Chapter 5

Introduction to Convection/Heat transfer coefficients

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out} + \dot{E}_{st}$$

Energy Conservation

Problems involving conduction: Chapters 2-3

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Transient problems: Chapter 5

Chapter 3:

- Obtained temperature profiles for 1-D, SS conduction, with and without generation
- We wrote the 1-D, SS problems in terms of resistances in series
- We defined an overall heat transfer coefficient, as the inverse of the total resistance

Obtained temperature as a function of time for cases where resistance to conduction was negligible

- In Chapters 1-5 we used Newton's law of convection:

$$q_{conv} = hA\Delta T$$

- ➢ h was provided
- ➢ we did not consider any temperature variations within the fluid

Chapter 6:

•We will use dimensional parameters to the boundary layer to find a functional dependence of h

•In subsequent chapters we will use this information to obtain temperature distributions within the fluid.

Introduction to Convection

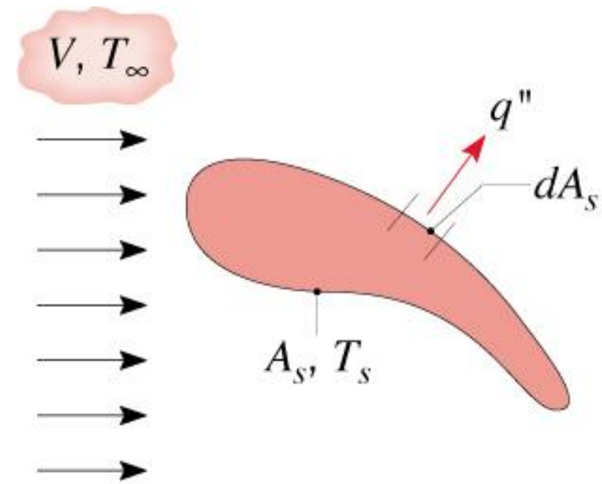
- Convection denotes energy transfer between a surface and a fluid moving over the surface.
- The dominant contribution is due to the bulk (or gross) motion of fluid particles.

- In this chapter we will
  - Discuss the physical mechanisms underlying convection
  - Discuss physical origins and introduce relevant dimensionless parameters that can help us to perform convection transfer calculations

Heat Transfer Coefficient

Recall Newton's law of cooling for heat transfer between a surface of arbitrary shape, area A_s and temperature T_s and a fluid:

$$q'' = h(T_S - T_\infty) \qquad (6.1)$$



- Generally flow conditions will vary along the surface, so q″ is a <u>*local*</u> heat flux and h a <u>*local*</u> convection coefficient.

- The total heat transfer rate is

$$q = \int_{A_S} q''\, dA_S = (T_S - T_\infty)\int_{A_S} h\, dA_S = \bar{h} A_S (T_S - T_\infty)$$

where

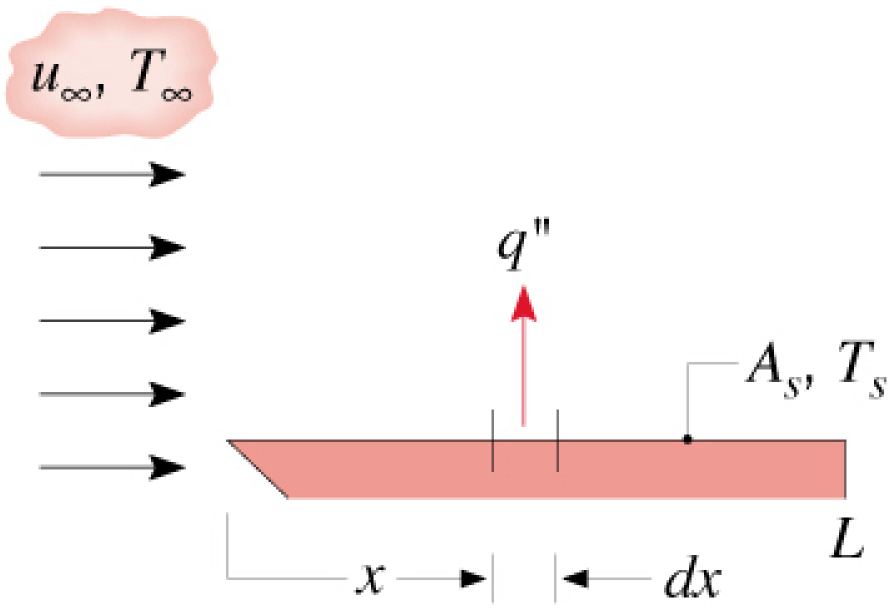
$$\boxed{\bar{h} = \frac{1}{A_S}\int_{A_S} h\, \mathrm{dA_S}} \qquad (6.2)$$

average heat transfer coefficient

Heat Transfer Coefficient

- For flow over a flat plate:

