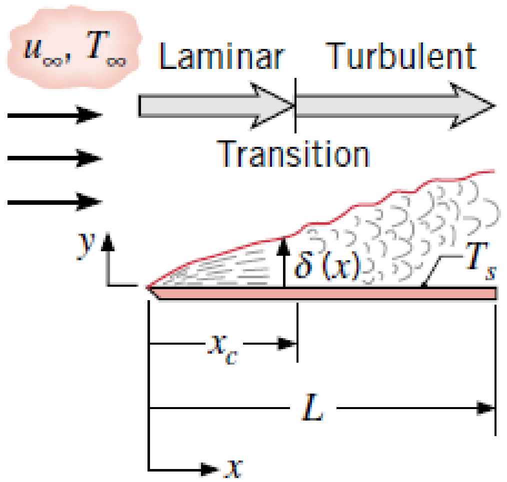
Chapter 6

Forced Convection

By: Wubishet Degife

7.1 Forced Convection (External flow)

- In this section, we will introduce correlations useful for estimating coefficients over a flat plate and curved surfaces of a cylinder and illustrated to compute convection heat rate.
- **<u>Flat plate Laminar flow</u>**
- In the absence of upstream disturbances, laminar boundary layer development begins at the leading edge (x=0),
- Transition to turbulence may occur at a
- downstream location (xc) for which the critic*a*
- Reynold number is Rexc=500000



- for laminar flow,

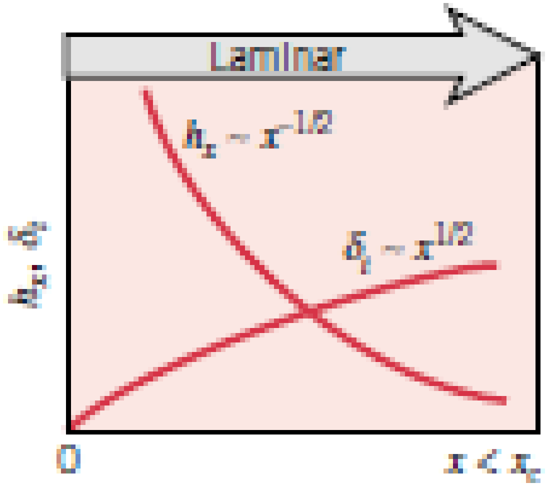
$$\delta = 5x\,\mathrm{Re}_x^{-1/2}$$

- Where the charactersitc length in the reynolds number is x, the distance from the leading edge

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu}$$

- The local Nusselt number is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \qquad [0.6 \le \mathrm{Pr} \le 50]$$



The expression for the average convection coefficient for any surface shorter than xc

- $$\bar{h}_x = \frac{1}{x}\int_0^x h_x\,dx = 0.332\left(\frac{k}{x}\right)\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\nu}\right)^{1/2}\int_0^x \frac{dx}{x^{1/2}}$$

$$\overline{\mathrm{Nu}}_x = \frac{\bar{h}_x x}{k} = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \qquad [0.6 \le \mathrm{Pr} \le 50]$$

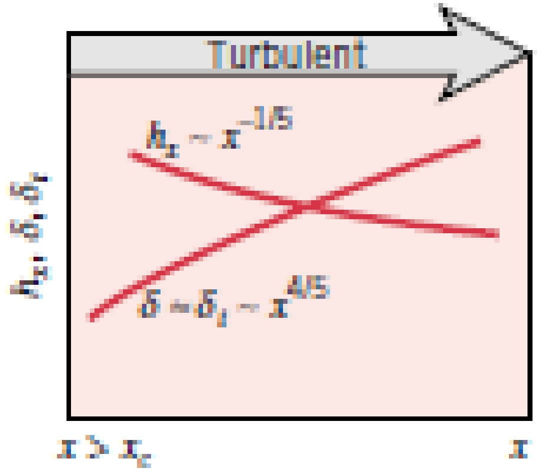
Turbulent flow

- For turbulent flows, the hydrodynamic boundary layer thickness

$$\delta = 0.37x\,\mathrm{Re}_x^{-1/5} \qquad [\mathrm{Re}_x \le 10^8]$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \qquad \begin{bmatrix}\mathrm{Re}_x \le 10^8 \\ 0.6 < \mathrm{Pr} < 60\end{bmatrix}$$

- The local



Mixed boundary Layer conditions

- If transition occurs towards the trailing edge of the plate, within the range 0.95<=(Xc/L)<=1, this equation will provide the reasonable approximation which will be influenced by conditions in both laminar and turbulent boundary layers

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{lam}\,dx + \int_{x_c}^{L} h_{turb}\,dx\right)$$

$$\overline{Nu}_L = [0.664\, Re_{x,c}^{1/2} + 0.037(Re_L^{4/5} - Re_{x,c}^{4/5})]Pr^{1/3}$$

- If representati

$$\overline{Nu}_L = (0.037\, Re_L^{4/5} - 871)Pr^{1/3} \qquad \left[\begin{array}{l} 0.6 < Pr < 60 \\ 5 \times 10^5 < Re_L \le 10^8 \\ Re_{x,c} = 5 \times 10^5 \end{array}\right]$$

00 is assumed ,
the equation w

- Sometimes, there are many practical applications where it is desirable to use turbulen … y layer at the leading edge, Rex,c=0.

