Chapter Three

Induction machine:

1.1 Introduction

The induction machine represents one of the most useful forms of AC powered electromechanical rotating machines. This device can be constructed with no physical connections to the rotor circuits. The rotor currents are generated because of the magnetic coupling between the stator and rotor. The alternating current is supplied to the stator *directly* and to the rotor by *induction* or *transformer action* from the stator. There are no moving contacts between the stator and the rotor.

An induction motor carries alternating current in both the stator and the rotor windings. An induction motor is a rotating transformer in which the secondary winding receives energy by induction while it rotates.

Working Principle of an Induction Motor

The motor which works on the principle of electromagnetic induction is known as the induction motor. The stator and rotor are two essential parts of the motor. The stator is the stationary part, and it carries the overlapping windings while the rotor carries the main or field winding. The windings of the stator are equally displaced from each other by an angle of 120°.

When the ***3-phase*** *stator winding is energized from a* ***3-phase*** *supply, a rotating magnetic field is produced which rotates around the stator at synchronous speed. The rotating magnetic field cuts the rotor conductors.*

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The conductors of the rotor are stationary. This stationary conductor cut the rotating magnetic field of the stator, and because of the electromagnetic induction, the EMF induces in the rotor. This EMF is known as the rotor induced EMF, and it is because of the electromagnetic induction phenomenon.

The induction motor is the single excited motor, i.e., the supply is applied only to the one part, i.e., stator.

The conductors of the rotor are short-circuited either by the end rings or by the help of the external resistance. The relative motion between the rotating magnetic field and the rotor conductor induces the current in the rotor conductors. As the current flows through the conductor, the flux induces on it. The direction of rotor flux is same as that of the rotor current.

Now we have two fluxes one because of the rotor and another because of the stator. These fluxes interact each other. On one end of the conductor the fluxes cancel each other, and on the other end, the density of the flux is very high. Thus, the high-density flux tries to push the conductor of rotor towards the low-density flux region. This phenomenon induces the torque on the conductor, and this torque is known as the electromagnetic torque.

The direction of electromagnetic torque and rotating magnetic field is same. Thus, the rotor starts rotating in the same direction as that of the rotating magnetic field.

The speed of the rotor is always less than the rotating magnetic field or synchronous speed.

The rotor starts moving without any additional excitation system and because of this reason the motor is called the self-starting motor. Thus, principle of 3 phase induction motor also explains why rotor rotates in same direction as the rotating field and why induction motor is self-starting.

The operation of the motor depends on the voltage induced on the rotor, and hence it is called the induction motor.

Consider the simplified view of a 3 phase induction motor shown below.

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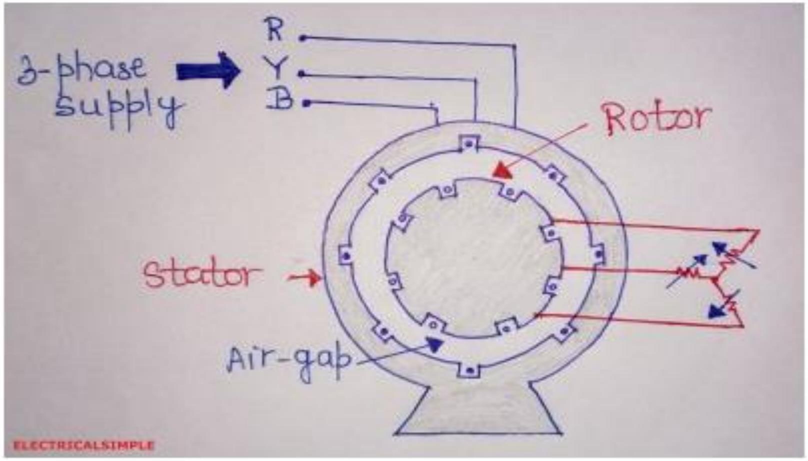

This stator winding is energized from a three phase supply. But, the rotor winding is not energized from any source and is short- circuited on itself.

When rotor winding is short-circuited with no resistance in series, it is called a squirrel cage induction motor and when rotor winding is shorted through a resistance in series, it is called slip ring induction motor.
Whatever be the type, working principle basically remains the same.

- ***The rotor conductors are short circuited, and hence rotor current is produced due to induced emf. That is why such motors are called as induction motors.***
  (This action is same as that occurs in transformers, hence induction motors can be called as rotating transformers.)
- Now, induced current in rotor will also produce alternating flux around it. This rotor flux lags behind the stator flux.
- As the cause of production of rotor current is the relative velocity between rotating stator flux and the rotor, the rotor will try to catch up with the stator RMF. Thus the rotor rotates in the same direction as

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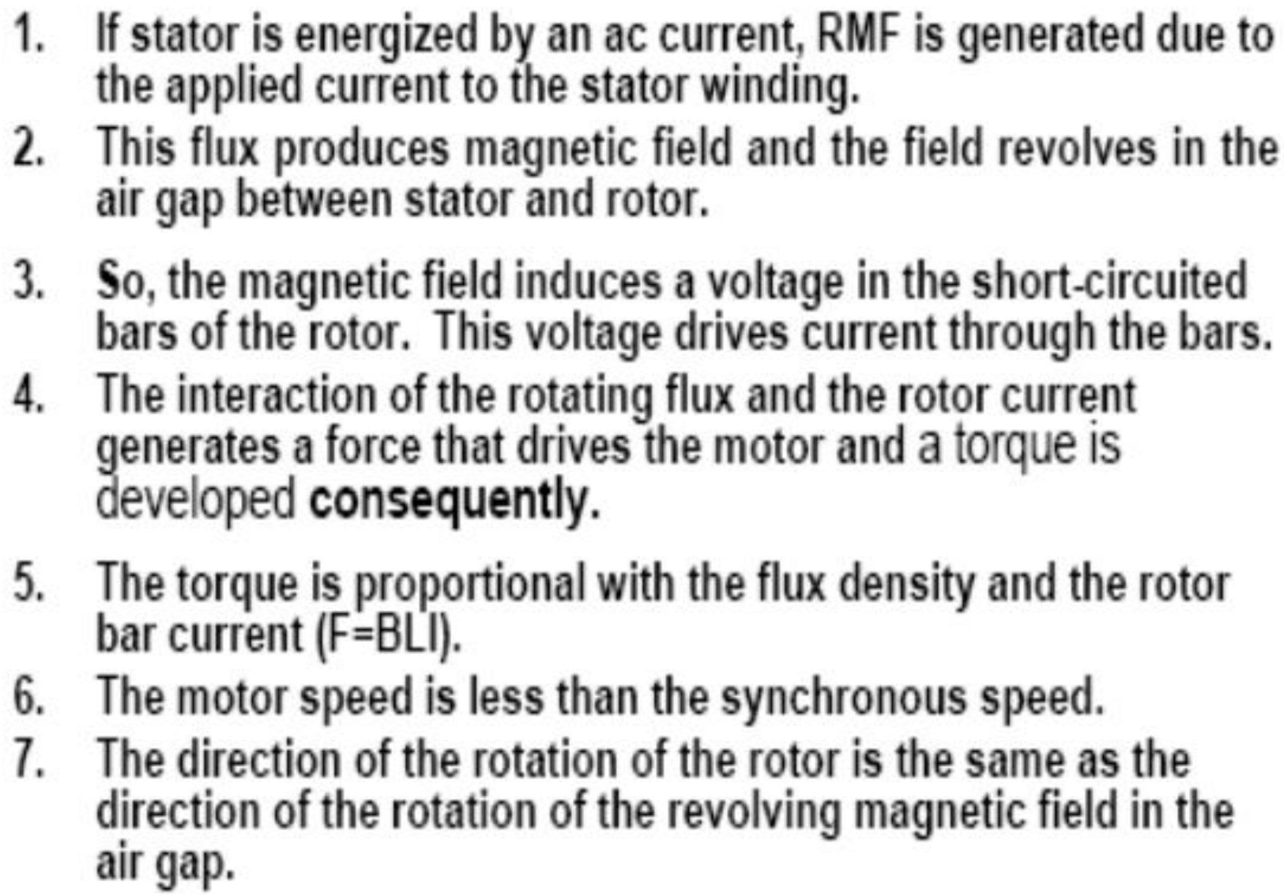
that of stator flux to minimize the relative velocity. However, the rotor never succeeds in catching up the synchronous speed. This is the **basic working principle of induction motor** of either type, single phase of 3 phase.

- In 3ϕ IM the power is transferred from stator to rotor winding through *Induction*
- In the induction machine both stator winding and rotor winding carry *alternating current*.



1. If stator is energized by an ac current, RMF is generated due to the applied current to the stator winding.
2. This flux produces magnetic field and the field revolves in the air gap between stator and rotor.
3. So, the magnetic field induces a voltage in the short-circuited bars of the rotor. This voltage drives current through the bars.
4. The interaction of the rotating flux and the rotor current generates a force that drives the motor and a torque is developed **consequently.**
5. The torque is proportional with the flux density and the rotor bar current (F=BLI).
6. The motor speed is less than the synchronous speed.
7. The direction of the rotation of the rotor is the same as the direction of the rotation of the revolving magnetic field in the air gap.