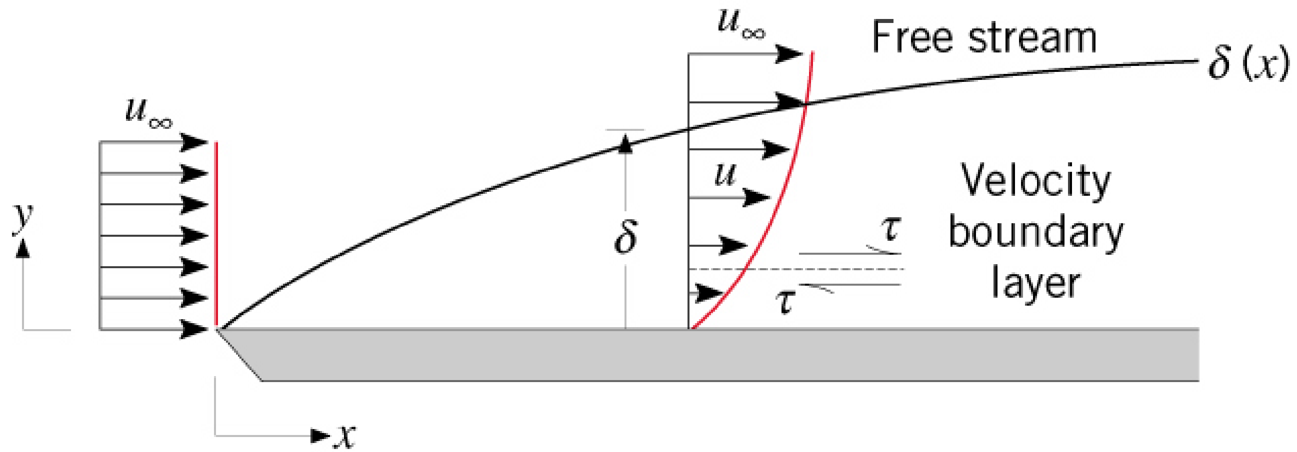
$$\bar{h} = \frac{1}{L}\int_0^L h\,dx \qquad (6.3)$$



➢ How can we estimate the heat transfer coefficient?

The Velocity Boundary Layer

Consider flow of a fluid over a flat plate:



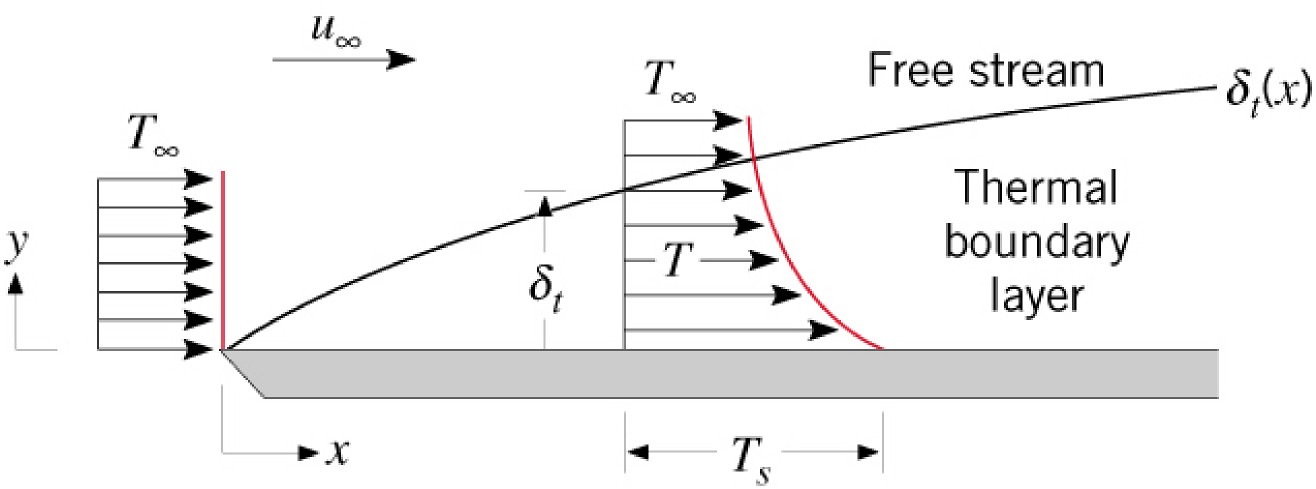
The flow is characterized by two regions:

