\Phi}{dt}$$

And

$$e_{ind} = -N\frac{d\Phi}{dt}$$

The negative sign simply reminds us that the induced current opposes the changing magnetic field that caused the induced current. The negative sign has no other meaning.

Furthermore, because the induced voltage equals the rate of change of flux linkages, an applied sinusoidal voltage has to produce a sinusoidally changing flux. Thus, the flux as a function of time is given as

$$\Phi = \Phi_m sin\omega t$$

Where

Φ_m is maximum flux value, ω is $2\pi f$, and f is the frequency

Then the induced voltage is given as

$$e_{ind} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(\Phi_m sin\omega t)$$

$$e_{ind} = -N\Phi_m \omega cos\omega t$$

$$\text{R.M.S value} = \frac{Peak\ value}{\sqrt{2}}$$

Then $$E = \frac{-N\Phi_m 2\pi f}{\sqrt{2}} = -\frac{2\pi}{\sqrt{2}} N\Phi_m f = 4.44Nf\,\Phi_m$$

$$\Phi_m = \frac{E}{4.44Nf}$$

Voltage Transformation Ratio(a)

- From emf Eqs. we get

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} = a$$

- The ratio of the primary voltage to the secondary voltage or primary turns to the secondary turns is called Transformation ratio (Turns ratio) (a)of a transformer.
    - If $N_2 > N_1$ i.e., a <1, then transformer is step-up transformer.
    - If $N_2 < N_1$ i.e., a >1, then transformer is step-down transformer.

- **Example:** A single phase transformer has 350 primary and 1050 secondary turns. The net cross-sectional area of the core is 55 cm^2. If the primary winding be connected to a 400 V, 50 Hz single phase supply, calculate (i) the maximum value of flux density in the core and (ii) the voltage induced in the secondary winding.

Solution

$V_1 = 400V \approx E_1, N_1 = 350, N_2 = 1050, A = 55\ cm^2 = 55 \times 10^{-4} m^2, f = 50Hz$

- Induced emf in the primary is given by

$$E_1 = 4.44 f \Phi_m N_1 = 4.44 f B_m A N_1$$

- Maximum value of flux density in the core,

$$B_m = \frac{400}{4.44 \times 50 \times 55 \times 10^{-4} \times 350} = 0.93T(wb/m^2)$$

- For an ideal transformer,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$E_2 = E_1 \times \frac{N_2}{N_1} = 400 \times \frac{1050}{350} = 1200V$$

Exercise 1

The required no-load voltage ratio in a single phase $50Hz$, core type transformer is $6600/500$. Find the number of turns in each winding, if the flux is to be $0.06Wb$

Exercise 2

The primary voltage of an iron core single-phase transformer $220V$. The number of primary and secondary turns of the transformer are 200 and 50 respectively. Calculate voltage of the secondary coil.

Exercise 3

The number of primary turns of a $30\ KVA$, $2200/220\ V, 50Hz$ single-phase transformer is 100. Find the (i) turns ratio (ii) mutual flux in the core, and (iii) full load primary and secondary currents.

2.4 Transformer Losses

- There are mainly two kinds of losses in a transformer, namely
  - Core loss or iron loss and
  - ohmic loss or copper loss

Core loss: due to alternating flux.

Core losses are caused by the alternating flux in the core of the transformer as this loss occurs in the core it is known as the core loss.

The core loss P_c occurring in the transformer iron, consists of two components, hysteresis loss P_h and eddy current loss P_e i.e.

$$P_c = P_h + P_e$$

- Hysteresis loss

The core of transformer is subjected to an alternating magnetizing force, and for each cycle of emf, a hysteresis loop is traced out. Power is dissipated in the form of heat known as hysteresis loss.

The hysteresis and eddy current losses in the core can be expressed by

$$P_h = K_h B_m^x f v$$

$$P_e = K_e B_m^2 f^2 t^2$$

Where K_h = hysteresis constant, depends on the core material.

K_e = eddy current constant depends on the thickness of laminations

B_m = maximum flux density

f = frequency of supply

t= thickness of the core

v= volume of the core

- The value of the exponent x (called Steinmetz's constant) varies from 1.5 to 2.5 depending upon the magnetic properties of the core material. Therefore, the total core loss is

$$P_c = K_h f v B^{1.6}_m + K_e f^2 B^2_m t^2$$

Ohmic Loss: due to winding resistances and rms currents

- When a transformer is loaded, ohmic loss (I^2R) occurs in both the primary and secondary winding resistances.

These losses are occur due to ohmic resistance of the transformer windings. If I1 and I2 are the primary and secondary current and R1 and R2 are the primary and secondary resistances then the copper loss in the windings will be $I_1^2 R_1 and\ I_2^2 R_2$ respectively.

Therefore, the total copper loss will be:

$$P_c = I_1^2 R_1 + I_2^2 R_2$$

2.5. Equivalent circuit of transformer

It is simply the circuit representation of the equation describing the performance of the device. The simplified equivalent circuit of a transformer is drawn by representing all the parameters of the transformer either on the secondary side or on the primary side.

Equivalent circuit of transformer should have to model the transformer copper and core losses the leakage flux and the magnetizing flux.

The equivalent circuit diagram of transformer is given below:-

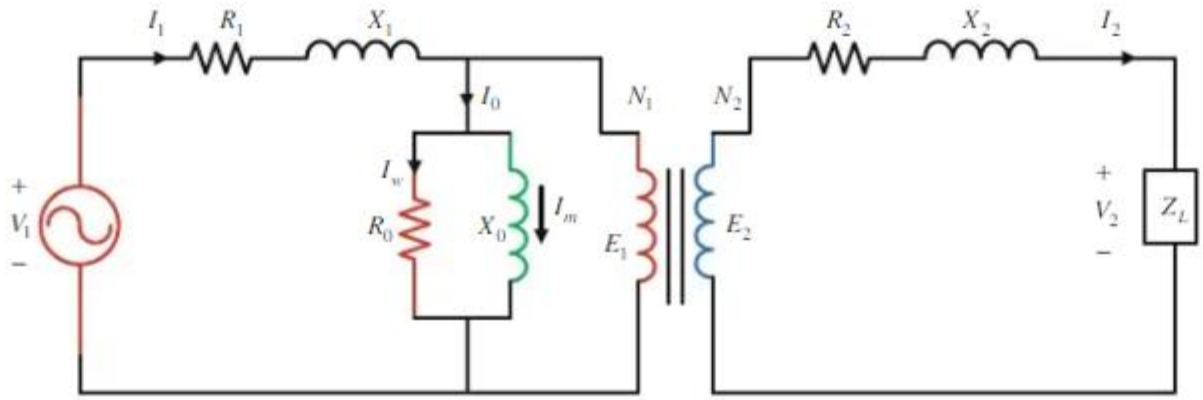


Fig.2.4: Equivalent circuit of transformer

Where,

$V_1, V_2 = rms\ value\ of\ therminal\ voltages$

$E_1, E_2 = rms\ value\ of\ internal\ nduced\ voltages$

$X_0 = magnetizing\ reactance$

$I_m = magnetizing\ current$

$I_w = core\ loss\ current$

$R_{1,}R_2 = primary\ and\ secondary\ \ winding\ resistances$
$I_0 = No\ load\ curren$

<u>Ideal Transformer:</u> Ideal transformer is a transformer, which is assumed to satisfy the following criteria:

- Zero leakage flux: Flux produced by the primary and secondary current are confined(kept or narrowed) within the core
- The winding have no resistances: no copper losses exist; induced voltages are equal to applied voltages.
- The core has infinite permeability: Reluctance of the core is zero; negligible current is required to establish magnetic flux
- Loss-less magnetic core: No hysteresis or eddy currents losses

2.5.1 Equivalent Circuit of Transformer Referred to Primary and secondary side

The complete equivalent circuit of transformer referred to primary is:

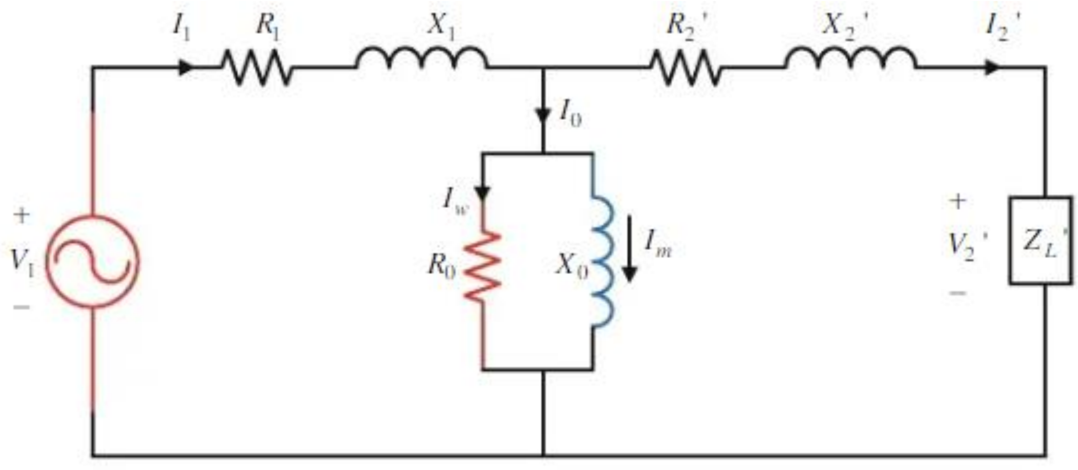


Fig.2.5 Equivalent circuit of transformer referred to primary side

$R_2' = a^2 R_2$ $\qquad x_2' = a^2 x_2$ $\qquad I_2' = \frac{I_2}{a}$

$Z_L' = a^2 Z_L$ $\qquad V_2' = aV_2$ $\qquad E_1 = aE_2$

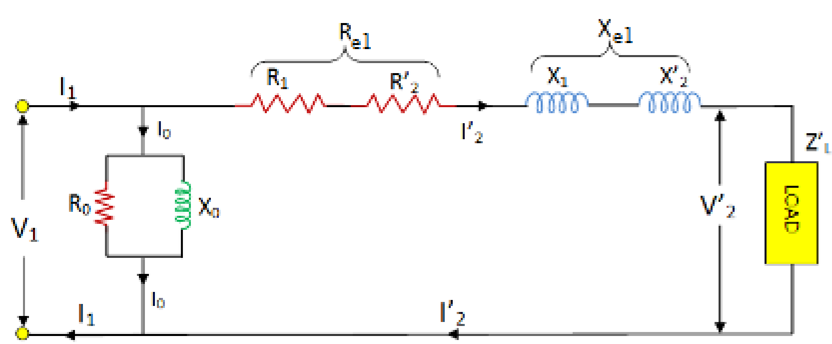
The equivalent resistance referred to primary side is given as

$$R_{e1} = R_1 + R_2' = R_1 + a^2 R_2$$

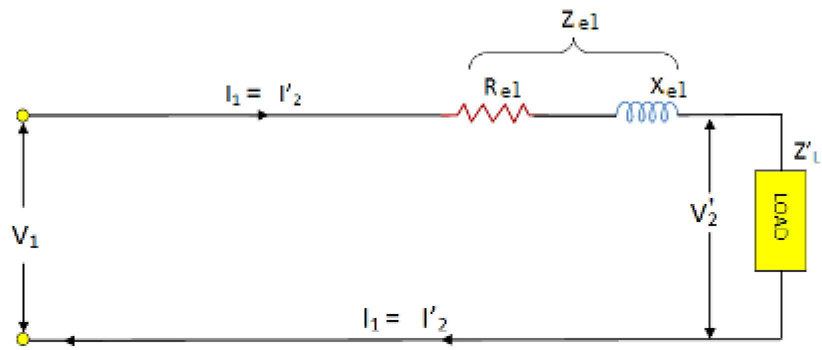
The equivalent reactance referred to primary side is given as

$$X_{e1} = X_1 + X_2' = X_1 + a^2 X_2$$

This can be simplified as,



At last, the circuit is simplified by omitting I_0 altogether as shown below



The complete equivalent circuit of transformer referred to secondary is:

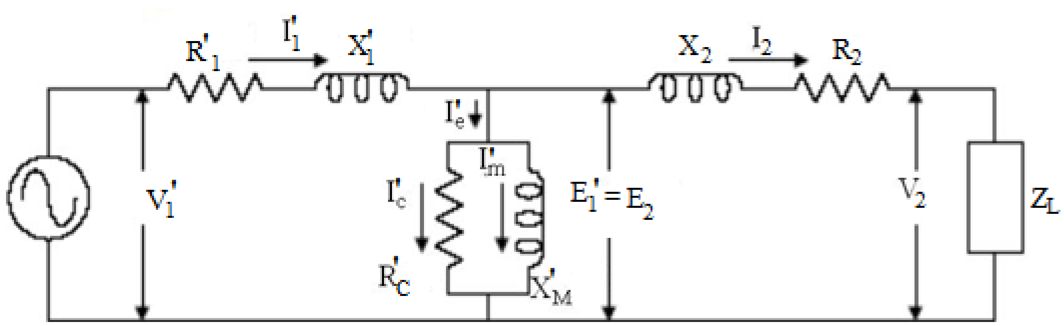


Fig.2.6 Equivalent circuit of transformer referred to secondary side

$V_1' = \frac{V_1}{a}$ $R_1' = \frac{R_1}{a^2}$ $I_1' = aI_1$ $x_1' = \frac{X_1}{a^2}$ $I_e' = aI_e$

$I_c' = aI_c$ $I_m' = aI_m$ $R_c' = \frac{R_c}{a^2}$ $x_M' = \frac{x_M}{a^2}$ $E_1' = \frac{E_1}{a}$

The equivalent resistance referred to secondary side is given as

$$R_{e2} = R_2 + R_1' = R_2 + \frac{R_1}{a^2}$$

The equivalent reactance referred to secondary side is given as

$$X_{e2} = X_2 + X_1' = X_2 + \frac{X_1}{a^2}$$

Approximate Equivalent Circuit of Transformer

Since I_o is very small compared to I_1, it is less than 5% of full load primary current, I_o changes the voltage drop insignificantly. Hence, it is good approximation to ignore the excitation circuit in approximate equivalent circuit of transformer.
All quantities have been referred to the same side of the transformer, and the ideal transformer may be omitted from the equivalent circuit.

1. The first step in the simplification process is to move the shunt magnetization branch from the middle of the T circuit to either the primary or secondary terminal, as shown in Fig below. This step neglects the voltage drop across the primary or secondary winding caused by the exciting current. The voltage drop caused by the load component of the current is still included, of course.

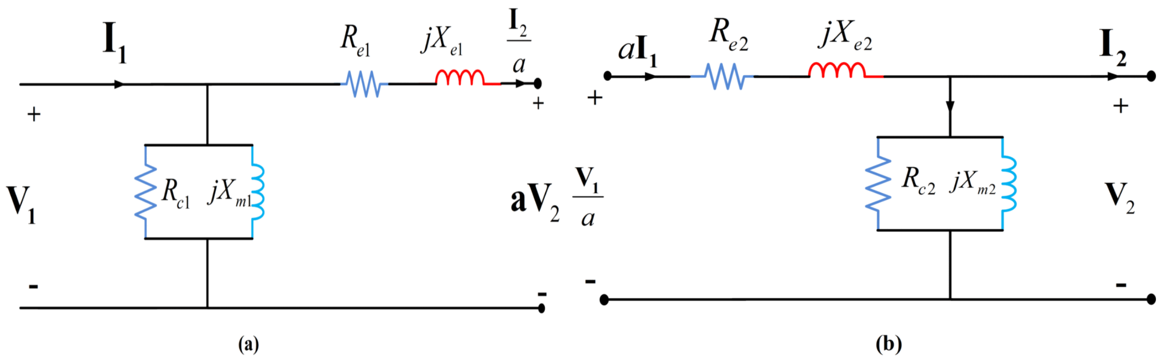


Fig.2.7 Approximate Equivalent Circuit referred to (a) primary (b) secondary.

2. The primary and secondary winding resistances are combined to give either the equivalent resistance referred to the primary side or the equivalent resistance referred to the secondary side.
3. Similarly, the primary and secondary winding reactance are combined to obtain either the equivalent reactance referred to the primary side or the equivalent reactance referred to the secondary side.
4. The next step in deriving the approximate equivalent circuit is the deletion of the shunt magnetizing branch completely. Thus, the transformer equivalent circuit reduces to a simple equivalent series impedance referred to either primary or secondary.