$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} \qquad \begin{bmatrix} Re_{x,c} = 0 \\ 0.6 \le Pr \le 50 \end{bmatrix}$$

- Where the boundary layer is assumed to be fully turbulent

The cylinder in a cross flow

- Free stream fluid is brought to rest at the forward stagnation point, and the thin hydrodynamic boundary layer begins to grow as the fluid moves toward the rear of the cylinder
- Depending upon the reynolds no , a transition from laminar to turbulent condition can occur

$$Re_D = \frac{\rho u_\infty D}{\mu} = \frac{u_\infty D}{\nu}$$

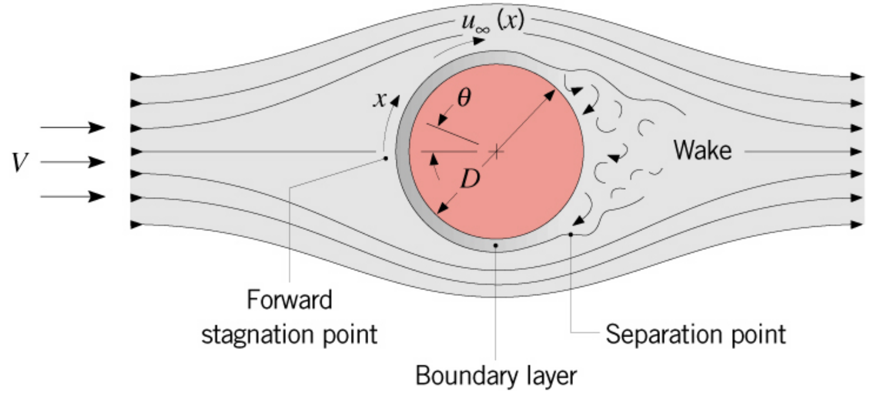
- Correlations are available for average nusselt number. The Hilpers correlation is one of the most widely used

- The churchill-Bernstein correlat ith a wide range of reynolds and prandtl no

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\, Re_D^m\, Pr^{1/3} \qquad [Pr \geq 0.7]$$

Cylinder in cross flow

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad [Re_D\,Pr > 0.2]$$



- Laminar boundary layer for $Re_D = \dfrac{\rho V D}{\mu} < 2\times 10^5$

Summary of Correlations ccf

1. Zhukauskas correlation:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4} \qquad 0.7 < \mathrm{Pr} < 500,\ 1 < \mathrm{Re}_D < 10^6 \qquad (7.9)$$

where C and m are listed in Table 7.4, (n=0.37 for 10≥Pr) and (n=0.36 for 10<Pr). Properties evaluated at T_∞, except Pr_s which is evaluated at T_s.

2. Churchill and Bernstein correlation, for all Re_D and Pr>0.2

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad (7.10)$$

Properties evaluated at film temperature

3. Hilpert correlation, can be used for cross flow around other non-circular shapes – see Table 7.2 for values of C and m

$$\overline{Nu}_D = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$$

Table 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

Table 7.4 Constants of Equation 7.53 for the circular cylinder in cross flow [16]

Re_D	C	m
1–40	0.75	0.4
40–1000	0.51	0.5
10^3–2×10^5	0.26	0.6
2×10^5–10^6	0.076	0.7

The sphere

- Whitaker correlation:

$$\overline{Nu}_D = 2 + (0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3})\,Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4} \qquad \begin{matrix} 0.71 < Pr < 380 \\ 3.5 < Re_D < 7.6\times10^4 \end{matrix} \qquad (7.12)$$

where properties are evaluated at T_∞ , except μ_s which is evaluated at T_s

- Correlation by Ranz and Marshall for heat transfer from freely falling liquid drops:

$$\overline{Nu}_D = 2 + 0.6\,Re_D^{1/2}\,Pr^{1/3} \qquad (7.13)$$

➢ At Re_D=0, equations (7.12) and (7.13) reduce to:

$$\overline{Nu}_D = 2$$

❖ Applicable for heat transfer to a stationary infinite medium around the surface

Procedures for calculation

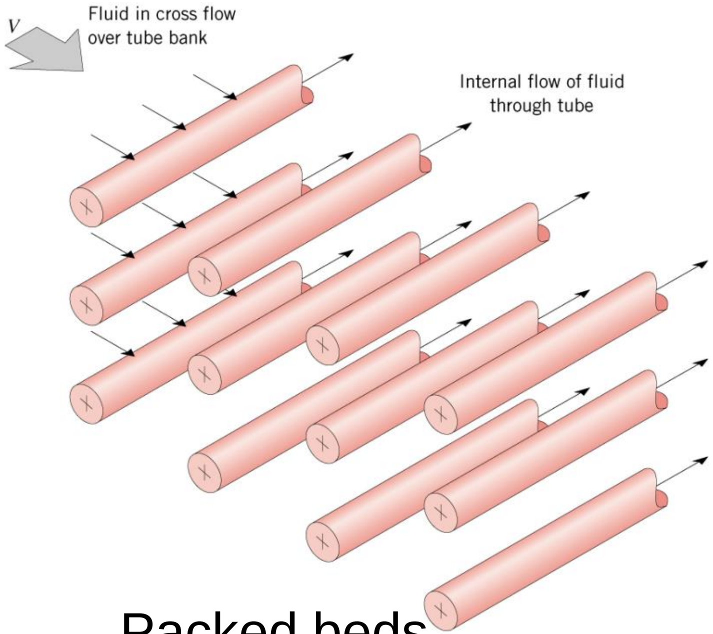
- Begin by recognizing the flow geometry (i.e. flat plate, sphere, cylinder etc.)
- Specify appropriate reference temperature for evaluation of fluid properties (usually film temperature, equation 7.2)
- Calculate Reynolds number – determine whether flow is laminar or turbulent
  - Reminder: Transition criteria:

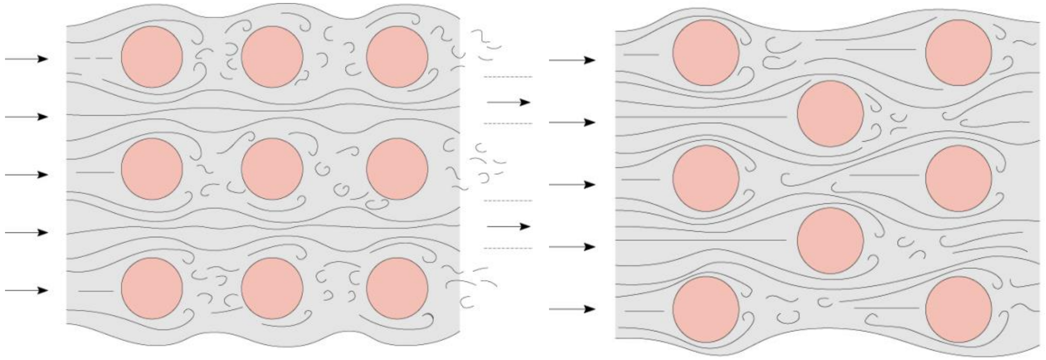
$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu} = 5\times10^5 \quad \textit{Flat plates} \qquad \mathrm{Re}_D = \frac{\rho V D}{\mu} < 2\times10^5$$

- Decide whether a local or average heat transfer coefficient is required
- Use appropriate correlation to determine heat transfer coefficient
- Proceed with other calculations, such as determination of heating or cooling rate

Examples

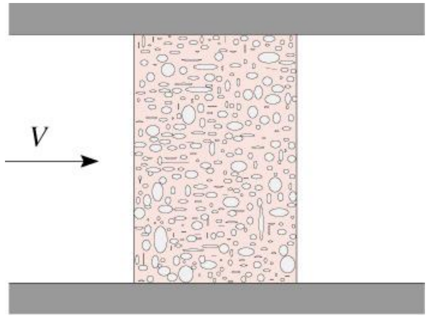
Other Applications



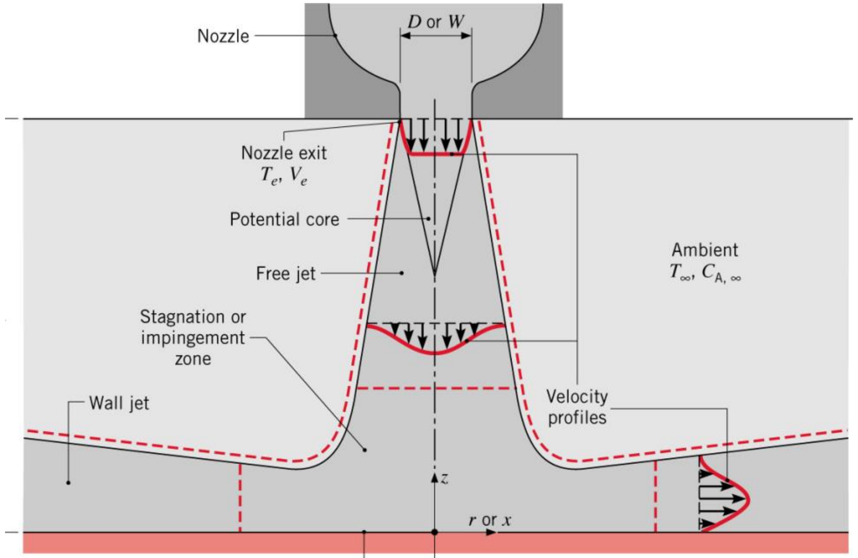


Flow around tube banks

Packed beds



Impinging jets



Flow across Banks of Tubes

- Several correlations exist (textbook section 7.6)
- Usually of the form

$$\overline{Nu}_D = C\,\mathrm{Re}_{D,\max}^{m}\,\mathrm{Pr}^{b}$$

where C and m can be found in tables

Summary

- In addition to heat transfer due to conduction, we considered for the first time heat transfer due to bulk motion of the fluid
- We discussed the concept of the boundary layer
- We defined the local and average heat transfer coefficients and obtained a general expression, in the form of dimensionless groups to describe them.
- we obtain expressions to determine the heat transfer coefficient for specific geometries

Tutorial Q

EXAMPLE 7–2 **Cooling of a Hot Block by Forced Air at High Elevation**

The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m × 6 m flat plate whose temperature is 140°C (Fig. 7-13). Determine the rate of heat transfer from the plate if the air flows parallel to the (*a*) 6-m-long side and (*b*) the 1.5-m side.

SOLUTION The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas.