- A thin fluid layer (boundary layer) in which velocity gradients and shear stresses are large. Its thickness δ is defined as the value of y for which u = 0.99 u_∞
- An outer region in which velocity gradients and shear stresses are negligible

For Newtonian fluids: $\tau_S = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$ and $C_f = \frac{\tau_S}{\rho u_\infty^2 / 2}$ where C_f is the local friction coefficient

The Thermal Boundary Layer

Consider flow of a fluid over an isothermal flat plate:



- The thermal boundary layer is the region of the fluid in which temperature gradients exist
- Its thickness is defined as the value of y for which the ratio: $\dfrac{T_S - T}{T_S - T_\infty} = 0.99$

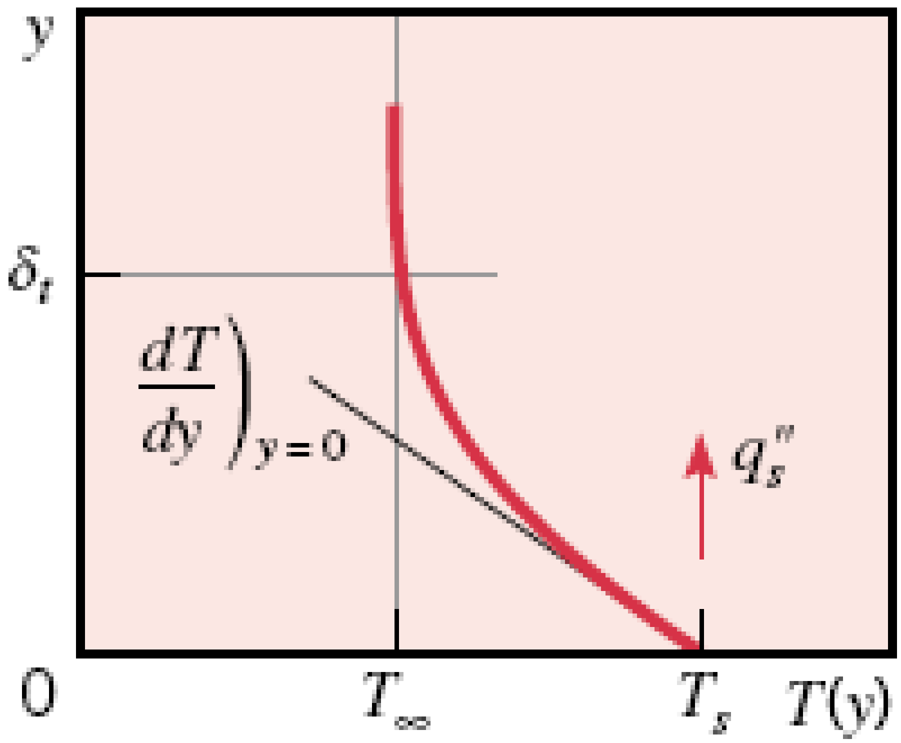
At the plate surface (y=0) there is no fluid motion – The local heat flux is:

$$q_S'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (6.4)$$

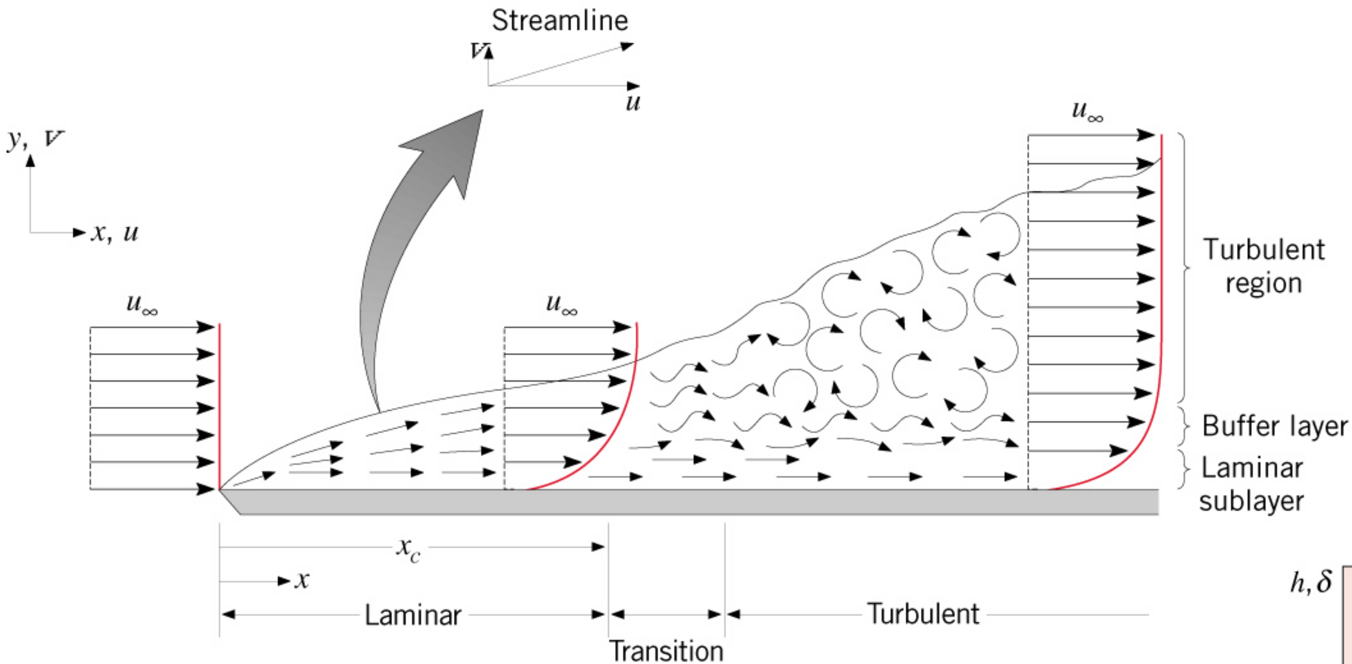
and

$$h = \frac{-k_f \, \partial T / \partial y \big|_{y=0}}{\mathrm{T_S} - T_\infty} \qquad (6.5)$$

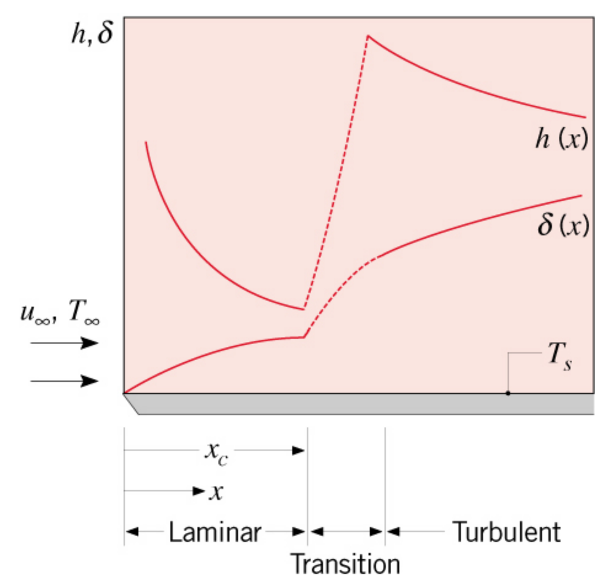
The Thermal Boundary Layer



Laminar and Turbulent Flow



$$Re_x = \frac{\rho u_\infty x}{\mu}$$



Transition criterion at $Re_{critical}$:

$$Re_{x,c} = \frac{\rho u_\infty x_c}{\mu} = 5 \times 10^5$$

Boundary Layers - Summary

- Velocity boundary layer (thickness $\delta(x)$) characterized by the presence of velocity gradients and shear stresses - *Surface friction, C_f*
- Thermal boundary layer (thickness $\delta_t(x)$) characterized by temperature gradients – *Convection heat transfer coefficient, h*