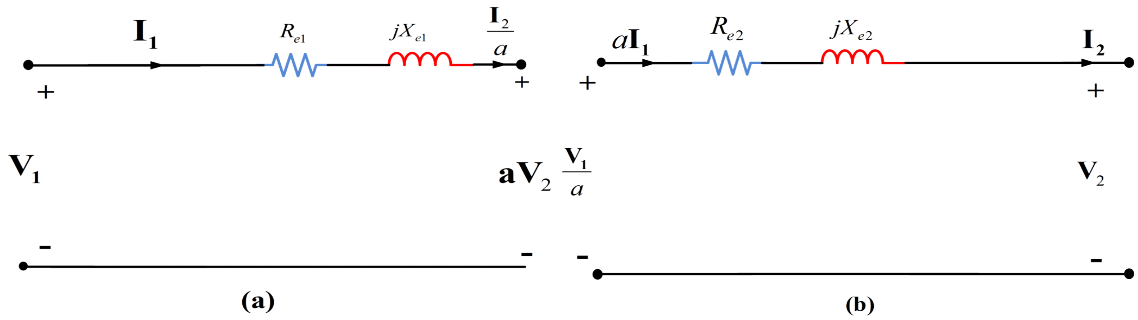


Fig.2.8 Simplified Approximate Equivalent Circuit

The important points for transferring parameters are

- R_1in the primary becomes $\frac{R_1}{a^2}$ when referred to the secondary,
- R_2in the secondary becomes $a^2R_2$when referred to the primary,
- X_1in the primary becomes $\frac{X_1}{a^2}$ when referred to the secondary, and
- X_2in the secondary becomes $a^2X_2$when referred to the primary,

Example: The number of primary and secondary turns of a single phase transformer are 300 and 30, respectively. The secondary coil is connected with a load impedance of 4Ω. Calculate the (i) turns ratio, (ii) load impedance referred to the primary, and (iii) primary current if the primary coil voltage is 220 V.

Solution

$$a = \frac{N_1}{N_2} = \frac{300}{30} = 10$$

$Z_L' = a^2Z_L = 10^2 \times 4 = 400\Omega$

$$I_1 = \frac{V_1}{Z_L'} = \frac{220}{400} = 0.05A$$

Exercise: A load impedance of 8Ω is connected to the secondary coil of a $400/200$ turns single-phase transformer. Determine the (i) turns ratio, (ii) load impedance referred to primary and (iii) primary current if the primary coil voltage is 120 V.

2.6. Voltage Regulation

The voltage regulation of a transformer is the change in the magnitude of secondary terminal voltage from no load to full load when the primary voltage is constant. The voltage regulation determines the ability of the transformer to provide the constant voltage for variable loads. It is usually expressed as a percentage of the full-load value as

$$\%\, VR = \frac{V_{2(NL)} - V_{2(FL)}}{V_{2(FL)}} \times 100$$

$$VR = \frac{E_2 - V_2}{E_2}$$

Where, E_2 – secondary terminal voltage at no load $V_{2(NL)} = E_2$

V_2 – secondary terminal voltage at full load

2.7. Efficiency

Due to the losses describe above the output power of a transformer is always less than the input power.

$$\eta = \frac{Output\ active\ power}{Input\ active\ power} \times 100\%$$

$$\eta = \frac{P_o}{P_{in}} \times 100\%$$

$$P_o = V_2 I_2 cos\theta_2 \qquad P_{in} = P_o + P_{Loss}$$

θ_2 *is the load power factor angle*

$\eta = \frac{V_2 I_2 cos\theta_2}{V_2 I_2 cos\theta_2 + P_{Loss}}$ $\quad$ *where* $P_{Loss} = P_{l(cu)} + P_{l(iron)}$

2.8. Auto Transformer

A transformer, having only one winding a part of which acts as a primary winding and the other as secondary is called an autotransformer. When the primary and secondary voltage are derived from the same winding, the transformer is called an autotransformer. In this type of transformer, a single coil is wound on to a steel core, the primary and secondary windings being part of one winding.

The autotransformer has a low cost, better regulation and low losses. The autotransformer is not used for interconnecting the high voltage and low voltage system. It is used in the places where slight variation is required. When 400000V (400kV) has to be transformed (stepped down) to 132kV, huge transformers are required.

- The advantages of autotransformer decreases as the ratio of transformation increases. Therefore, an autotransformer has advantages only for low values of transformation ratio.

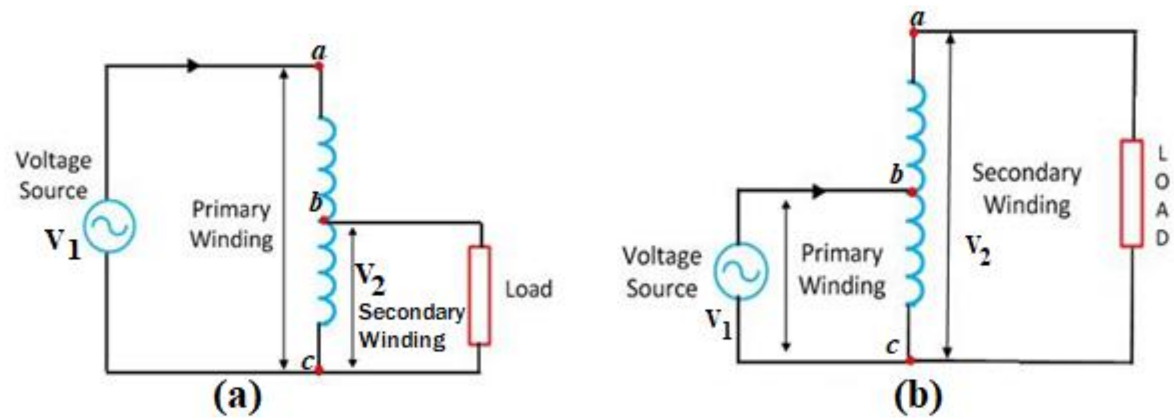
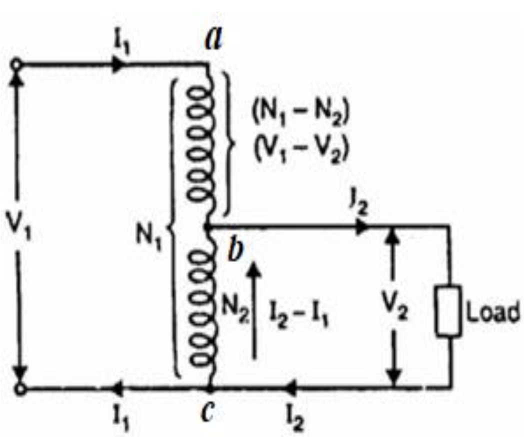


Figure. Autotransformers: (a) step-down; (b) step-up.



- Winding a-c - N_1 turns - primary winding
- winding b-c - N_2 turns - secondary winding
- Input current is I_1
- Output current is I_2
- Portion a-b of the wdg has N_1 - N_2 turns and V_1 - V_2 voltage.
- The current through the common portion of the winding is I_2 - I_1.

Step-down autotransformer:

The input voltage V1 is connected to the complete winding (a-c) and the load RL is connected across a portion of the winding, that is, (b-c).The voltage V2 is related to V1 as in the conventional two-winding transformer, that is,

$$\frac{\mathrm{V_1 - V_2}}{\mathrm{V_2}} = \frac{\mathrm{N_1 - N_2}}{\mathrm{N_2}}$$

$$(\mathrm{N_1 - N_2})\mathrm{V_2} = (\mathrm{V_1 - V_2})\mathrm{N_2}$$

$$\mathrm{V_2N_1 - V_2N_2 = V_1N_2 - V_2N_2}$$

$$V_2N_1 = V_1N_2$$

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = a$$

$$V_2 = V_1 \times \frac{N_{bc}}{N_{ac}}$$

Where $N_{bc}$and N_{ac} are the number of turns on the respective windings. The ratio of voltage transformation in an autotransformer is the same as that for an ordinary transformer, namely,

$$a = \frac{N_{ac}}{N_{bc}} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

with a>1 for step-down.

When a voltage V_1 is applied to the primary of the autotransformer, the induced voltages are related by

$$\frac{E_1}{E_2} = \frac{E_{ac}}{E_{bc}} = \frac{N_{ac}}{N_{bc}} = a$$

Step-up autotransformer:

The input voltage V1 is connected to the portion of winding (b-c) and the load is connected across a the winding (a-c).The voltage V2 is related to V1 as in the conventional two-winding transformer, that is,

$$V_2 = V_1 \times \frac{N_{ac}}{N_{bc}}$$

Where N_{ac} and N_{bc} are the number of turns on the respective windings.

$$a = \frac{N_{bc}}{N_{ac}} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

with a<1 for step-up.