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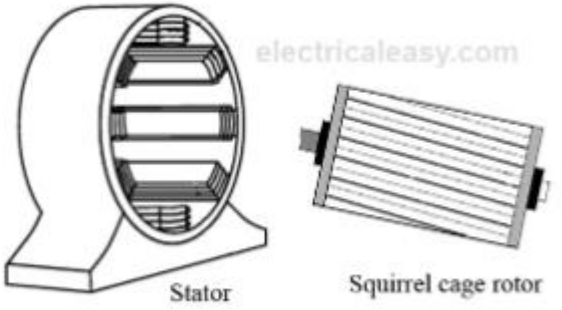
3.1. Classification of induction Motors

- **As regards of their types of input supply**

  A. Single phase induction motors

  B. Three phase induction motors



3-phase induction motors are the most widely used motors in the industries since this type of motor does not require additional starting devices. Because of this 3-phase induction motor is self-starting motor.

Advantage of induction machines

1. It has very simple and extremely rugged, almost unbreakable construction (especially squirrel cage type)
2. Its cost is low and it is very reliable

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3 It has sufficiently high efficiency.
4 It has a reasonably good power factor
5 It requires minimum of maintenance
6 It starts up from rest and needs no extra starting motor and has not to be synchronized. Its starting arrangement is simple especially – for squirrel- cage type motor

7 ability to operate dirty and explosive conditions

Disadvantage of Induction machine

1) Its speed cannot be varied without sacrificing some of its efficiency.
2) Just like a d.c. shunt motor, its speed decreases with increase in load
3) The speed is not easily controlled
4) Large starting current
5) They run at low and lagging power factor when lightly loaded.
6) Its starting torque is somewhat inferior to that of a d.c shunt motor

Construction of IM

- ***A 3 phase induction motor consists of two major parts:***
  1. Stator
  2. Rotor

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Stator: as its name indicates stator is stationary part of IM.

The **stator** of three phase induction motor is made up of numbers of slots to construct a 3 phase winding circuit which we connect with 3 phase AC source. We arrange the three-phase winding in such a manner in the slots that they produce rotating magnetic field when we switch on the three-phase AC supply source.

- Stator winding is placed in the stator of induction motor and 3ϕ-supply is given to it.
- Consists of steel frame that support a hollow cylindrical core**.**

- A three-phase winding is put in slots punched out on the inner surface of the stator frame.
- *Greater the number of poles, lesser the speed and vice versa.*

- The stator windings, when supplied with 3-phase currents , produce a magnetic flux which is of constant magnitude but which revolves (or rotates) at synchronous speed.

$$N_s = \frac{120.f}{P}$$

- This revolving magnetic flux induces an emf in the rotor by mutual induction

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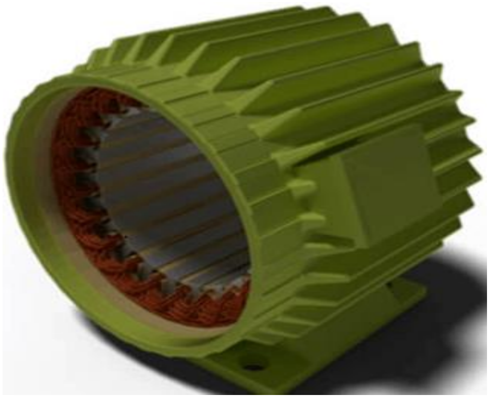

Fig: Stator

Rotor

The **rotor** of three phase induction motor consists of a cylindrical laminated core with parallel slots that can carry conductors. The conductors are heavy copper or aluminum bars fitted in each slot and short-circuited by the end rings. The rotor is a rotating part of IM.

○ It is connected to the mechanical load through the shaft composed of punched laminations providing space for the rotor winding

- aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)
- The frequency of the rotor flux is very low; as a result thicker laminations can be used without excessive iron losses.

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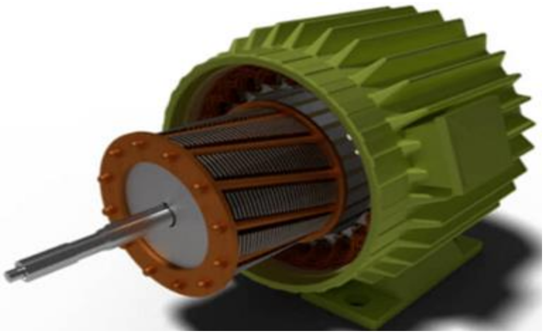

Fig: Rotor inside Stator

Two types of rotor

1. **Squirrel cage rotor**: The rotor consists of Cu. Bars which are short circuited on both sides.

- It is not possible to add external resistance in the rotor circuit

2. **Slip ring rotor**: The rotor consists of 3-Φ distributed winding. The winding is usually connected in star.

The common terminals of three phase windings are connected together and other three are brought outside and connected to slip rings.

- It is possible to add external resistance in the rotor circuit

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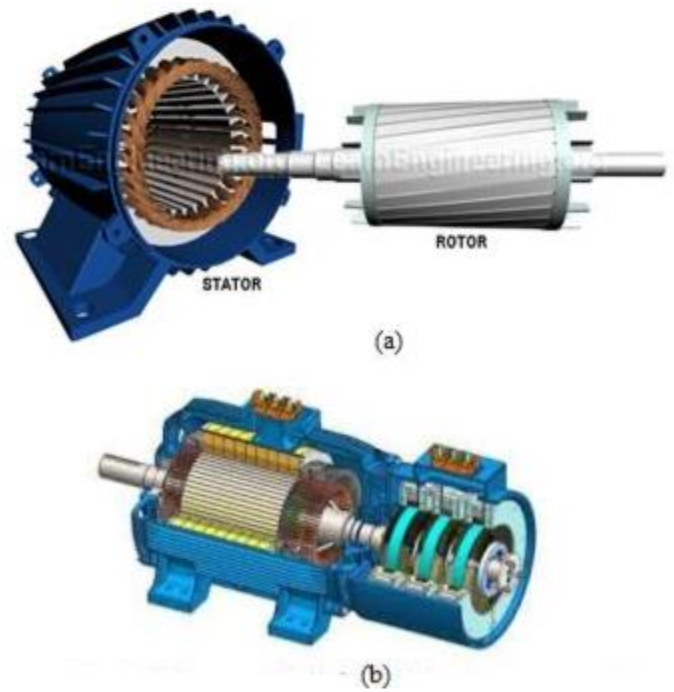

Fig: Three phase induction motor (a) squirrel cage rotor (b) slip ring rotor.

Synchronous Speed:

The rotational speed of the rotating magnetic field is called as synchronous speed.

$$N_s = \frac{120 \times f}{P} \quad \text{(RPM)}$$

Where, f = frequency of the supply
P = number of poles

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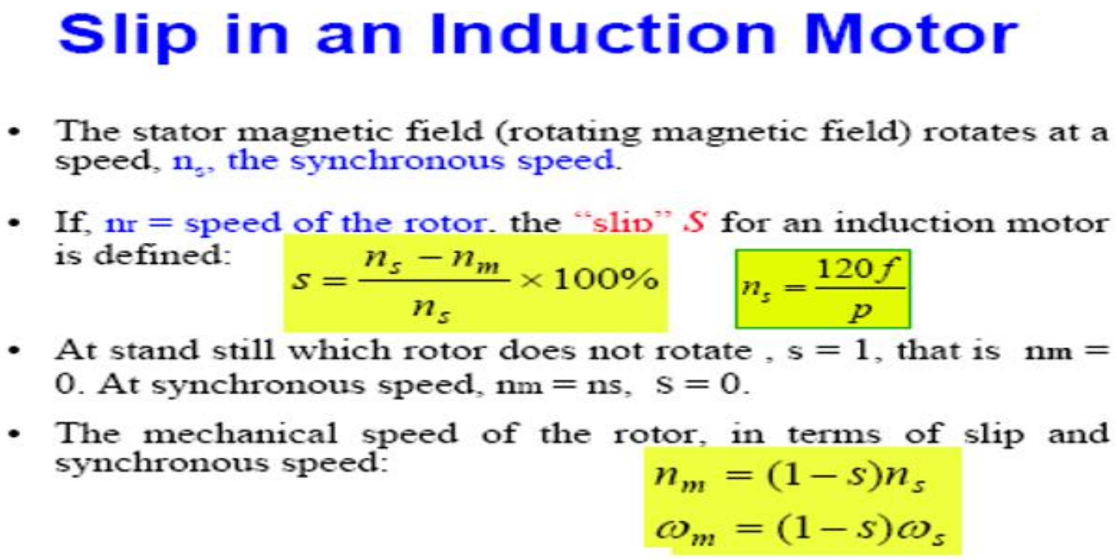
Slip:

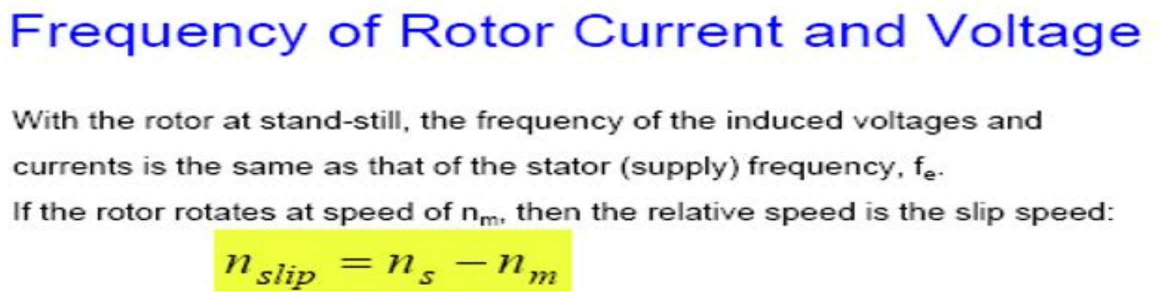
***The difference between the stator (synchronous speed N_s) and actual speed (N) of the rotor (rotor speeds) is called the slip*.**

$$\% \text{ slip} \quad s = \frac{Ns - N}{Ns} \times 100$$



Slip in an Induction Motor

- The stator magnetic field (rotating magnetic field) rotates at a speed, n_s, the synchronous speed.
- If, nr = speed of the rotor, the "slip" S for an induction motor is defined: $$s = \frac{n_s - n_m}{n_s} \times 100\% \qquad n_s = \frac{120f}{p}$$
- At stand still which rotor does not rotate , s = 1, that is nm = 0. At synchronous speed, nm = ns, S = 0.
- The mechanical speed of the rotor, in terms of slip and synchronous speed: $$n_m = (1-s)n_s$$ $$\omega_m = (1-s)\omega_s$$



Frequency of Rotor Current and Voltage

With the rotor at stand-still, the frequency of the induced voltages and currents is the same as that of the stator (supply) frequency, f_e.

If the rotor rotates at speed of n_m, then the relative speed is the slip speed:

$$n_{slip} = n_s - n_m$$

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n_{slip} is the speed responsible for the induction.

But $n_m = n_s(1 - s)$ by definition of slip.

Hence, $n_{slip} = n_s - n_s(1 - s)$, thus the frequency of the induced voltages and currents is, $f_r = sf_e$.

Example no. 1:

A three-phase, 20 hp, 208 V, 60 Hz, six pole, wye connected induction motor delivers 15 kW at a slip of 5%.

Calculate:

a) Synchronous speed

b) Rotor speed

c) Frequency of rotor current

Solution

- Synchronous speed: $n_s = 120\, f / p = (120*60) / 6 = 1200$ rpm
- Rotor speed: $n_r = (1\text{-}s)\, n_s = (1- 0.05)\,(1200) = 1140$ rpm
- Frequency of rotor current: $f_r = s\, f = (0.05)\,(60) = 3$ Hz

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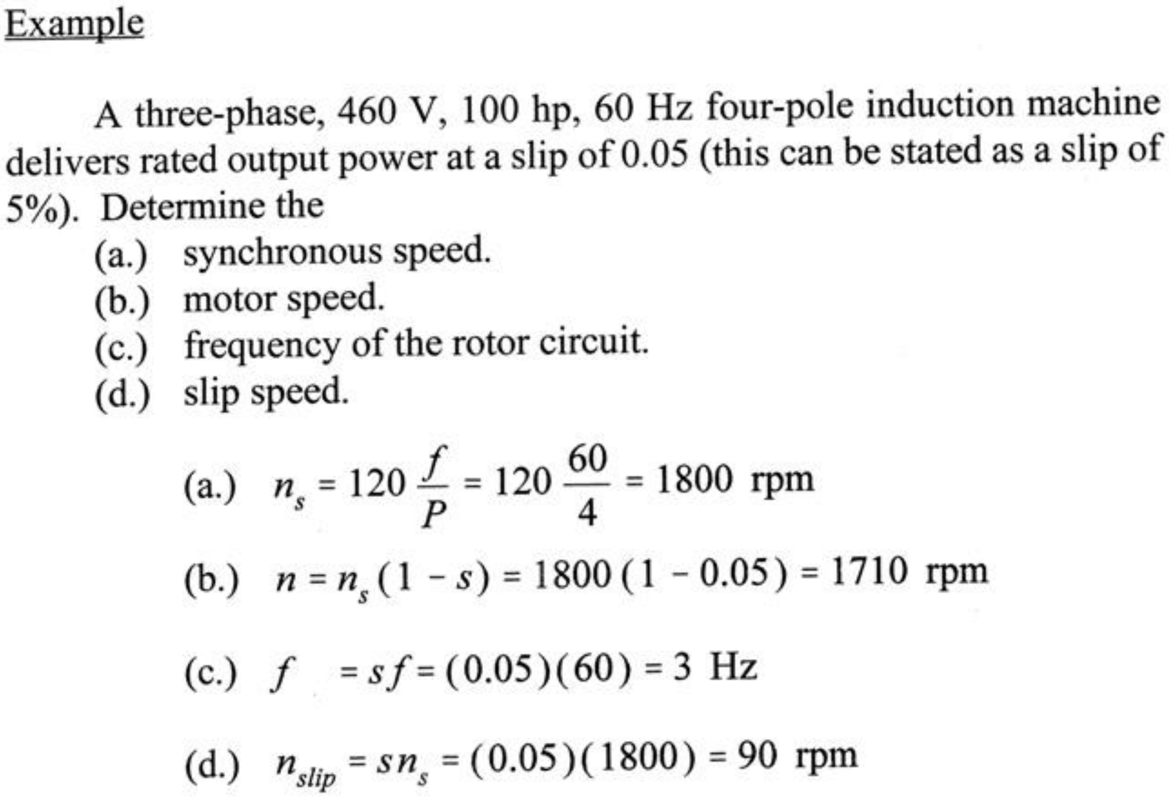

Example

A three-phase, 460 V, 100 hp, 60 Hz four-pole induction machine delivers rated output power at a slip of 0.05 (this can be stated as a slip of 5%). Determine the

(a.) synchronous speed.
(b.) motor speed.
(c.) frequency of the rotor circuit.
(d.) slip speed.

(a.) $n_s = 120\frac{f}{P} = 120\frac{60}{4} = 1800$ rpm

(b.) $n = n_s(1 - s) = 1800(1 - 0.05) = 1710$ rpm

(c.) $f = sf = (0.05)(60) = 3$ Hz

(d.) $n_{slip} = sn_s = (0.05)(1800) = 90$ rpm

Principle of rotating magnetic field

When balanced three phase currents flow through balanced three phase windings, rotating magnetic field of constant magnitude is developed.

The three phase currents are said to be balanced if the three phase currents have time phase difference of 120° with respect to one another and equal in magnitude.

The magnitude of this rotating field is constant and is equal to 1.5ϕm. Where ϕm is the maximum flux due to any phase.

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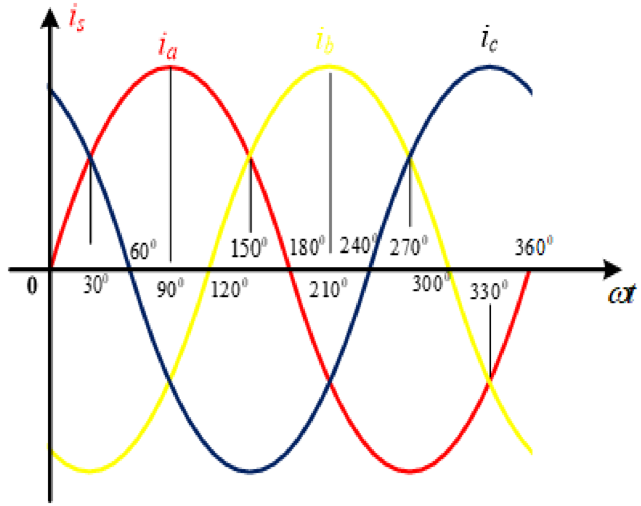
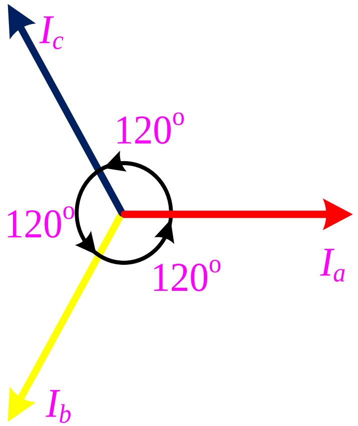


a) Three phase balanced currents



b) Phasor diagram

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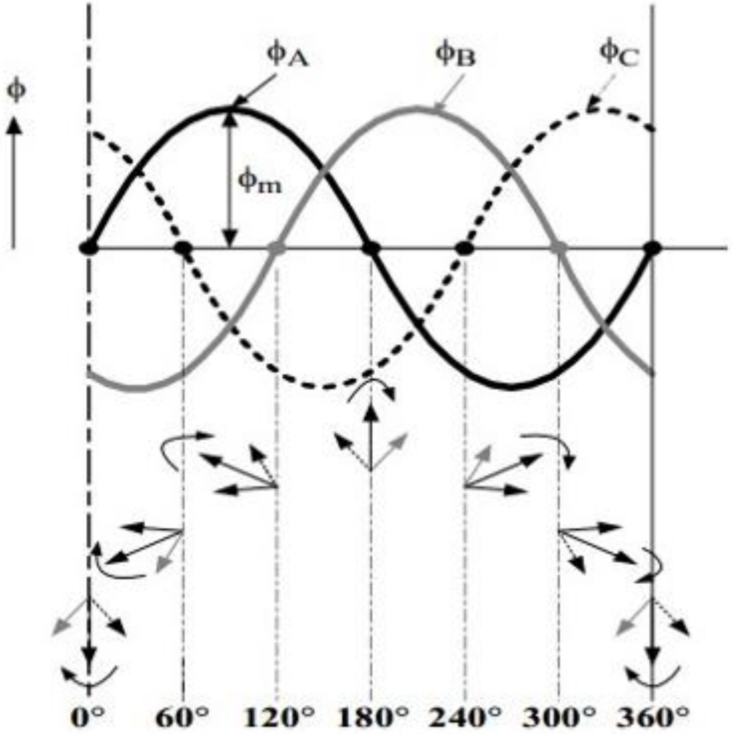