Properties The properties k, μ, C_p, and Pr of ideal gases are independent of pressure, while the properties ν and α are inversely proportional to density and thus pressure. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (140 + 20)/2 = 80°C$ and 1 atm pressure are (Table A–15)

$$k = 0.02953 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu_{@\,1\text{ atm}} = 2.097 \times 10^{-5} \text{ m}^2/\text{s}$$

The atmospheric pressure in Denver is P = (83.4 kPa)/(101.325 kPa/atm) = 0.823 atm. Then the kinematic viscosity of air in Denver becomes

$$\nu = \nu_{@\,1\text{ atm}}/P = (2.097 \times 10^{-5} \text{ m}^2/\text{s})/0.823 = 2.548 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis (*a*) When air flow is parallel to the long side, we have L = 6 m, and the Reynolds number at the end of the plate becomes

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8 \text{ m/s})(6 \text{ m})}{2.548 \times 10^{-5} \text{ m}^2/\text{s}} = 1.884 \times 10^6$$

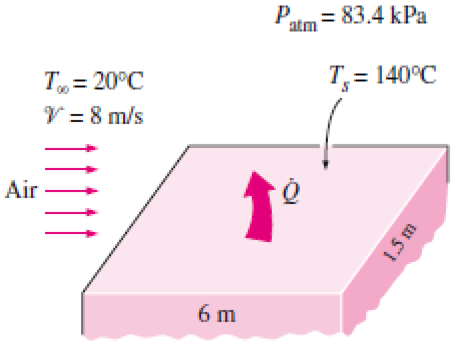


FIGURE 7–13
Schematic for Example 7-2.

which is greater than the critical Reynolds number. Thus, we have combined laminar and turbulent flow, and the average Nusselt number for the entire plate is determined to be

$$\begin{aligned} \text{Nu} = \frac{hL}{k} &= (0.037\ \text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \\ &= [0.037(1.884 \times 10^6)^{0.8} - 871]0.7154^{1/3} \\ &= 2687 \end{aligned}$$

Then

$$\begin{aligned} h &= \frac{k}{L}\text{Nu} = \frac{0.02953\ \text{W/m}\cdot{}^\circ\text{C}}{6\ \text{m}}(2687) = 13.2\ \text{W/m}^2\cdot{}^\circ\text{C} \\ A_s &= wL = (1.5\ \text{m})(6\ \text{m}) = 9\ \text{m}^2 \end{aligned}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2\ \text{W/m}^2\cdot{}^\circ\text{C})(9\ \text{m}^2)(140 - 20)^\circ\text{C} = \mathbf{1.43 \times 10^4\ W}$$

Note that if we disregarded the laminar region and assumed turbulent flow over the entire plate, we would get Nu = 3466 from Eq. 7–22, which is 29 percent higher than the value calculated above.

(*b*) When air flow is along the short side, we have $L = 1.5$ m, and the Reynolds number at the end of the plate becomes

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8\ \text{m/s})(1.5\ \text{m})}{2.548 \times 10^{-5}\ \text{m}^2/\text{s}} = 4.71 \times 10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate, and the average Nusselt number is

$$\text{Nu} = \frac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} = 0.664 \times (4.71 \times 10^5)^{0.5} \times 0.7154^{1/3} = 408$$

Then

$$h = \frac{k}{L}\text{Nu} = \frac{0.02953\ \text{W/m} \cdot {}^\circ\text{C}}{1.5\ \text{m}}(408) = 8.03\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (8.03\ \text{W/m}^2 \cdot {}^\circ\text{C})(9\ \text{m}^2)(140 - 20)^\circ\text{C} = \mathbf{8670\ W}$$

which is considerably less than the heat transfer rate determined in case (*a*).

Discussion Note that the *direction* of fluid flow can have a significant effect on convection heat transfer to or from a surface (Fig. 7-14). In this case, we can increase the heat transfer rate by 65 percent by simply blowing the air along the long side of the rectangular plate instead of the short side.

EXAMPLE 7–6 Cooling of a Steel Ball by Forced Air

A 25-cm-diameter stainless steel ball (ρ = 8055 kg/m³, C_p = 480 J/kg · °C) is removed from the oven at a uniform temperature of 300°C (Fig. 7–24). The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.

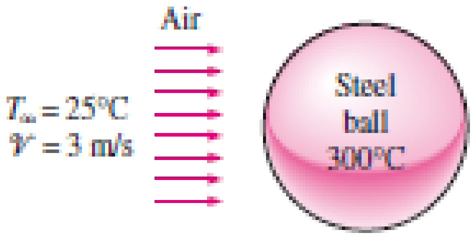


FIGURE 7–24
Schematic for Example 7–6.

SOLUTION A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas. **4** The outer surface temperature of the ball is uniform at all times. **5** The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

Properties The dynamic viscosity of air at the average surface temperature is $\mu_s = \mu_{@\,250°C} = 2.76 \times 10^{-5}$ kg/m · s. The properties of air at the free-stream temperature of 25°C and 1 atm are (Table A-15)

$$k = 0.02551 \text{ W/m}\cdot°\text{C} \qquad \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu = 1.849 \times 10^{-5} \text{ kg/m}\cdot\text{s} \qquad \text{Pr} = 0.7296$$

Analysis The Reynolds number is determined from

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(3 \text{ m/s})(0.25 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 4.802 \times 10^4$$

The Nusselt number is

$$\begin{aligned}\text{Nu} = \frac{hD}{k} &= 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\,\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \\ &= 2 + [0.4(4.802 \times 10^4)^{1/2} + 0.06(4.802 \times 10^4)^{2/3}](0.7296)^{0.4} \\ &\quad \times \left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1/4} \\ &= 135\end{aligned}$$

Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551\ \text{W/m}\cdot{}^\circ\text{C}}{0.25\ \text{m}}(135) = \mathbf{13.8\ W/m^2\cdot{}^\circ C}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25\ \text{m})^2 = 0.1963\ \text{m}^2$$

$$\dot{Q}_{ave} = hA_s(T_{s,ave} - T_\infty) = (13.8\ \text{W/m}^2\cdot{}^\circ\text{C})(0.1963\ \text{m}^2)(250 - 25)^\circ\text{C} = 610\ \text{W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

$$m = \rho V = \rho\tfrac{1}{6}\pi D^3 = (8055\ \text{kg/m}^3)\,\tfrac{1}{6}\pi(0.25\ \text{m})^3 = 65.9\ \text{kg}$$

$$Q_{total} = mC_p(T_2 - T_1) = (65.9\ \text{kg})(480\ \text{J/kg}\cdot{}^\circ\text{C})(300 - 200)^\circ\text{C} = 3{,}163{,}000\ \text{J}$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t \approx \frac{Q}{\dot{Q}_{ave}} = \frac{3{,}163{,}000\ \text{J}}{610\ \text{J/s}} = 5185\ \text{s} = \mathbf{1\ h\ 26\ min}$$

Discussion The time of cooling could also be determined more accurately using the transient temperature charts or relations introduced in Chapter 4. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naive to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between one and two hours.

6.2 Forced Convection

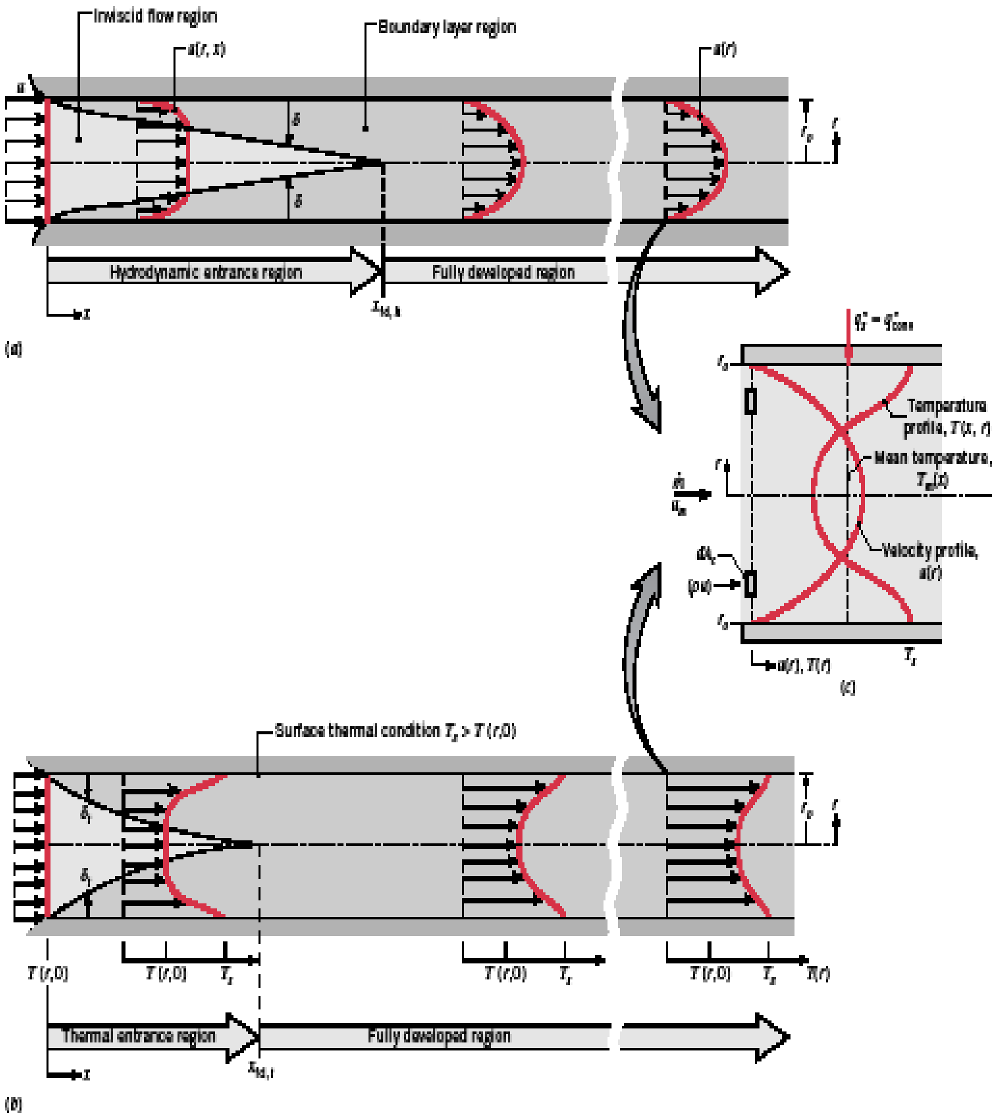
Internal Flow

Introduction

- When flow enters a tube, a hydrodynamic boundary layer forms in the *entrance region,* growing in thickness to eventually fill the tube.
- Beyond this location, referred to as the *fully developed region,* the velocity profile no longer changes in the flow direction.
- We begin by considering thermal boundary layer formation in the entrance and fully developed regions, and how the convection coefficient is determined from the resulting temperature profile.
- We will introduce empirical correlations to estimate convection coefficients for laminar and turbulent flows in the fully developed region, deferring consideration of correlations for the entrance region.