When a voltage V_1 is applied to the primary of the autotransformer, the induced voltages are related by

$$\frac{E_1}{E_2} = \frac{E_{bc}}{E_{ac}} = \frac{N_{bc}}{N_{ac}} = a$$

Disadvantage of autotransformers:

- There is a direct connection between the primary and secondary. Therefore, the output is no longer isolated from the input.
- It is not safe for stepping down a high voltage to a low voltage.

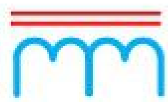
Key Differences between Autotransformer and Transformer

1. An autotransformer has only one winding which acts both as a primary and the secondary whereas the conventional transformer has a two separate windings, i.e., the primary and the secondary winding.
2. The auto-transformer works on the principle of self-induction where as the conventional transformer works on the principle of mutual induction
3. The autotransformer is smaller, whereas the conventional transformer is larger.
4. The voltage regulation and efficiency of an auto-transformer is much better than the conventional transformer
5. The efficiency of an auto-transformer is more, whereas the efficiency of conventional transformer is less
6. Since autotransformer has only one winding. Thus, less conductor is required for winding as compared to the conventional transformer.
7. The primary and secondary windings of the autotransformer are not electrically insulated whereas the windings of the conventional transformer are electrically insulated from each other.

8. The leakage flux and resistance of an autotransformer are low because it has only one winding whereas it is high in the conventional transformer.
9. The losses in the autotransformer are less as compared to the conventional transformer.
10. Output voltage of an autotransformer is variable and whereas conventional transformers has constant output voltage.

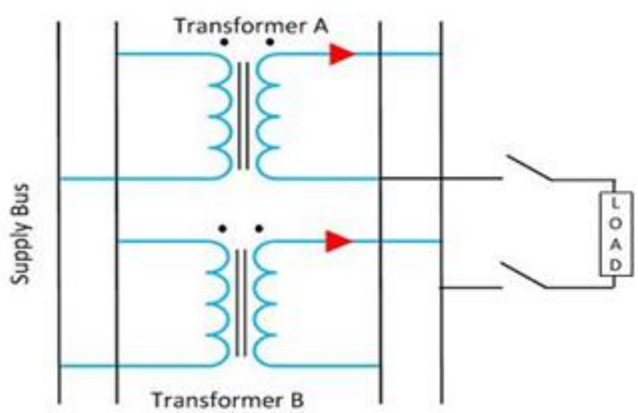
Similarities: The autotransformer and the conventional transformer both work on the principle of electromagnetic induction.

Symbol of autotransformer is:



2.9. Parallel Operation of Transformer

The Transformer is said to be in Parallel Operation when their primary windings are connected to a common voltage supply, and the secondary windings are connected to a common load. The connection diagram of the parallel operation of a transformer is shown in the figure below.



The parallel operation of a transformer has some advantages likes it increases the efficiency of the system, makes the system more flexible and reliable. However, it increases the short-circuit current of the transformers.

Conditions for parallel operation of Transformers

The two main necessary conditions are

- **Polarities** of the transformers must be same.
- **Turn Ratio** of the transformer should be equal.

Reasons for Parallel Operation:

- It is impractical and uneconomical to have a single large transformer for heavy and large loads.
- If the transformers are connected in parallel, so there will be scope in future, for expansion of a substation to supply a load beyond the capacity of the transformer already installed.
- If there will be any breakdown of a transformer in a system of transformers connected in parallel, there will be no interruption of power supply, for essential services.
- If any of the transformer from the system is taken out of service for its maintenance and inspection, the continuity of the supply will not get disturbed.

Example 1:

A transformer has 600 turns on the primary and 3000 turns on the secondary side. If the input voltage to the primary is 220 V, determine

a) The transformation ratio of the transformer

b) The secondary output voltage of the transformer

Solution:

a) $a = \frac{N_1}{N_2} = \frac{600}{3000} = 0.2$ b) $\frac{N_1}{N_2} = \frac{V_1}{V_2} \Longmapsto V_2 = \frac{N_2}{N_1} \times V_1 = \frac{3000}{600} \times 220 = 1100\, V$

Example 2:

A 200 KVA, 6600/400 V, 50Hz single phase transformer has 80 turns on the secondary side. Calculate

a) The primary and secondary currents

b) The number of turns of the primary

Solution:

Ignoring losses, the input apparent power equals the output power

a) $S_{in} = V_1 I_1 = 200\ KVA \rightarrow\ I_1 = \frac{S_{in}}{V_1} = \frac{200KVA}{6600} = 30.3A$

$S_0 = V_2 I_2 = 200KVA\ \rightarrow I_2 = \frac{200KVA}{400} = 500A$

b) $\frac{N_1}{N_2} = \frac{V_1}{V_2}\ \rightarrow\ N_1 = \frac{V_1}{V_2} \times N_2 = \frac{6600}{400} \times 80 = 1320\ turns$

Example 3:

A single-phase transformer has the following parameters,

$R_1 = 0.3\Omega$ $X_1 = 2\Omega$ $\mathrm{R}_2 = 1.5\Omega$ $\mathrm{X}_2 = 4\Omega$ $R_c = 6K\Omega$

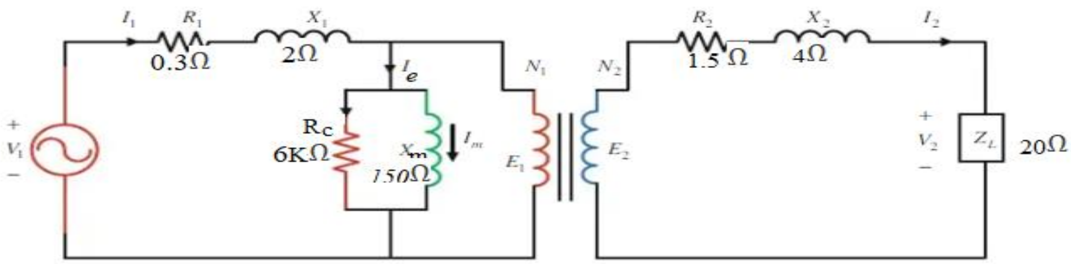
$\mathrm{X_m} = 150\Omega$ $\mathrm{N}_1 = 400\ turns$ $N_2 = 800\ turns\ \&\ load\ impedance\ Z_L = 20\Omega$

a) Draw the complete equivalent circuit without approximation.

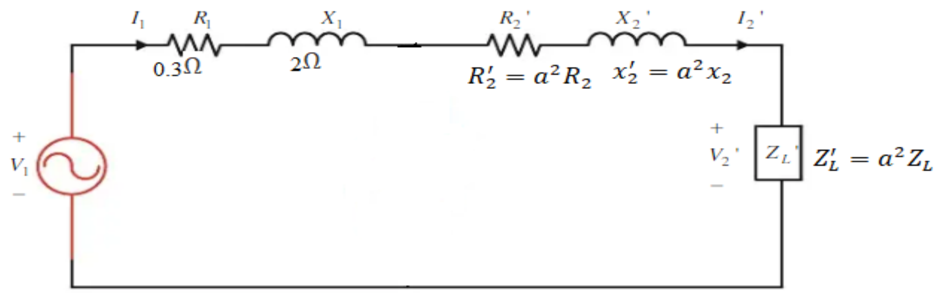
b) Draw the approximate equivalent circuit referred to the primary ignoring the excitation component.