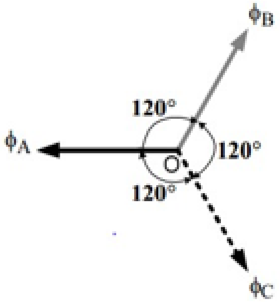

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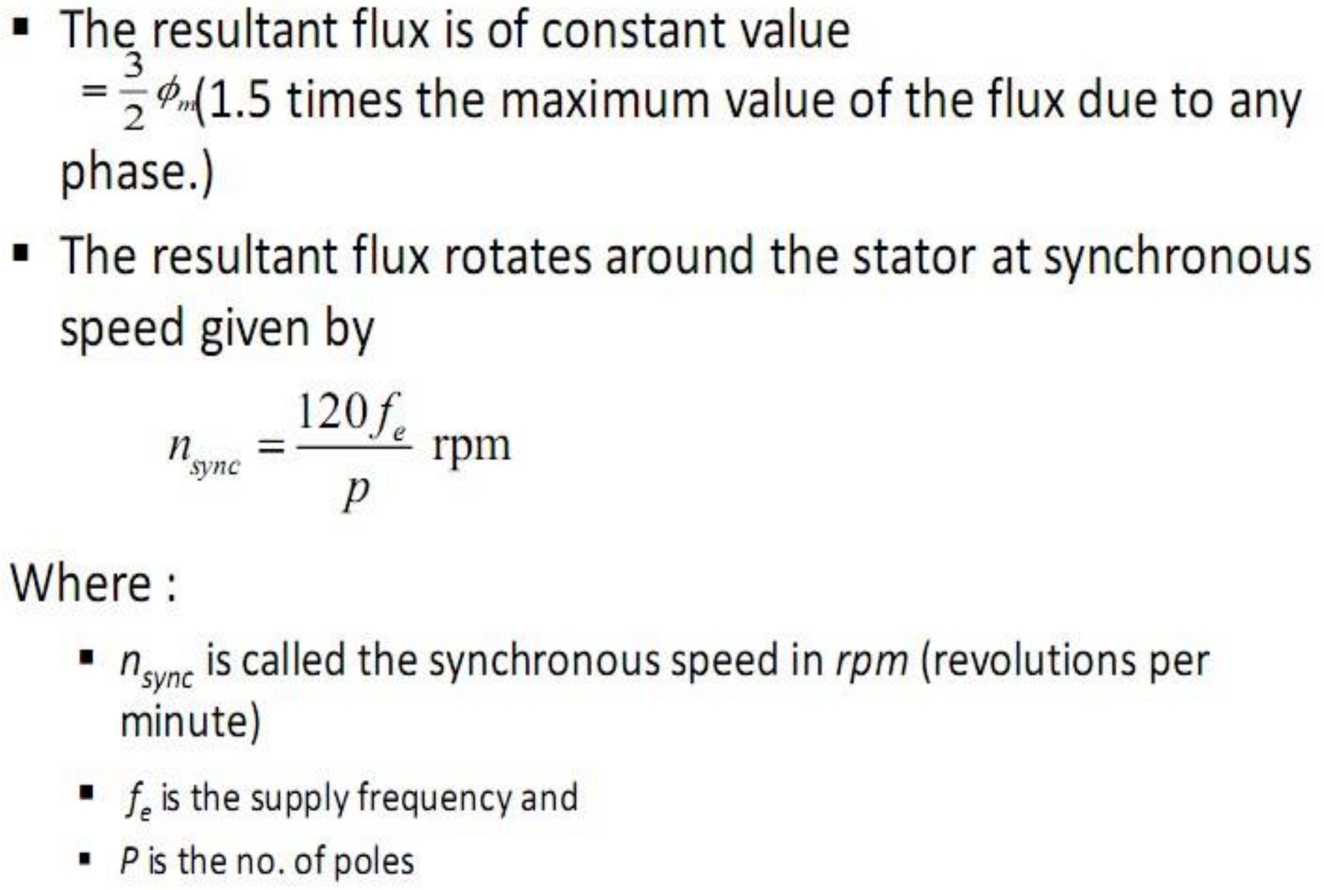

- The resultant flux is of constant value $= \frac{3}{2}\phi_m$ (1.5 times the maximum value of the flux due to any phase.)
- The resultant flux rotates around the stator at synchronous speed given by

$$n_{sync} = \frac{120 f_e}{p} \text{ rpm}$$

Where :

- n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)
- f_e is the supply frequency and
- P is the no. of poles

N.B The direction rotation of a poly phase Induction motor depends on the motor connection to the power lines. Rotation can be readily reversed by interchanging any two input leads.

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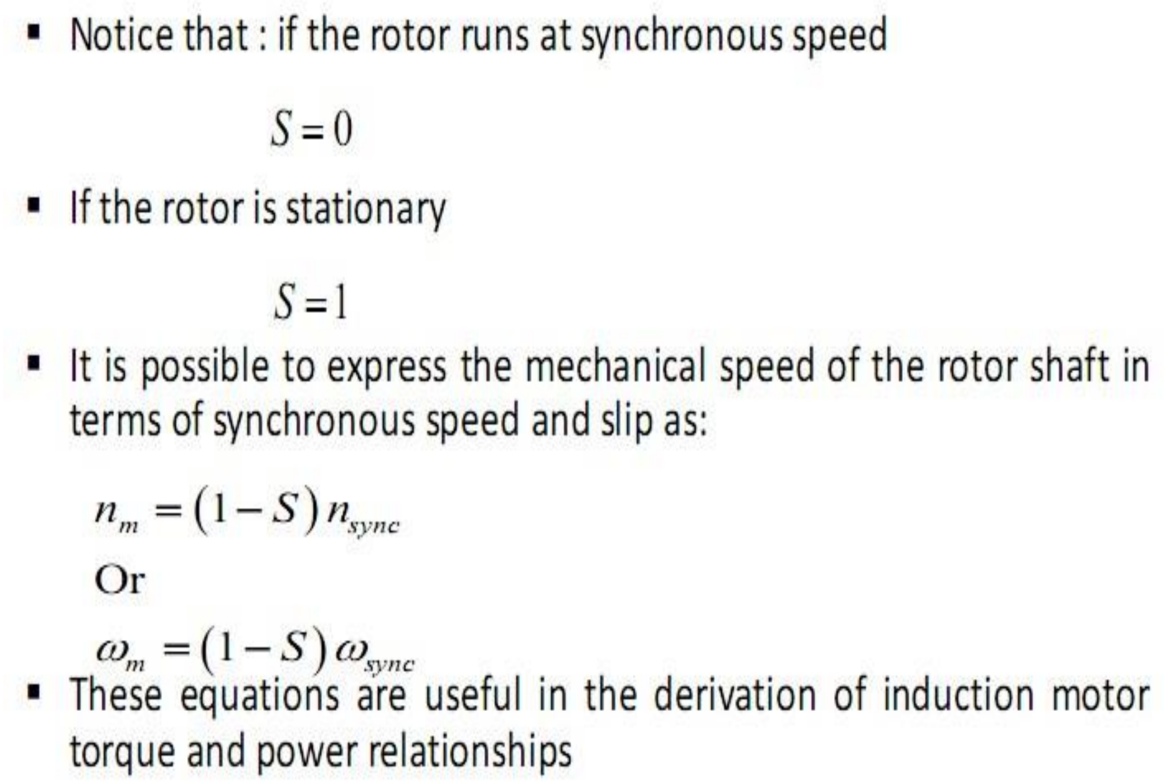

- Notice that : if the rotor runs at synchronous speed

$$S = 0$$

- If the rotor is stationary

$$S = 1$$

- It is possible to express the mechanical speed of the rotor shaft in terms of synchronous speed and slip as:

$$n_m = (1 - S)\, n_{sync}$$

Or

$$\omega_m = (1 - S)\, \omega_{sync}$$

- These equations are useful in the derivation of induction motor torque and power relationships

1
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2

IM and Transformer

- **Transformer**: voltage applied to the primary windings produce an induced voltage in the secondary windings
- **Induction motor**: voltage applied to the stator windings produce an induced voltage in the rotor windings
  - The difference is that, in the case of the induction motor, the secondary windings can move.
  - Due to the rotation of the rotor (the secondary winding of the IM), the induced voltage in it does not have the same frequency of the stator (the primary) voltage.