Hydrodynamic and Thermal Considerations

- The development of the boundary layer for *laminar* flow in a circular tube is represented in Fig 1
- Because of viscous effects, the uniform velocity profile at the entrance will gradually change to a parabolic distribution as the boundary layer begins to fill the tube in the entrance region.



- Beyond the *hydrodynamic entrance length, Xfd,h*, the velocity profile no longer changes, and we speak of the flow as ***hydrodynamically fully developed***.
- The extent of the entrance region, as well as the shape of the velocity profile, depends upon Reynolds number, which for internal flow has the form

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} = \frac{4\dot{m}}{\pi D \mu}$$

- In a fully developed flow $Re_{D,c} \approx 2300$ *ritical Reynolds number* corresponding to the onset of turbulence is
- For *laminar flow* (Re*D*≤2300), the *hydrodynamic entry length* has the form
- While for *turbul*... he entry length is approximately inde $\left(\frac{x_{fd,h}}{D}\right)_{lam} \le 0.05\,Re_D$ eynolds number and that, as a first approximation

$$10 \le \left(\frac{x_{fd,h}}{D}\right)_{turb} \le 60$$

- For *laminar flow,* the thermal entry length may be expressed as:-

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} \leq 0.05\,Re_D\,Pr \qquad [Re_D < 2300]$$

- For *turbulent f*… early independent of Prandtl num… approximation the *thermal entrance length* is:-

$$\left(\frac{x_{fd,t}}{D}\right)_{turb} = 10 \qquad [Re_D \geq 10{,}000]$$

The Mean Temperature

- As shown in Fig.*c*, the temperature and velocity profiles at a *particular* location in the flow direction *x* each depend on radius, *r*.
- The *mean* temperature of the fluid, also referred to as the average or bulk temperature, shown on the figure as *Tm*(*x*), is defined in terms of the energy transported by the fluid as it moves past location *x*.
- For incompressible flow, with constant specific heat C*p*, the *mean temperature* is found from

$$T_m = \frac{\int_{A_c} uT dA_c}{u_m A_c}$$

where u_m is the mean velocity. For a circular tube, $dA_c = 2\pi r dr$, and it follows that

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} uTr\,dr$$

- The ***mean temperature*** is the fluid reference temperature used for determining the convection heat rate with Newton's law of cooling and the overall energy balance.
- Newton's law of cooling, also referred to as the ***convection rate equation,*** is expressed as

$$q_s'' = q_{conv}'' = h(T_s - T_m)$$

Fully Developed Conditions.

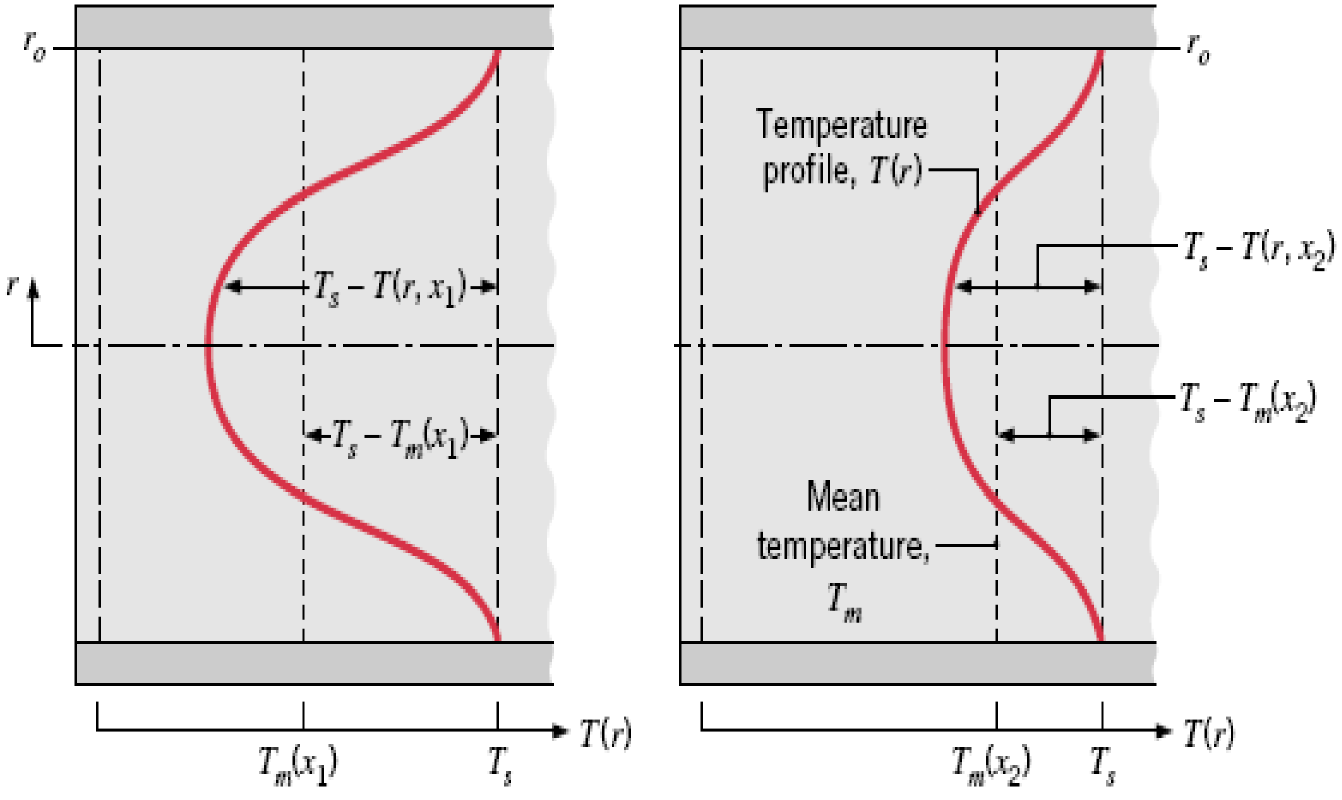
- The temperature profile can be conveniently represented as the dimensionless ratio