Solution

a)

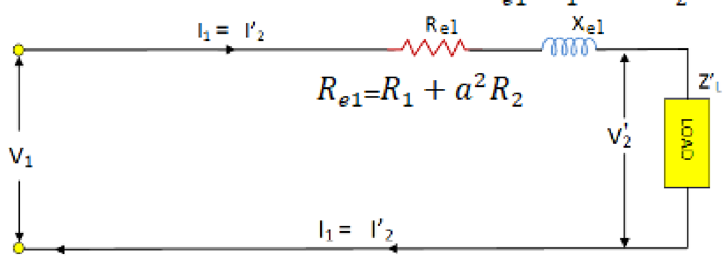


b)



This can be further simplified as,

X_{e1}=$X_1 + a^2X_2$



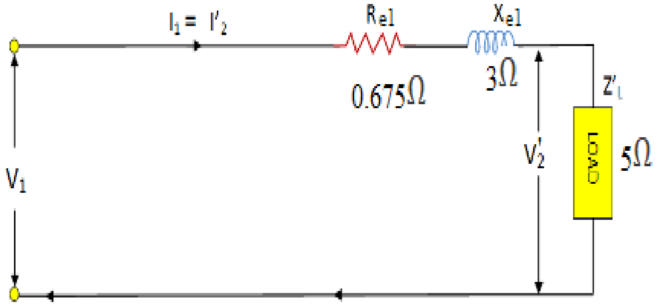
$$a = \frac{N_1}{N_2} = \frac{400}{800} = 0.5$$

$R_2' = a^2R_2 = 0.5^2 \times 1.5\Omega = 0.375\Omega$

$X_2' = a^2X_2 = 0.5^2 \times 4\Omega = 1\Omega$

$\mathrm{Z}_\mathrm{L}' = \mathrm{a}^2\mathrm{Z}_\mathrm{L} = 0.5^2 \times 20\Omega = 5\Omega$

Lastly



$$R_{eq1} = R_1 + R_2' = 0.3\Omega + 0.375\Omega = 0.675\Omega$$

$$X_{eq1} = X_1 + X_2' = 2\Omega + 1\Omega = 3\Omega$$

Example 4:

A 20 KVA 2000/200 V single-phase transformer has a primary and secondary resistance of 2.1Ω and 0.026Ω respectively. If the total iron loss is 200W, determine the efficiency of the transformer at full load with a load power factor 0.5 lagging.

Solution:

$\eta = \frac{P_o}{P_{in}} \times 100\%$ $\quad S = 20\ KVA$ $\quad cos\theta_{pf} = 0.5$

$$P_o = V_2 I_2 cos\theta_{pf} = Scos\theta_{pf} = 20{,}000 \times 0.5 = 10KW$$

$$P_{in} = P_o + P_{loss} = P_o + P_{L(ir)} + P_{L(cu)} \qquad P_{L(ir)} = 200W$$

$$P_{L(cu)} = I_1^2 R_1 + I_2^2 R_2$$

Now we have to find I_1 and I_2 $\quad since\ \ S = V_1 I_1 = V_2 I_2$

$I_1 = \frac{S}{V_1} = \frac{20000}{2000} = 10\ A$ and $I_2 = \frac{S}{V_2} = \frac{20000}{220} = 91\ A$

$$P_{L(cu)} = I_1^2 R_1 + I_2^2 R_2 = 425\ W$$

$$P_{in} = P_o + P_{L(ir)} + P_{L(cu)} = 10{,}625\ W$$

Therefore, $\eta = \frac{P_o}{P_{in}} \times 100\% = \frac{10000}{10625} \times 100\% = 94\%$

Example5:

A 23kVA, 2300/230 V, 60 Hz, step-down transformer has the following resistance and leakage-reactance values: $R_1 = 4\Omega$, $R_2 = 0.04\Omega$, $X_1 = 12\Omega$ and $X_2 = 0.12\Omega$. The transformer is operating at 75% of its rated load. If the power factor of the load is 0.866 leading, determine the efficiency of the transformer.

Solution:

Since the transformer is operating at 75% of its rated load, the effective value of the secondary winding current is

$$I_2 = \frac{23000}{230} = 100\ A \times 0.75 = 75A$$

The load current at a leading power factor of 0.866, in phasor form, is

$$I_2 = 75\angle 30^0 A$$

The secondary winding impedance is $Z_2 = R_2 + jX_2 = 0.04 + j0.12\ \Omega$

The induced emf in the secondary winding is

$$E_2 = V_2 + I_2 Z_2 = 230 + (75\angle 30^0)(0.04 + j0.12) = 228.287\angle 2.33^0\ V$$

Since $a = \frac{2300}{230} = 10$

Then $E_1 = aE_2 = 10 \times 228.287\angle 2.33^0\ V = 2282.87\angle 2.33^0\ V$

$$I_1 = \frac{I_2}{a} = 7.5\ \angle 30^0\ A$$

The primary winding impedance is $Z_1 = R_1 + jX_1 = 4 + j12\ \Omega$

Then $V_1 = E_1 + I_1 Z_1 = 2282.87\angle 2.33^0\ V + (7.5\ \angle 30^0)(4 + j12\ )$

$$V_1 = 2269.578\angle 4.7^0\ V$$

$$P_o = V_2 I_2 cos\theta_2 = (230)(75\angle 30^0)(0.866) = 14{,}938.5\ W$$

$$P_{in} = V_1 I_1 cos\theta_1 = (2269.578\angle 4.7^0)(7.5\angle{-30^0})(0.90408) = 15{,}489.86985\ W$$

$$\eta = \frac{P_o}{P_{in}} = \frac{14{,}938.5}{15{,}489.87} \times 100\% = 96.44\%$$

Example 6:

The secondary winding of a transformer has a terminal voltage of

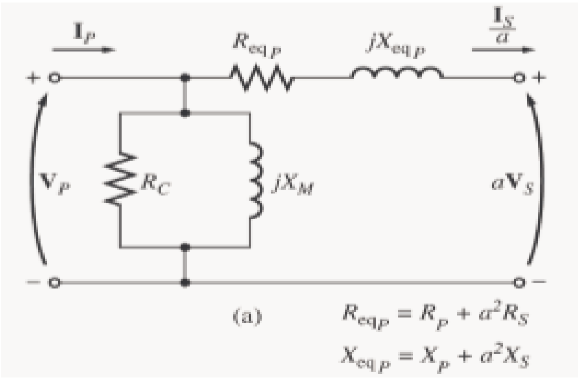
$V_s(t) = 282.8 \sin 377$ V. The turn ratio of the transformer is 100:200. If the secondary current of the transformer is $I_s(t) = 7.07 \sin{(377\text{t} - 36.87^0)}$A, what is the primary current of this transformer? What are its voltage regulation and efficiency? The impedances of this transformer referred to the primary side are

$R_{eq1} = 0.20\Omega$ R_c =300Ω

X_{eq1} =0.750Ω $X_m = 80\Omega$

Solution:

The approximate equivalent circuit of transformer referred to the primary side is



The secondary voltage and current are given as

$$V_s = \frac{282.8}{\sqrt{2}}\angle 0^0\ V = 200\angle 0^0$$

$$I_s = \frac{7.07}{\sqrt{2}}\angle -36.87^0\ A = 5\angle -36.87^0\ A$$

The secondary voltage referred to the primary side is

$$V_s' = aV_s = 100\angle 0^0\ V$$

Then primary voltage is given by

$$V_p = V_s' + I_p\left(R_{eqp} + jX_{eqp}\right) = aV_s + I_s'\left(R_{eqp} + jX_{eqp}\right)$$

But, $I_s' = \frac{I_s}{a} = 10\angle -36.87^0$

$$V_p = 100\angle 0^0 + 10\angle -36.87^0(0.2\Omega + j0.75\Omega) = 106.2\angle 2.6^0\ V$$

The excitation current of this transformer is

$$I_{ex} = I_c + I_m = \frac{106.2\angle 2.6^0 V}{300\Omega} = 0.354\angle 2.6^0 + 1.328\angle -87.4^0$$

$$I_{ex} = 1.37\angle -72.5\ A$$

Therefore, the total primary current of this transformer is

$$I_p = I_s' + I_{ex} = 10\angle -36.87^0 + 1.37\angle -72.5^0 = 11.1\angle -41^0 A$$

The voltage regulation of the transformer at this load is

$$VR = \frac{V_P - aV_s}{aV_s} \times 100\% = \frac{106.2 - 100}{100} \times 100\% = 6.2\%$$

The input power to this transformer is

$$P_{in} = V_p I_p cos\theta = (106.2\ V)(11.1\ A)cos 43.6^0 = 854\ W$$

The output power from this transformer is

$$P_0 = V_s I_s cos\theta = (200)(5)cos(36.87)^0 = 800\ W$$

Therefore, the transformer's efficiency is

$$\eta = \frac{P_o}{P_{in}} \times 100\% = \frac{800}{854} \times 100\% = 93.7\%$$

Example 7:

A single-phase, 10-kVA, 440/110-V, two- winding transformer is connected as an autotransformer to supply a load at 550 V from a 440 V supply as shown below. Calculate the following.