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- What would be the frequency of the rotor's induced voltage at any speed n_m?

$$f_r = Sf_e$$

- When the rotor is blocked (s=1) , the frequency of the induced voltage is equal to the supply frequency

- On the other hand, if the rotor runs at synchronous speed (s = 0), the frequency will be zero

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Effect of Slip on the Rotor Circuit

- When the rotor is stationary, s = 1. Under these conditions, the per phase rotor e.m.f. E_2 has a frequency equal to that of supply frequency f.
- At any slip s, the relative speed between stator field and the rotor (slip speed) is decreased.

  Consequently, the rotor e.m.f. and frequency are reduced proportionally to **sEs** and **sf** respectively.
- At the same time, per phase rotor reactance X_2, being frequency dependent, is reduced to $\mathbf{sX_2}$.

Thus at any slip s,

	At Standstill, $N_r = 0$	**At slip s N_r = Ns (1-s)**
Rotor EMF	E_2	sE_2
Rotor frequency, f_2	f	sf
Rotor reactance	X_2	sX_2

E_1: Induced EMF in stator, E_2: Induced EMF in rotor at standstill,

f: Supply frequency, X_2: Rotor reactance at standstill

At standstill, the induction motor is same as transformer

At standstill, the stator and rotor induced EMFs are related as $\frac{E_1}{E_2} = \frac{N_1}{N_2}$

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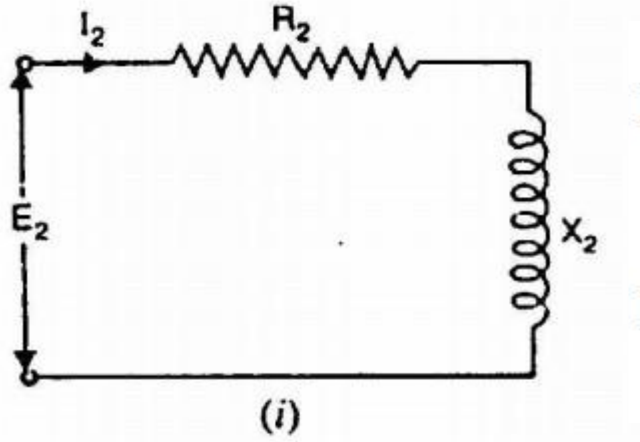
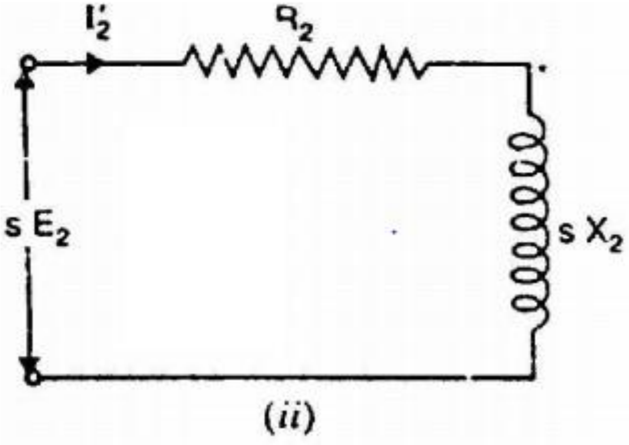
At standstill. Fig. ((i)) shows one phase of the rotor circuit at standstill.



$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

$$\text{Rotor p.f., } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

When running at slip s. Fig. ((ii)) shows one phase of the rotor circuit when the motor is running at slip s.



$$\text{Rotor current, } I'_2 = \frac{sE_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor p.f., } \cos\phi'_2 = \frac{R_2}{Z'_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

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Rotor Torque

The torque T developed by the rotor is directly proportional to:

i. rotor current

ii. rotor e.m.f.

iii. power factor of the rotor circuit

$$T \propto E_2 I_2 \cos\phi_2$$

$$T = K E_2 I_2 \cos\phi_2$$

where I_2 = rotor current at standstill
E_2 = rotor e.m.f. at standstill
$\cos\phi_2$ = rotor p.f. at standstill

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Let E_2 = rotor e.m.f. per phase at standstill
X_2 = rotor reactance per phase at standstill
R_2 = rotor resistance per phase

Rotor impedance/phase, $Z_2 = \sqrt{R_2^2 + X_2^2}$ ...at standstill

Rotor current/phase, $I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

Rotor p.f., $\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

$\therefore$ Starting torque, $T_s = K E_2 I_2 \cos\phi_2$

$$= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

It can be shown that $K = 3/2\pi N_s$.

$$T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

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Starting Torque (T_s)

- The torque developed at the instant of starting of a motor is called as starting torque.
- Starting torque may be greater than running torque in some cases, or it may be lesser.
- Starting torque is the torque produced by induction motor when it starts.
  - We know that at the start the rotor speed, N_r is zero.

So, the equation of starting torque is easily obtained by simply putting the value of s = 1 in the equation of torque of the induction motor.

- The starting torque is also known as standstill torque.

Condition for Maximum Starting Torque

- It can be proved that starting torque will be maximum when rotor resistance /phase is equal to standstill rotor reactance/phase.

Now $$T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \qquad \text{(i)}$$

Differentiating eq. (i) w.r.t. R_2 and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{\left(R_2^2 + X_2^2\right)^2} \right] = 0$$

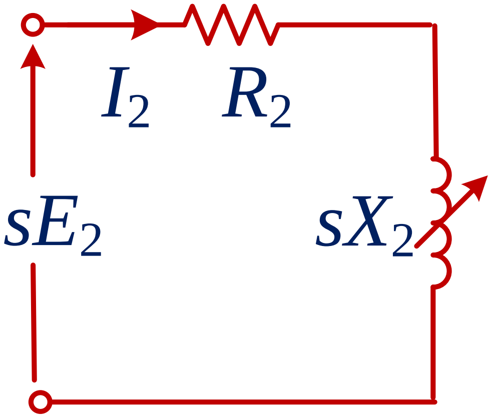
or $$R_2^2 + X_2^2 = 2R_2^2$$

or $$R_2 = X_2$$

- Under the condition of maximum starting torque, ϕ = 45° and rotor power factor is 0.707 lagging
- As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when R2 = X2.
- If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

Torque under Running Conditions

$$T_r = \frac{3}{2\pi N_s} \cdot \frac{s\,E_2^2\,R_2}{R_2^2 + (s\,X_2)^2} = \frac{3}{2\pi N_s} \cdot \frac{s\,E_2^2\,R_2}{(Z'_2)^2}$$



Rotor current at a speed of N rpm or at a slip of s *is*

$$I_2 = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}} \dots\dots\dots\dots(1)$$

Rotor pf at a speed of N rpm or at a slip of s *is*

$$cos\theta_2 = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}} \dots\dots\dots(2)$$

The torque developed is proportional to power transferred to rotor circuit

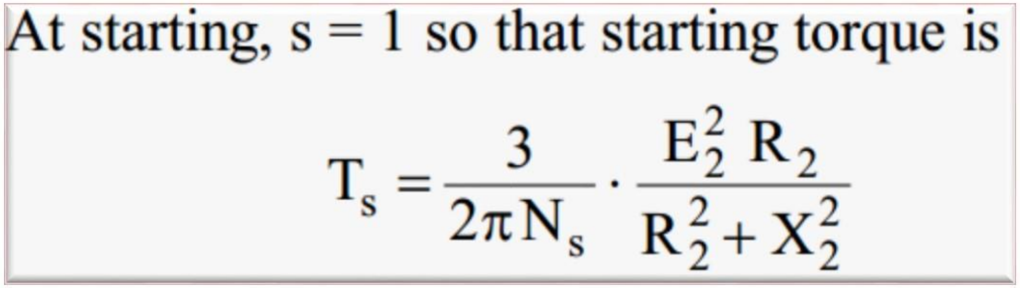
$$T_m \propto E_2 I_2 \cos\theta_2 \dots. (3)$$

Substituting (1) and (2) in (3)

$$T_m = \frac{kSE_2^2R_2}{R_2^2 + (SX_2)^2}$$

It may be seen that running torque is:

- directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.
- directly proportional to square of supply voltage (E_2 α V) .



At starting, s = 1 so that starting torque is

$$T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Maximum Torque under Running Conditions

$$T_r = \frac{K_2\, s\, R_2}{R_2^2 + s^2 X_2^2} \qquad \text{(i)}$$

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate exp. (i) w.r.t. s and equate the result to zero i.e.,

$$\frac{dT_r}{ds} = \frac{K_2\left[R_2\left(R_2^2 + s^2 X_2^2\right) - 2sX_2^2(s\,R_2)\right]}{\left(R_2^2 + s^2 X_2^2\right)^2} = 0 \;\longrightarrow\; R_2 = s\, X_2$$

$$T_r \propto \frac{s\, R_2}{R_2^2 + s^2 X_2^2} \qquad \text{… from exp. (i) above}$$

Putting $R_2 = s\, X_2$ in the above expression $\longrightarrow$ $T_m \propto \dfrac{1}{2\, X_2}$

Where $\boldsymbol{K_2 = K_1 E_2^2}$

Slip corresponding to maximum torque, $s = R_2/X_2$.