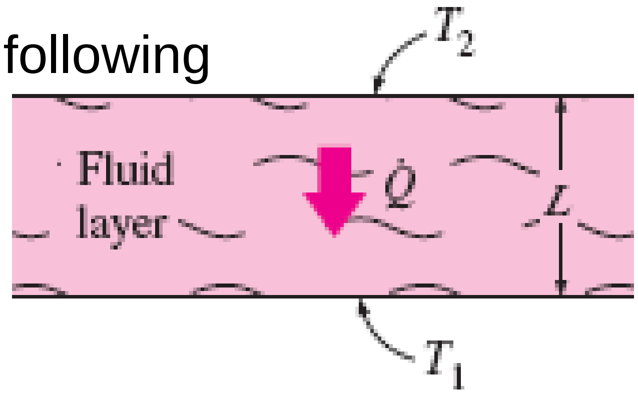
- Need to determine the heat transfer coefficient, h

$$h = \frac{-k_f \; \partial T / \partial y \big|_{y=0}}{\mathrm{T_S} - T_\infty}$$

- Must know T(x,y), which depends on velocity field

Nusselt Number

Consider the following



$$\Delta T = T_2 - T_1$$

Heat transfer through the fluid layer will be by convection, when the fluid involves some motion, and by conduction when the fluid layer is motionless

Taking the ratio of convection to conduction heat transfer through the fluid layer

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

Nusselt Number

So the nusselt number represents the enhancement of the heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer

If Nu=1, heat transfer through the layer is by pure conduction

Prandtl number

- The relative thickness of the velocity and thermal boundary layer,

$$\Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

The prandtl number of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate.

For laminar flow the boundary layer thickness is given by. n=1/3

$$\frac{\delta}{\delta_t} = \Pr^n$$

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oil relative to the velocity boundary layer

Reynolds number

The transition from laminar to turbulent flow depends on surface geometry, surface roughness, free stream velocity, surface temperature and type of fluid

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu}$$

The flow regimes depends mainly on the ratio of the inertia forces to viscous forces in the fluid, dimensionless quantity

At large reynold numbers, the inertia forces are large relative to the viscous forces and thus the viscous forces cannot prevent the random and rapid fluctuation of the fluid

At small reynolds number the visous forces are large enough to overcome the inertia forces and to keep the fluid line

Example

Experimental results for the *local* heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$$h_x(x) = ax^{-0.1}$$

where a is a coefficient ($W/m^{1.9} \cdot K$) and x (m) is the distance from the leading edge of the plate.

(a) Develop an expression for the ratio of the *average* heat transfer coefficient $\bar{h}_x$ for a plate of length x to the *local* heat transfer coefficient h_x at x.

(b) Show qualitatively the variation of h_x and $\bar{h}_x$ as a function of x.

Example 6.1

Experimental results for the local heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$h_x(x) = ax^{-0.1}$

where a is a coefficient ($W/m^{1.9}.K$) and x(m) is the distance from the leading edge of the plate.

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx$$

1. Develop an expression for the ratio of the average heat transfer coefficient $\bar{h}_x$ for a plate of length x to the local heat transfer coefficient $h_x(x)$.
2. Sketch the variation of h_x and $\bar{h}_x(x)$ *with x*.