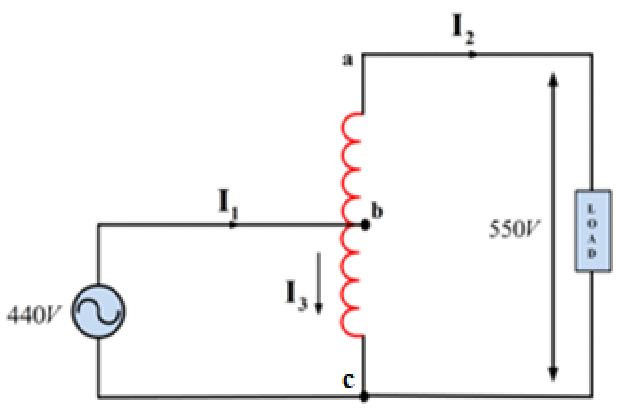
a) KVA rating as an autotransformer

b) Apparent power transferred by conduction

c) Apparent power transferred by electromagnetic induction

Solution:

The single –phase, a two-winding transformer is reconnected as an autotransformer as shown in Fig. below. The current ratings of the windings are given by



$$I_{ab} = \frac{10{,}000}{110} = 90.9\ A$$

$$I_{bc} = \frac{10{,}000}{440} = 22.7\ A$$

At full load , the primary and secondary currents are

$$I_2 = 90.9\ A \quad and \quad I_1 = I_2 + I_3 = 113.6\ A$$

Therefore, the KVA rating of the autotransformer is

$$KVA_1 = (440)(113.6) = 50\ KVA$$

$$KVA_2 = (550)(90.9) = 50\ KVA$$

Transformer Tests

The performance of a transformer can be calculated on the basis of its equivalent ckt which contains four main parameters. These are:

- The equivalent resistance Ro1 as referred to primary (secondary Ro2)
- The equivalent leakage reactance Xo1 and Xo2
- The core loss conductance Go (resistance Ro)
- The magnetizing susceptance Bo (reactance Xo)

The above parameters can be easily determined by the two tests.

- Open ckt test
- Short ckt test

a. Open ckt test

One winding of the transformer is left open while the other is excited by applying the rated voltage. The frequency of the applied voltage must be the rated frequency of the transformer. Although it does not matter which side of the transformer is excited, it is safer to conduct the test on the low-voltage side. Another justification for performing the test on the low-voltage side is the availability of the low-voltage source in any test facility. Figure 2.11 shows the connection diagram for the open-circuit test with ammeter, voltmeter, and wattmeter inserted on the low-voltage side.

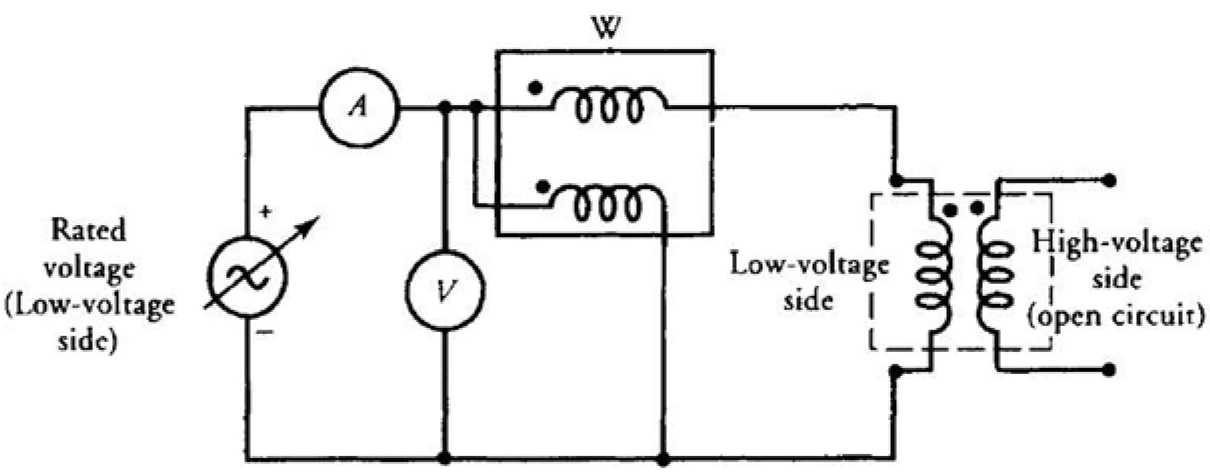


Fig 2.11.A two-winding transformer wired with instruments for open-circuit test

b. Short circuit Test

This test is designed to determine the winding resistances and leakage reactances. The short-circuit test is conducted by placing a short circuit across one winding and exciting the other from an alternating-voltage source of the frequency at which the transformer is rated. Since the short circuit constrains the power output to be zero, the power input to the transformer is low. The low power input at the rated current implies that the applied voltage is a small fraction of the rated voltage.

Therefore, extreme care must be exercised in performing this test. In this case, the wattmeter records the copper loss at full load.

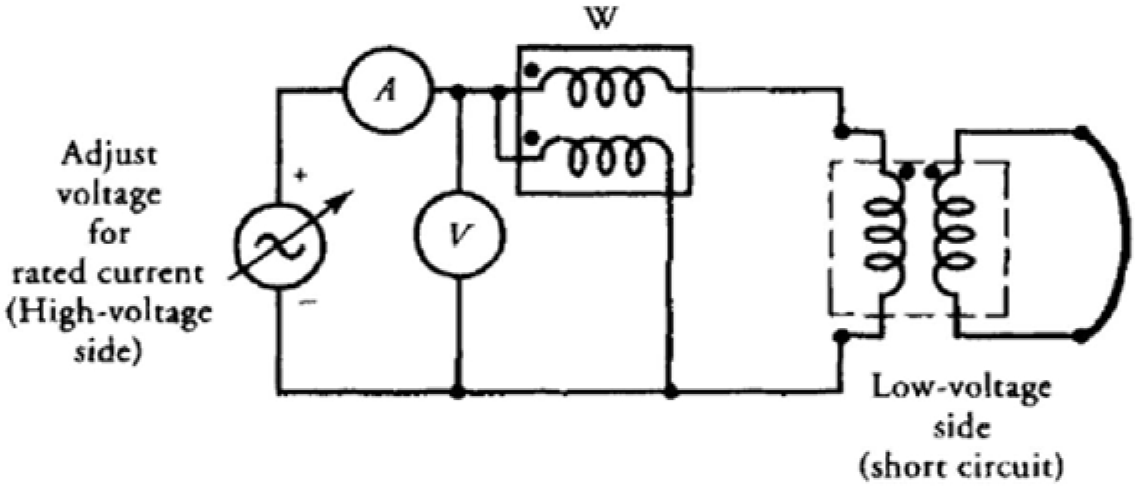


Figure 2.12 a two-winding transformer wired for short-circuit test.

2.6. THREE PHASE TRANSFORMOR

Transformer power levels range from low-power applications, such as consumer electronics power supplies to very high power applications, such as power distribution systems. For higher power applications, three-phase transforms are commonly used. The typical construction of a three-phase transformer is shown in Figure. The detailed analysis of this circuit is not straightforward since there are numerous combinations of flux paths linking various windings. For this reason, the three-phase transformer will be modeled as three independent single-phase transformers herein.