It can be shown that:

$$T_m = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2\,X_2}\ \text{N - m}$$

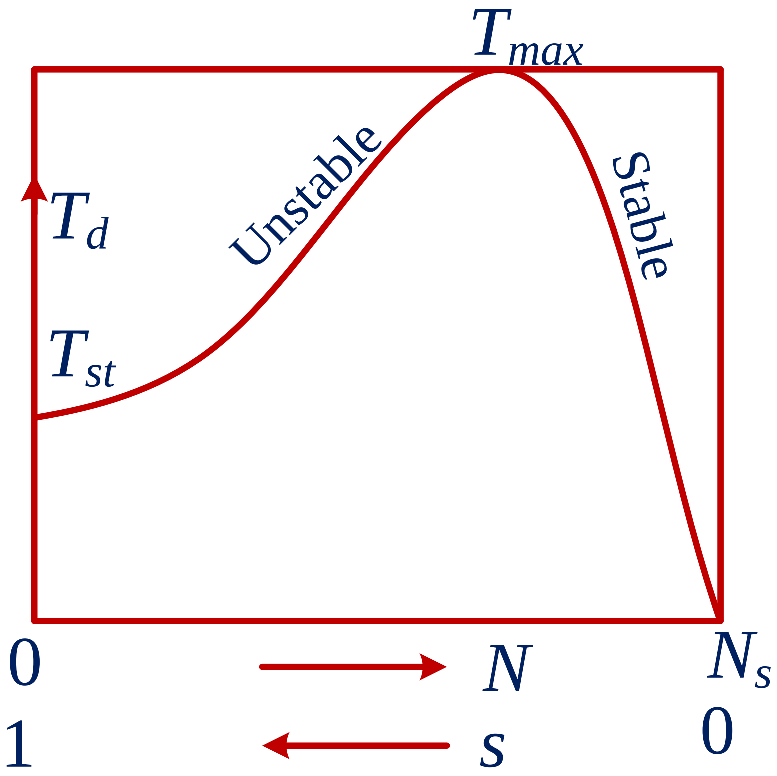
$$\boldsymbol{T_m = \frac{kE_2^2}{2X_2}}$$

The torque will be maximum when slip $s = R_2 / X_2$
From the above equation it is concluded that

1. The maximum torque is directly proportional to square of rotor induced emf at the standstill.
2. The maximum torque is inversely proportional to rotor reactance.
3. The maximum torque is independent of rotor resistance.
4. The slip at which maximum torque occur depends upon rotor resistance, R_2. So, by varying the rotor resistance, maximum torque can be obtained at any required slip.

- To obtain maximum torque at starting ($s = 1$), the rotor resistance must be made equal to rotor reactance at standstill.

Torque speed characteristics:



Torque developed is

$$\boldsymbol{T_d = \frac{kSE_2^2R_2}{R_2^2 + (SX_2)^2}}$$

For normal speeds:

The rotor runs nearer to synchronous speed, slip is small,

In the denominator, $R_2^2 >> (sX_2)^2$

$$\boldsymbol{T_d = \frac{kSE_2^2}{R_2}}$$

As E_2, R_2 and k are constant, $T_d \propto s$ linear characteristics

At Low Speed and Starting:

Slip, s is nearer to unity, $R_2^2 << (sX_2)^2$

$$\boldsymbol{T_d = \frac{KE_2^2R_2}{S(X_2)^2}}$$

As E_2, R_2, k and X_2 are constant $T_d \propto \frac{1}{s}$, Rectangular hyperbola

Torque at Standstill (Starting):

At standstill (starting) $s = 1$

$$\boldsymbol{T_{st} = \frac{kE_2^2R_2}{R_2^2 + (X_2)^2}}$$

Speed Control of 3-Phase Induction Motors

$$\boldsymbol{N_m = (1 - S)N_s = (1 - S)\frac{120f}{P}}$$

The speed N_m of an induction motor can be varied by changing:

i. Supply frequency f

ii. Number of poles P on the stator and

iii. Slip s.

Power Stages in Induction Motor:

Power supplied to the stator, $P_1=3V_1I_1\text{Cos}\theta_1$ V_1: Phase voltage, I_1: Phase Current

(In terms of line parameters: $P_1 = \sqrt{3}V_{L1}I_{L1}cos\theta_1$)

Losses in stator:

1. Stator Cu. Loss: $3I_1^2R_1$ (The current I_1 depends on mechanical load applied)
2. Stator core loss (Depends on voltage and frequency, and constant): $3I_c^2R\text{c}$

The power transferred to the rotor, $P_2 = P_1 -$ stator loss: $P_2 = 3E_2I_2\cos\theta_2$

Rotor Cu. Loss: $3I_2^2R_2$

The remaining power, P_m (called mechanical power) is converted into mechanical power and gives rise to gross torque, T_g

The torque developed in induction motor, $Td = \frac{Pm}{\omega m}$ $\omega m = \frac{2\pi N}{60}$

Out of T_g, some torque is lost due to friction and windage loss.

The power output, $P_{out} = P_m$ – Friction & windage losses.

The torque at the shaft, $T_{sh} = \frac{Pout}{\omega m}$

Efficiency $= \frac{Output}{Input} * 100;$ $\eta = \frac{P_{out}}{P_{in}} * 100$

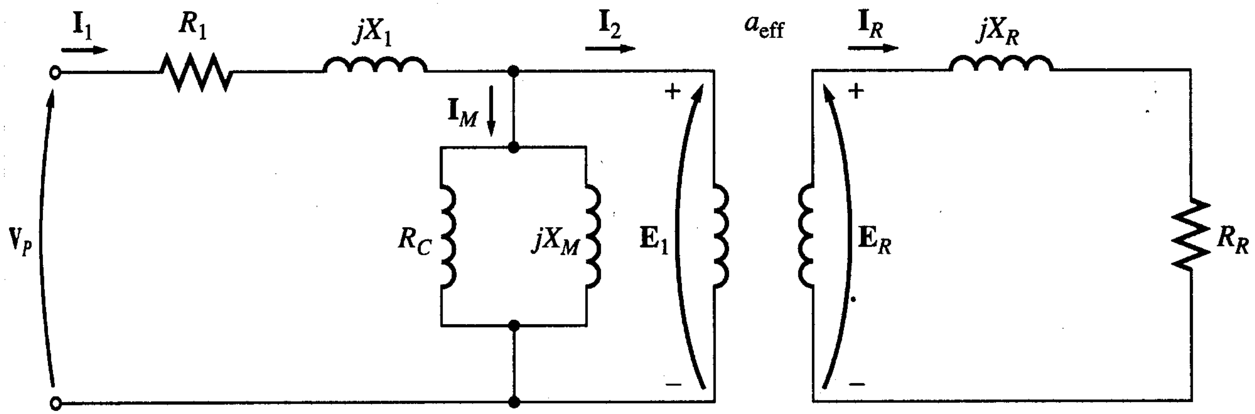
Horse power

- Unit used to measure mechanical power is the horse power.
- It is used to refer to the mechanical output power of the motor.
- Since, as an electrical engineers, deal with watts as a unit to measure electrical power, there is a relation between horse power and watts.

1hp = 746wattas=0.746kw

Equivalent Circuit of an Induction Motor

- The equivalent circuit of an induction motor is similar to that of the transformer. The only difference is that the secondary winding of IM is dynamic or it rotates.



- When the rotor is locked (or blocked), i.e. s =1, the largest voltage and rotor frequency are induced in the rotor.(@ standstill)

- On the other side, if the rotor rotates at synchronous speed, i.e. s = 0, the induced voltage and frequency in the rotor will be equal to zero.

$$\boldsymbol{E_R = SE_{R0}}$$

Where E_{R0} is the largest value of the rotor's induced voltage obtained at s = 1(locked rotor)

- The same is true for the frequency, i.e. in three phase IM the frequency of rotor winding is change due to the change of slip value.

$$\boldsymbol{f_r = Sf_e}$$

- It is known that

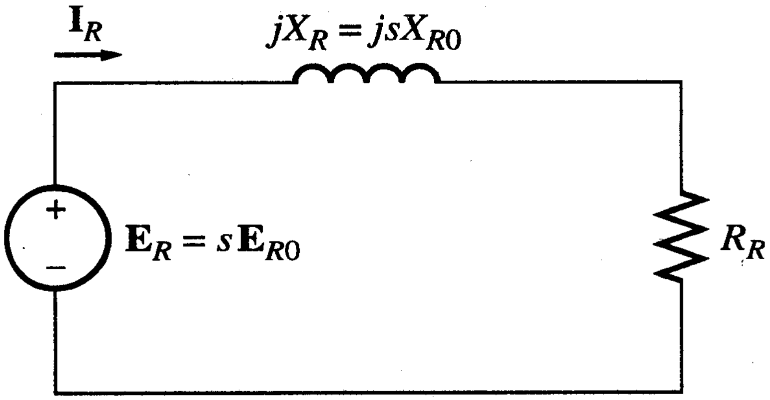
$$X = \omega L = 2\pi f L$$

- So, as the frequency of the induced voltage in the rotor changes, the reactance of the rotor circuit also changes

$$X_r = \omega_r L_r = 2\pi f_r L_r = 2\pi s f_e L_r = sX_{r0}$$

Where X_{r0} is the rotor reactance at the supply frequency (at blocked rotor)