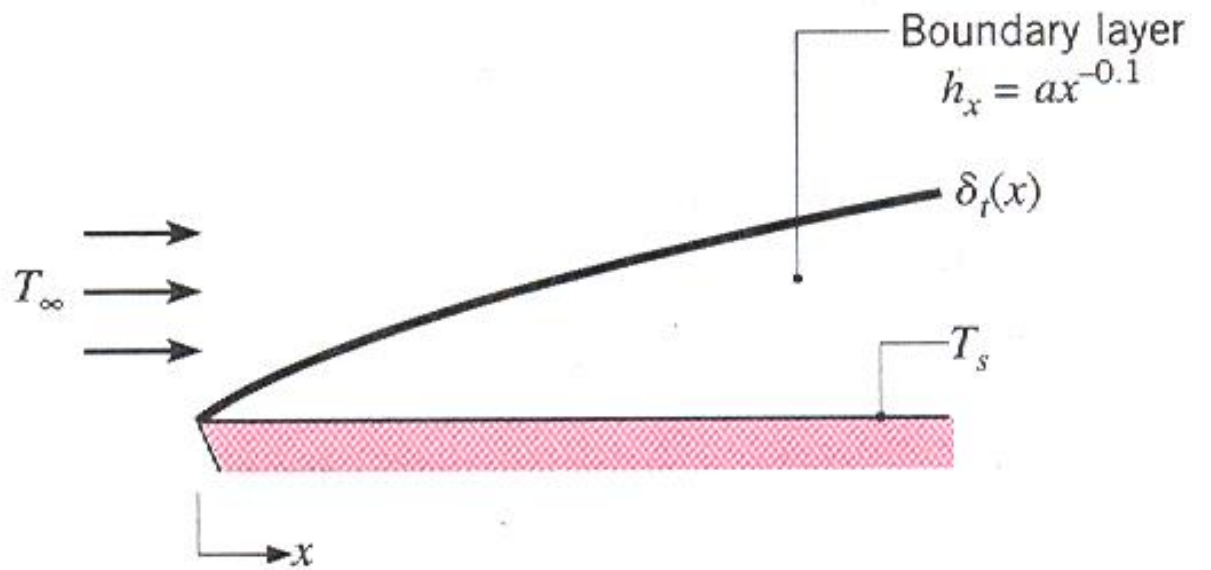


Figure for example 6.1

Solution

1. The average value of the convection heat transfer coefficient over the region from 0 to x is

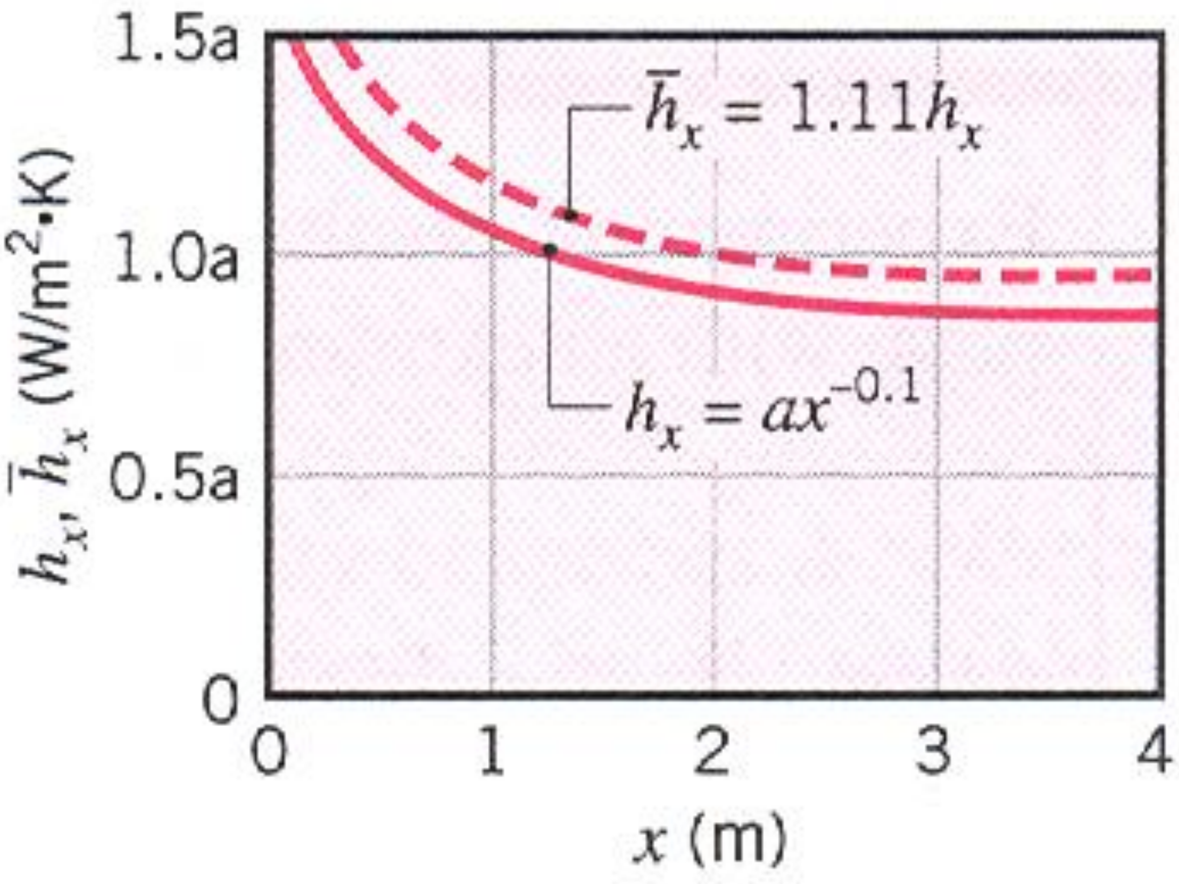
$$\bar{h}_x = \bar{h}_x(x) = \frac{1}{x}\int_0^x h_x(x)$$