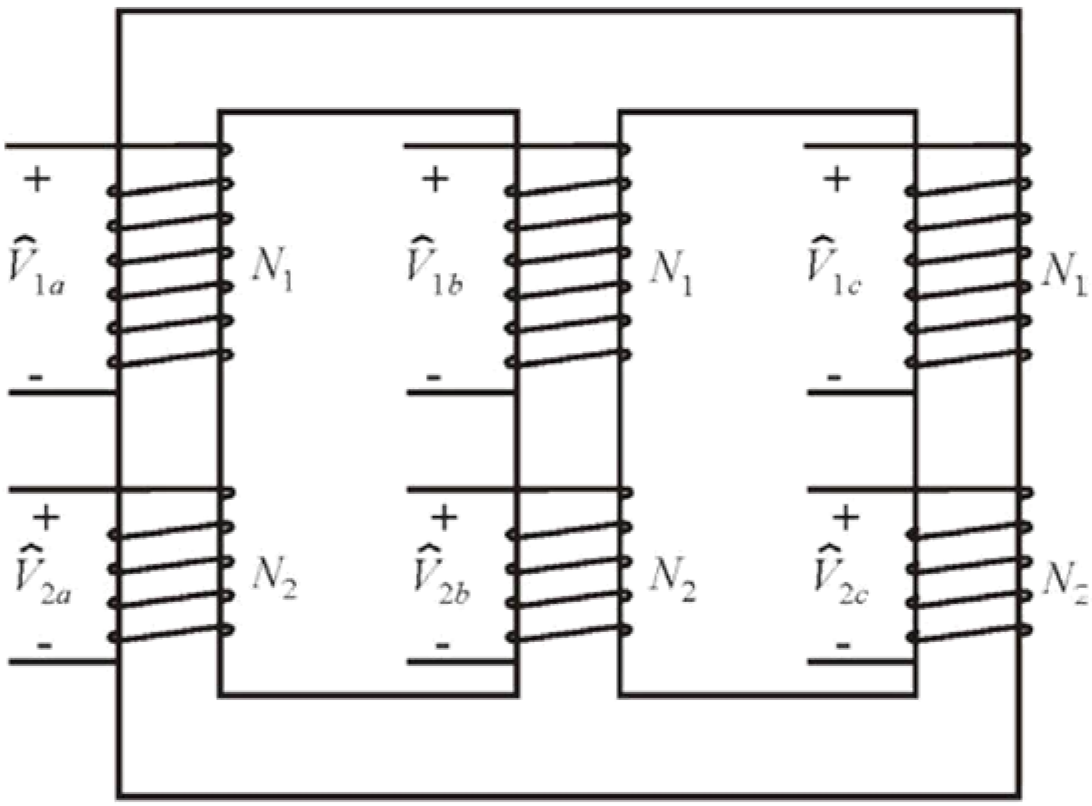


Fig 2.16 Three phase transformer

For practical calculations, it is reasonable to model the three-phase transformer as three ideal transformers as shown in Figure 2.17. Since these transformers are ideal, the secondary voltages are related to the primary voltages by the turn's ratio:

$$\hat{V}_{2a} = \frac{N_2}{N_1}\hat{V}_{1a}$$

$$\hat{V}_{2b} = \frac{N_2}{N_1}\hat{V}_{1b}$$

$$\hat{V}_{2c} = \frac{N_2}{N_1}\hat{V}_{1c}$$

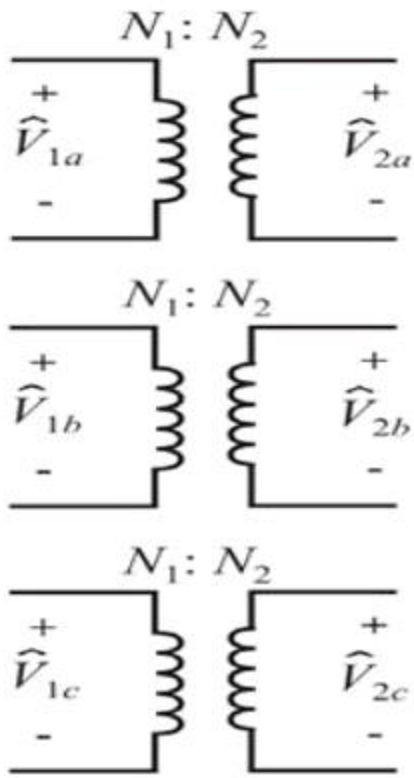


Fig 2.17Three phase transformer ideal model

Considering the individual transformers of Figure 2.17 and that both delta and wye connections are possible for the primary and secondary windings, there are four possible combinations of transformer connections. The most straightforward combinations are the wye-to-wye connection and the delta-to-delta connections shown in Figures 2.20 (S) and (B) respectively. In these cases, the line to-line voltages on the secondary side are directly proportional to those on the primary side through the turn‘s ratio. Therefore, the following relationship holds for both connections.

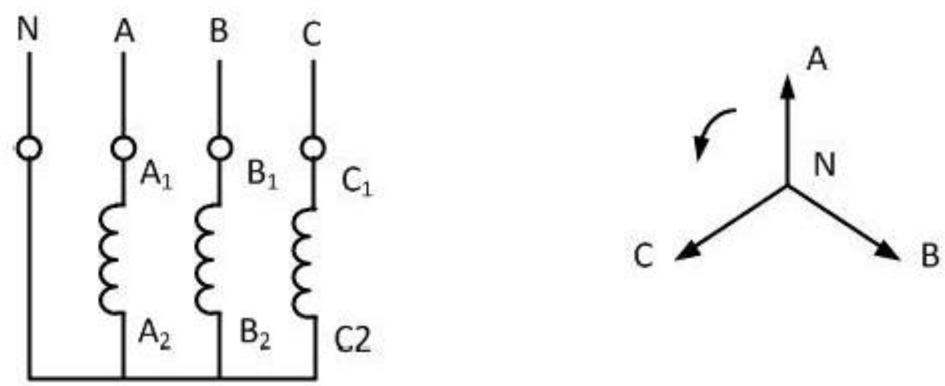
$$\hat{V}_{ab} = \frac{N_2}{N_1}\hat{V}_{AB}$$

Possible Transformer connection

- Star-star or wye-wye connection
- Delta-Delta connection (Δ-Δ)
- Star-Delta connection (Y-Δ)
- Delta-Star connection (Δ-Y))
- Open delta connection (V-V)
- Scott transformer

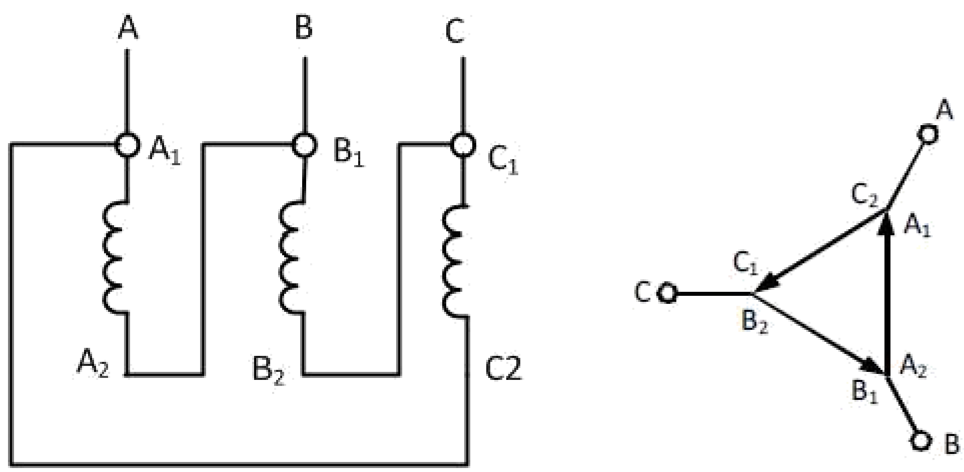
Three-phase circuits are the most economical for a.c. power transmission and distribution. As a consequence, three-phase transformers are the most widely used in power systems. A three phase transformer may be a single unit (all windings wound around the same core, immersed in one tank) or it may be made up of three single-phase units. In practice the choice between one or another type is governed mainly by economic reasons, transportation, future expansion, reliability, etc. See references.

The basic types of polarity connections (Y and Δ) of three-phase circuits are illustrated in Figs. 2.18 and 2.19. Except for the open-delta connection which would be used mainly as a stopgap solution, a three-phase transformer may have its primary and secondary windings connected in any combination of the basic connections shown in Figs. 2.18 and 2.19.



(a) Connection Diagram *(b) Phasor Diagram*

Figure 2.18 Y Connection



(a) Connection Diagram *(b) Phasor Diagram*

Figure 2.19 Δ Connection

In Figures 2.20 (A) and (B), the ideal transforms are schematically rotated by 120o from each other to represent a 120o electrical displacement in the voltages. The voltage labels on the individual coils correspond to the labels in Figure 2.17. An equation similar to (4) can be derived to relate the primary and secondary currents if desired.