- Then, we can draw the rotor equivalent circuit as follows



Where E_R is the induced voltage in the rotor and R_R is the rotor resistance

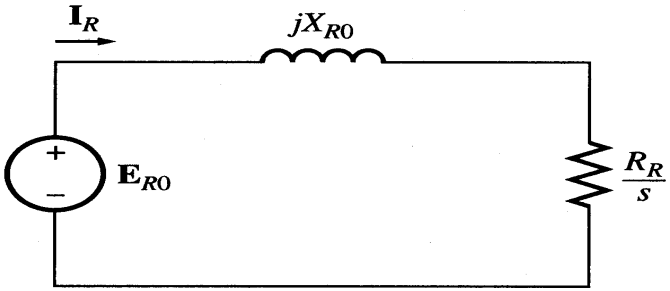
- Now we can calculate the rotor current as $I_R = \frac{E_R}{(R_R + jX_R)} = \frac{sE_{R0}}{(R_R + jsX_{R0})}$

- Dividing both the numerator and denominator by s so nothing changes we get

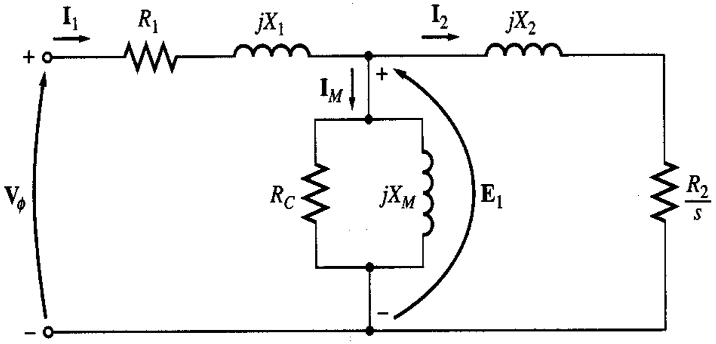
$$I_R = \frac{E_{R0}}{(\frac{R_R}{s} + jX_{R0})}$$

Where E_{R0} is the induced voltage and X_{R0} is the rotor reactance at blocked rotor condition (s = 1)

- Now we can have the rotor equivalent circuit



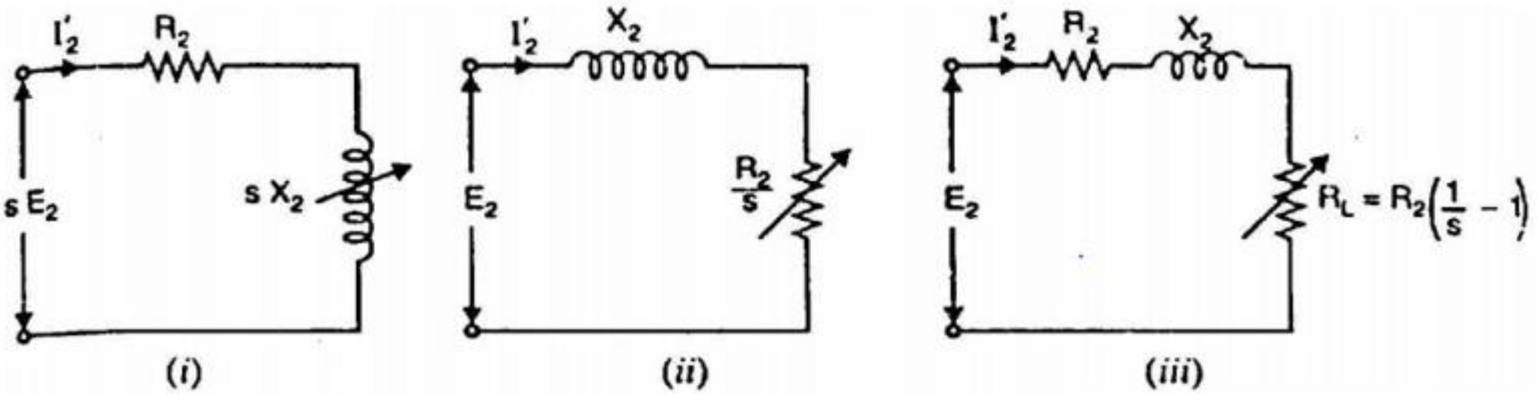
- Now as we managed to solve the induced voltage and different frequency problems, we can combine the stator and rotor circuits in one equivalent circuit



Where $X_2 = a_{eff}^2 X_{Ro}$ $R_2 = a_{eff}^2 R_R$

$I_{22} = \frac{I_R}{a_{eff}^2}$

$E_1 = a_{eff}^2 E_{R0}$ $a_{eff} = \frac{N_S}{N_R}$



$$I'_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$I'_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}}$$

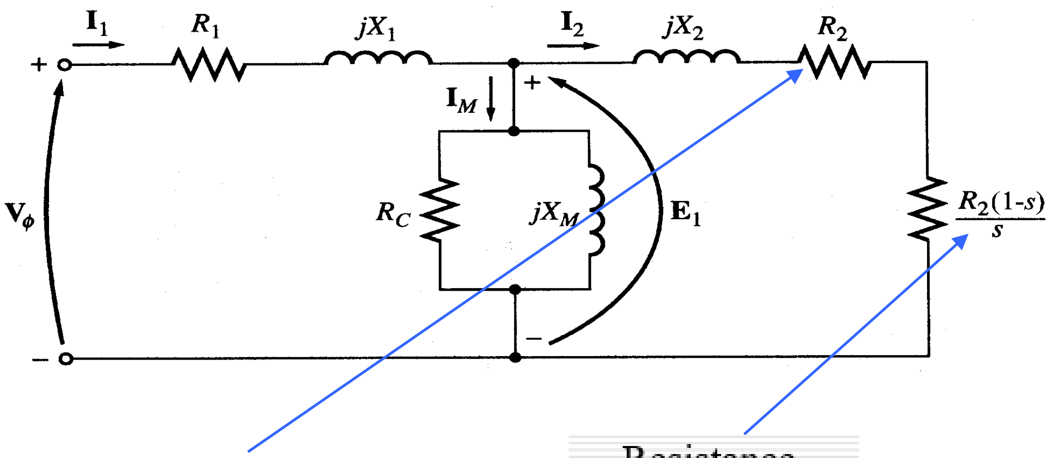
$$\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$$

The power delivered to this load represents the total mechanical power developed in the rotor.

i. The first part R_2 is the rotor resistance/phase, and represents the rotor C_u loss.

ii. The second part $R_2\ (\frac{1}{S} - 1)$ is a variable-resistance load.