Substituting the given value for the local heat transfer coefficient gives

$$\bar{h}_x = \frac{1}{x}\int_0^x a x^{-0.1} dx = \frac{a}{x}\int_0^x x^{\;\;0.1} dx = \frac{a}{x}\left(\frac{x^{+0.9}}{0.9}\right) = 1.11 a x^{-0.1}$$

or $$\bar{h}_x = 1.1 h_x$$

2. The variation of h_x and $\bar{h}_x$ with x is as follows.



Variation of h_x and $\bar{h}_x$ with x.

Empirical Correlation

- Empirical correlations are needed based on surface geomentry and flow conditions. The most general correlation for forced convection.

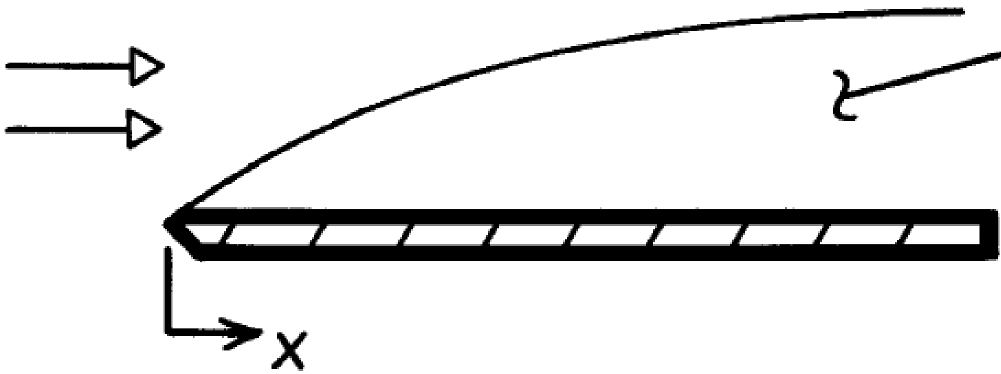
$$\overline{\mathrm{Nu}}_x = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^n$$

- Where C, m,n are independent of the fluid, but dependent on the surface geometry and flow conditions (laminar Vs Turbulent)

Tutorial Q1

For laminar free convection from a heated vertical surface, the local convection coefficient may be expressed as $h_x = Cx^{-1/4}$, where h_x is the coefficient at a distance x from the leading edge of the surface and the quantity C, which depends on the fluid properties, is independent of x. Obtain an expression for the ratio $\bar{h}_x/h_x$, where $\bar{h}_x$ is the average coefficient between the leading edge ($x = 0$) and the x location. Sketch the variation of h_x and $\bar{h}_x$ with x.

solution



ANALYSIS: The average value of h_x between 0 and x is

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{C}{x}\int_0^x x^{-1/2} dx$$

$$\bar{h}_x = \frac{C}{x} 2x^{1/2} = 2Cx^{-1/2}$$

$$\bar{h}_x = 2h_x.$$

solution

Hence, $$\frac{\bar{h}_x}{h_x} = 2.$$ <

COMMENTS: Both the local and average coefficients decrease with increasing distance x from the leading edge, as shown in the sketch below.