  (T_s-T)/(T_s -T_m).

- Literatures indicate that although the temperature profile $T(r)$ continues to change with x, the *relative shape* of the profile given by this *temperature ratio* is independent of x for fully developed conditions.
- The requirement for such a condition is mathematically stated as:-

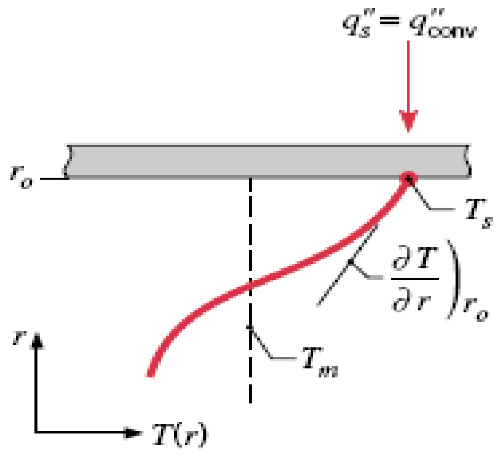
$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right]_{fd,t} = 0$$

- Where *Ts* is the tube surface temperature, *T* is the local fluid temperature, and *Tm* is the mean temperature as shown in Fig. 2*a*.



(*a*)

- Since the temperature ratio is independent of *x*, the derivative of this ratio with respect to *r* must also be independent of *x*.
- Evaluating this derivative at the tube surface (note that *Ts* and *Tm* are constants in sofar
with respect to *r* is concerned), we ob



$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=r_o} = \frac{-\partial T/\partial r|_{r=r_o}}{T_s - T_m} \neq f(x)$$

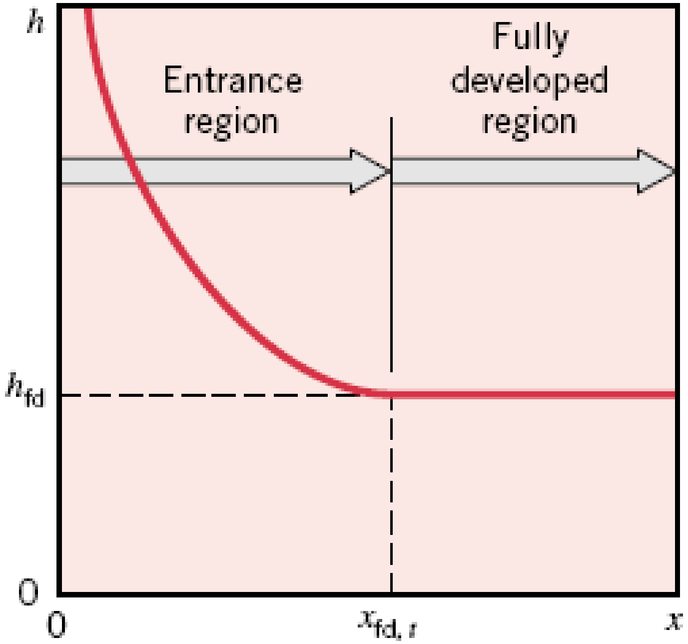
Substituting for $\partial T/\partial r$ urier's law,

$$q_s'' = k\frac{\partial T}{\partial r}\bigg|_{r=r_o}$$

and for q_s'' m Newton's law of cooling, we obtain

$$\frac{h}{k} \neq f(x) = \text{constant}$$

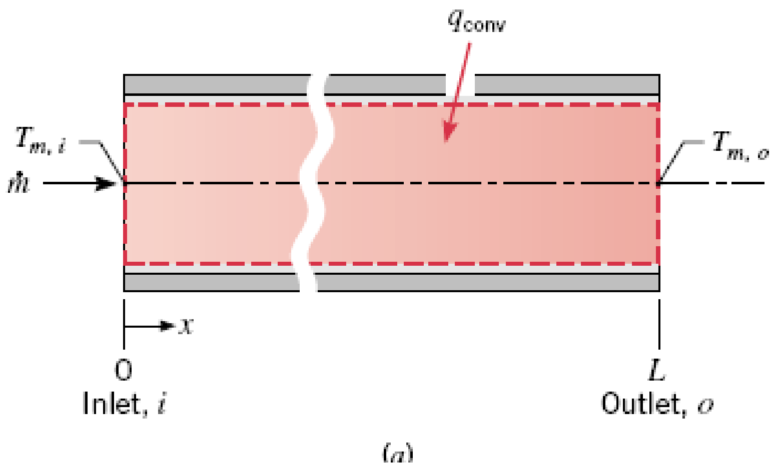
- Hence, *in the thermally fully developed flow of a fluid with constant properties, the local convection coefficient is a constant, independent of x*.