- We can rearrange the equivalent circuit as follows



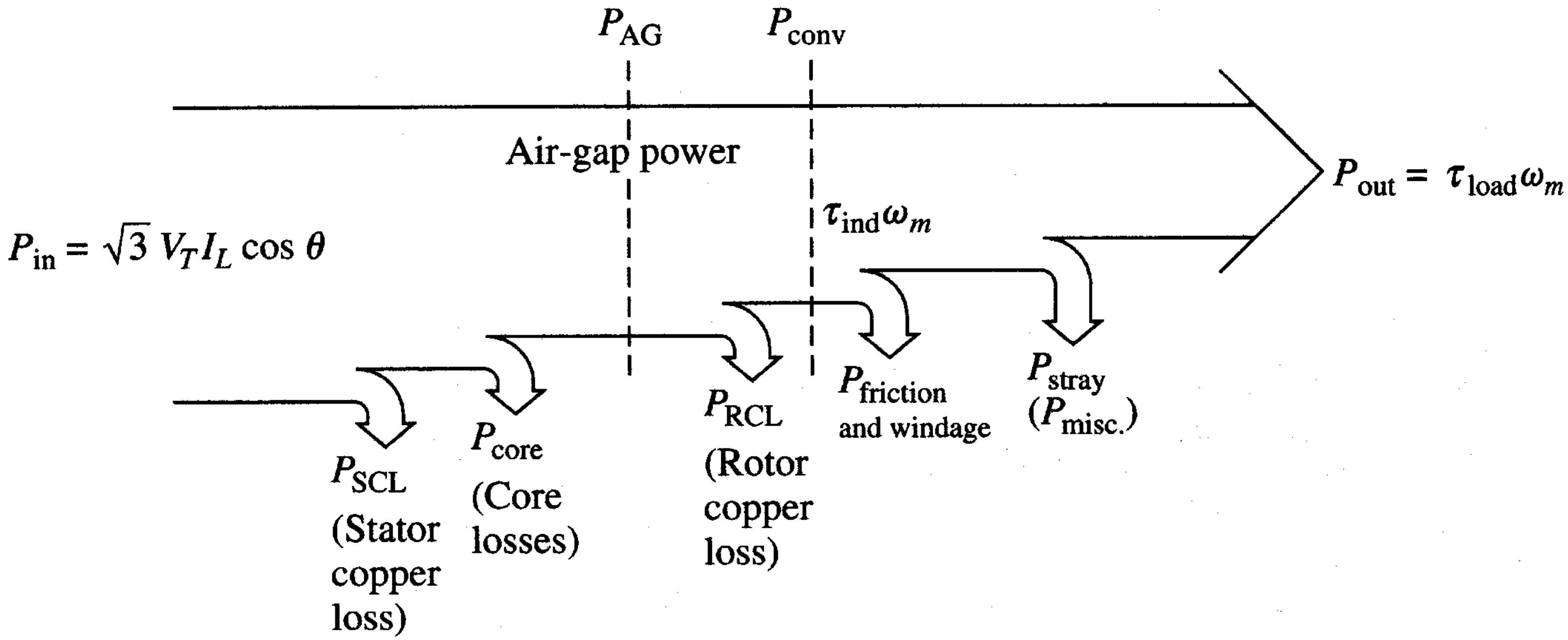
Actual rotor resistance

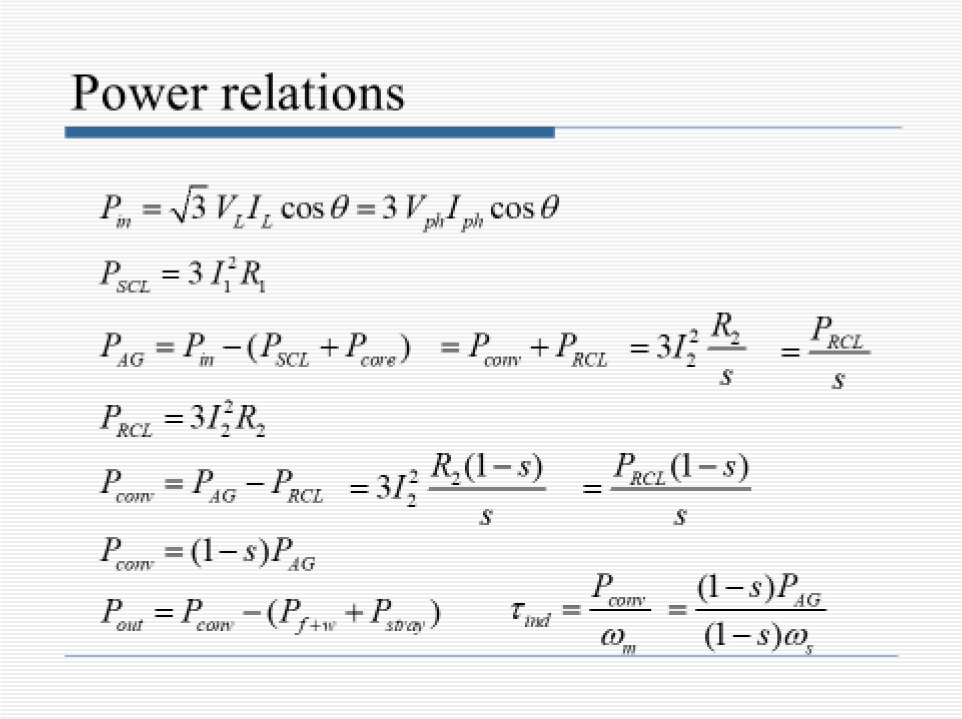
Resistance equivalent to mechanical load

- When $N_m = N_s$ the slip S=zero then $R_L = \infty$ this means that current cannot pass through this load resistance. Therefore, this load is open circuited.
- When N_m=zero the slip S=one then R_L=zero this means that there is no load resistance, therefore this load terminal is short-circuited.

Power flow in induction motor

Power-Flow diagram:





Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3\, V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{conv} + P_{RCL} \quad = 3I_2^2 \frac{R_2}{s} \quad = \frac{P_{RCL}}{s}$$

$$P_{RCL} = 3I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL} \quad = 3I_2^2 \frac{R_2(1-s)}{s} \quad = \frac{P_{RCL}(1-s)}{s}$$

$$P_{conv} = (1-s)P_{AG}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray}) \qquad \tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_s}$$

Example1: A 480V, 60Hz, 50hp, 3-phase induction motor is drawing 60A at 0.85 PF lagging. The stator copper losses are 2kW, and the rotor copper losses are 700W. The friction and windage losses are 600W, the core losses are 1800W, and the stray losses are negligible. Find:

i. The air gap power P_{AG}
ii. The power converted $P_{\mathbf{conv}}$
iii. The output power P_{out}
iv. The efficiency of the motor

Solution:

The input power $P_{in} = \sqrt{3}V_L I_L cos\theta_1 = \sqrt{3} * 480 * 60 * 0.85 = 42.4Kw$

$P_{Ag} = P_{in} - P_{SCL} - P_{core}$*=42.4-2-1.8=38.6Kw*

$P_{conv} = P_{Ag} - P_{RCL} = 38.6 - 0.7 = 37.9Kw$

$P_{out} = P_{conv} - P_{F\&W} = 37.9 - 0.6 = 37.3Kw$

This is the same as 50hp*0.746=37.3Kw

$$\eta = \frac{P_{out}}{P_{in}} = \frac{37.3}{42.4} * 100 = 88\%$$

Example2: A three-phase, two-pole, 60-Hz induction motor is observed to be operating at a speed of 3502 r/min with an input power of 15.7 kW and a terminal current of 22.6 A. The stator- winding resistance is 0.20 Ω/phase. Calculate the I^2R power dissipated in rotor.

Solution:

The stator cupper loss $P_{SCL} = 3I_1^2R_1 = 3 * 22.6^2 * 0.2 = 306W$

$$P_{Ag} = P_{in} - P_{SCL} = 15.7 - 0.306 = 15.4Kw$$

$$N_s = \frac{120 * f}{P} = 3600\, rpm$$

$$S = \frac{3600 - 3502}{3600} = 0.0272$$

$$P_{RCL} = SP_{Ag} = 0.0272 * 15.4Kw = 419W$$

Example3: A 3- Φ, 4-pole, 400 V, 50 Hz, star connected induction motor develops shaft power output of 30.39 kW at 1455 rpm. Friction and windage losses are 1.2 kW, power factor is 0.85 and stator losses total 2 kW. Calculate a) Rotor Cu. loss b) Efficiency c) Input current.

<u>Solution:</u>

Shaft power output, P_{out} = 30.39 kW

Mechanical power output, P_m=P_{conv} = P_{out} + F&W loss = 31.59 kW

The slip of the motor, s = = $\frac{\mathrm{Ns-N}}{\mathrm{Ns}}$= $\frac{1500-1455}{1500}$ = 0.03

$$P_{conv} = P_{Ag} - P_{RCL} = 3I_2^2\frac{R_2}{S} - 3I_2R_2 = P_{RCL}(\frac{1-S}{S})$$

$$P_{RCL} = \frac{0.03 * 31.59}{1 - 0.03} = 0.977kw$$

The rotor input, $P_{Ag} = P_m + \ P_{RCL}$ = 32.567 kW

The stator input, $P_{in} = P_{Ag} + \ P_{stator\ losses} = 34.567$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{30.39}{34.567} * 100 = 88.49\%$$

The stator input, $P_{in} = \sqrt{3}V_{L1}I_{L1}cos\theta_1$ = 34.567 kW

The stator line current, I_{L1} = 58.69 A

Example4: A 4 pole, 50 Hz, 1425rpm, 3-phase induction motor has a voltage of 520V between two slip rings on open circuit. The Y connected rotor has standstill impedance of (0.4 + j2) Ω/phase. Determine

a) Full load torque
b) T_{max} and corresponding speed

<u>Solution</u>

$$T_{fl} = \frac{ksE_2^2R_2}{R_2^2 + (sX_2)^2}$$

$k = \frac{3}{2pns}$ n_s: Synchronous speed in RPS = $N_s/60$

E_2: Rotor induced EMF/phase at standstill

R_2: Rotor resistance

X_2: Rotor standstill reactance

s_{fl}: Slip at full load

The voltage between the slip rings is the line voltage in the rotor circuit

Per phase induced EMF at standstill, $E_2 = \frac{520}{\sqrt{3}} = 300.22V$

Synchronous speed, $N_s = \frac{120f}{p} = 1500$ rpm. k= 0.0191

Slip at full load, $s_{fl} = \frac{Ns-N}{Ns} = 0.05$

Torque at full load, $T_{fl} = \frac{0.019*0.05*300.22^2*0.4}{(0.4^2+(0.05*2)^2)} = 201.47\ N\_m$

The maximum torque,

$$T_{max} = \frac{kE_2^2}{2X_2} = \frac{0.019 * 300.22^2}{2 * 2} = 427.5\ N\_m$$

The slip at maximum torque, $S_{Tmax} = \frac{R_2}{X_2} = 0.2$

The speed at maximum torque, $N_{Tmax} = (1 - S_{Tmax})N_s = 1200\ rpm$

Induction Motor Rating

The nameplate of a 3-phase induction motor provides the following information:

- Horsepower
- Line voltage
- Line current
- Speed
- Frequency
- Temperature rise

✓ The horsepower rating is the mechanical output of the motor when it is operated at rated line voltage, rated frequency and rated speed.

✓ The speed given on the nameplate is the actual speed of the motor at rated full load; it is not the synchronous speed

Methods of Starting 3-Phase Induction Motors (*No more than 5 pages*)

1. **Direct-on-line starting**
2. **Stator resistance starting**
3. **Autotransformer starting**
4. **Star-delta starting**
5. **Rotor resistance starting**

Assignment 2