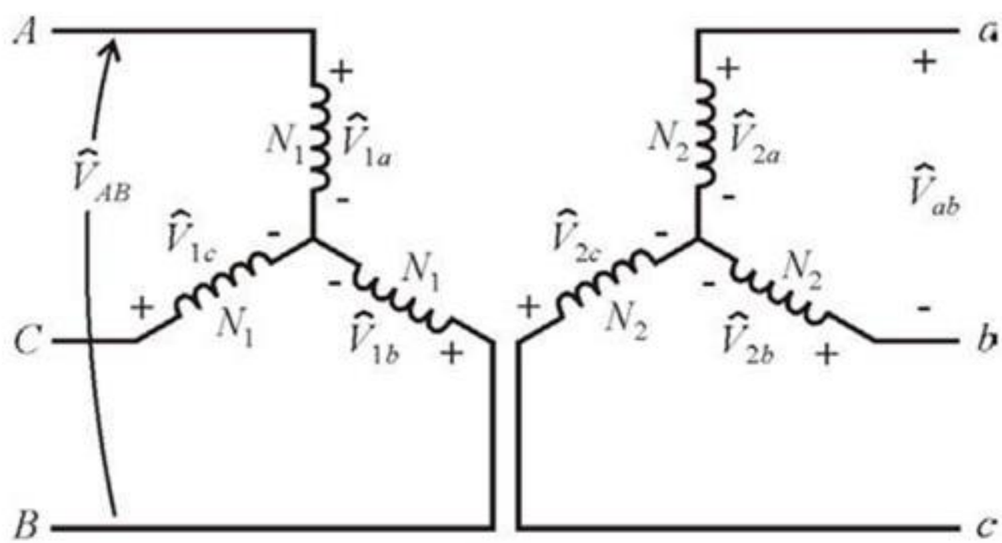


Fig 2.20 (A) Wye-Wye connection

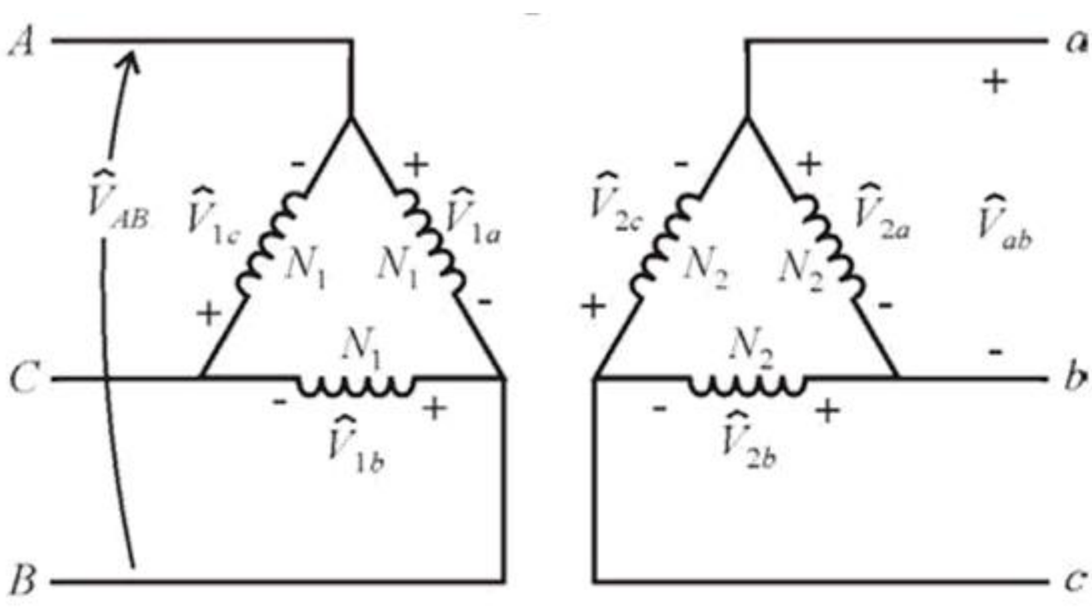


Fig 2.20(B) Delta-Delta connections

The delta-to-wye connection is shown in Figure 2.20 (C). From the ideal transformer equations, it can be determined that

$$\hat{V}_{ab} = \frac{N_2}{N_1}\left(\hat{V}_{1a} - \hat{V}_{1b}\right) = \frac{N_2}{N_1}\left(\hat{V}_{BC} + \hat{V}_{AB}\right)$$

In this analysis A-B-C sequence is assumed. If a source does not have this sequence the input lines can be relabeled so that it does. With this sequence, (5) is equivalent to

As can be seen, the secondary line-to-line voltage leads the primary by 30o. A factor of 3 is also inserted in the magnitude calculation as well. The wye-to-delta connection is shown in Figure 6.

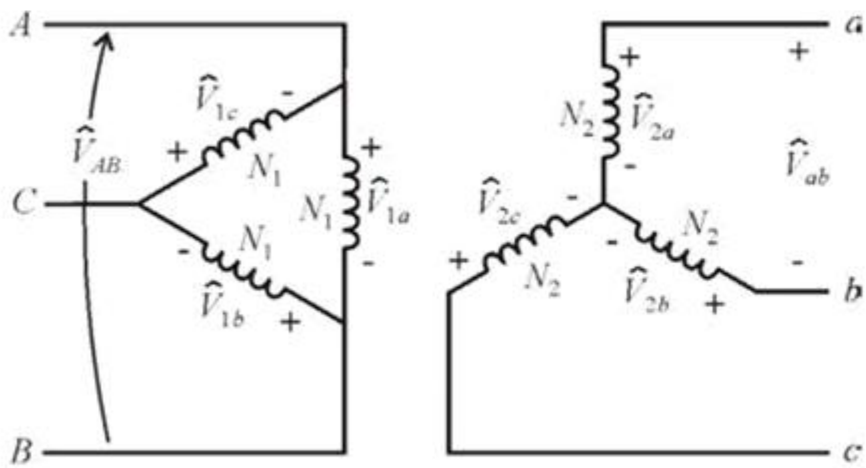


Fig 2.20 (C) Delta- Wye connections

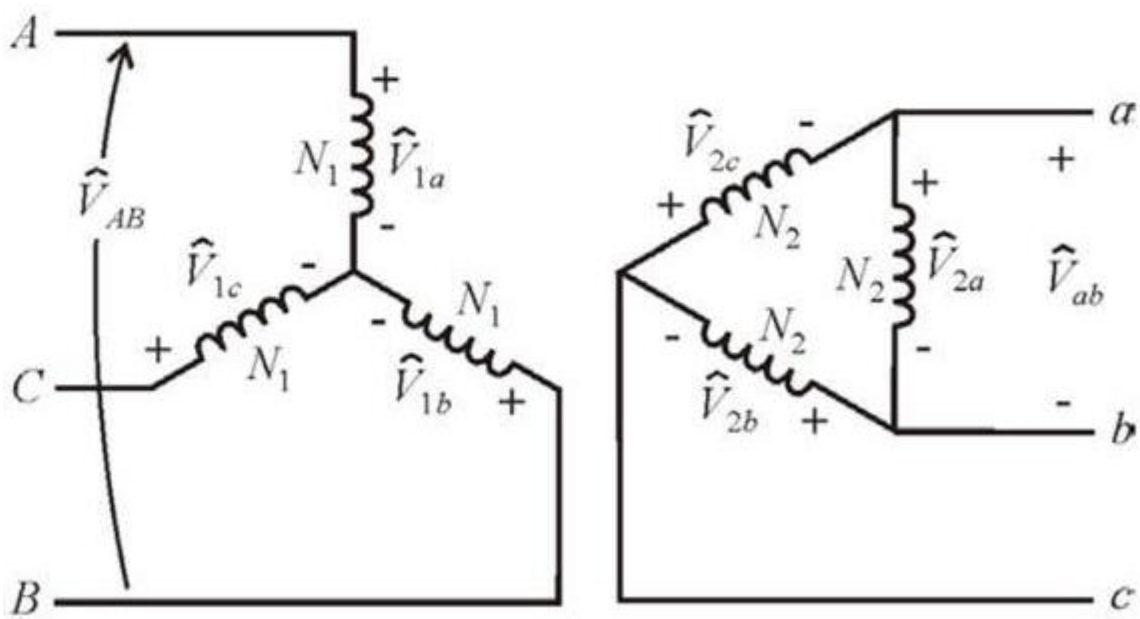


Fig 2.20 (D) Wye - Delta connections

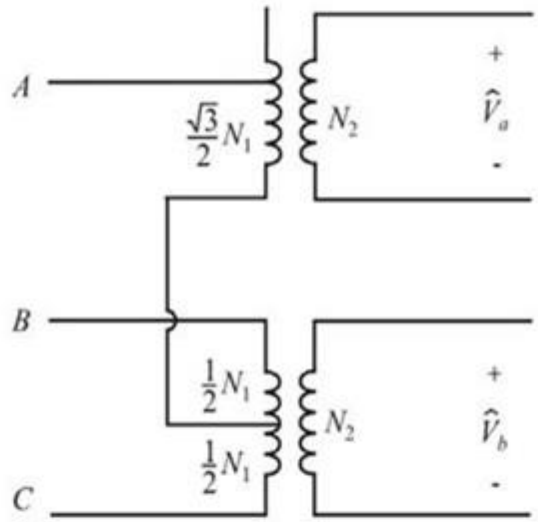


Fig 2.21 Scott-Transformer connection