(b)

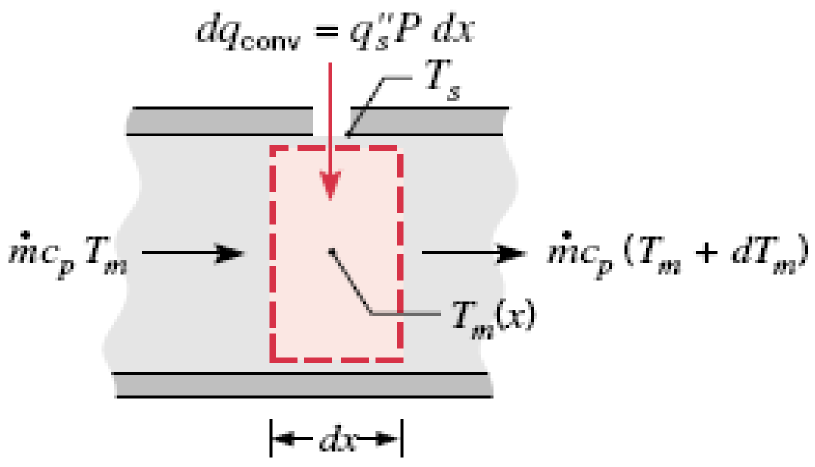
Energy Balances and Methods of Heating

- ***Overall Tube Energy Balance:-*** Consider the tube flow of Fig.4*a*. Fluid moves at a constant flow rate and convection heat transfer occurs along the wall surface. Assuming that fluid kinetic and potential energy changes are negligible, there is no shaft work, and regarding *cp* as constant, the energy rate balance yields:-



$$q_{conv} = \dot{m} c_p (T_{m,o} - T_{m,i})$$

Energy Balance on a Differential Control Volume.



(b)

$$dq_{conv} = \dot{m}c_p dT_m$$

$$dq_{conv} = q_s'' P dx$$

$$q_s'' P dx = \dot{m}c_p dT_m$$

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}c_p} \qquad [\text{surface heat flux, } q_s'']$$

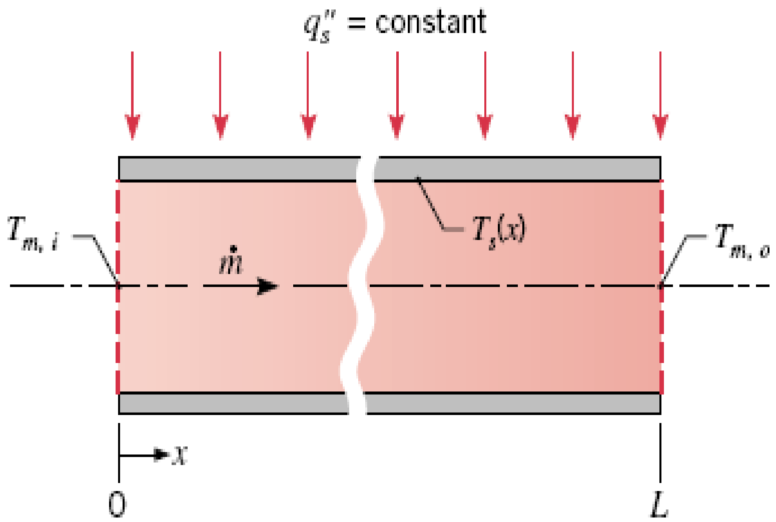
where P is the surface perimeter.

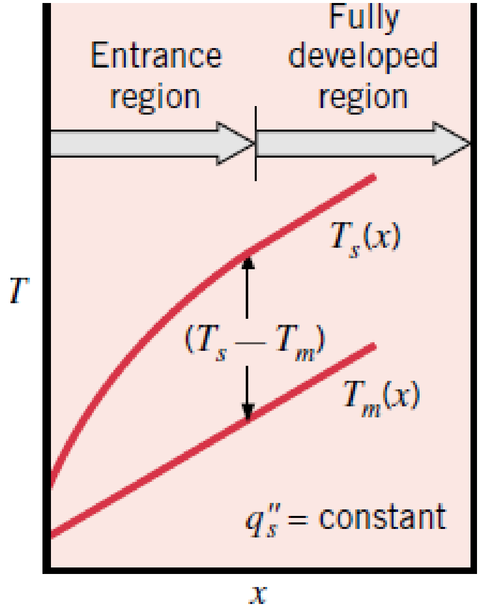
$$q_s'' = h(T_s - T_m),$$

$$\frac{dT_m}{dx} = \frac{P}{\dot{m}c_p} h(T_s - T_m) \qquad [\text{surface temperature, } T_s]$$

- The solutions to the last eqn. for *Tm*(*x*) depend upon the ***surface thermal condition.***
- We will now consider two special cases of interest: *constant surface heat flux* (*qs*) and *constant surface temperature* (*Ts*). It is common to find one of these conditions existing in practical applications to a reasonable approximation.

Thermal Condition: Constant Surface Heat Flux



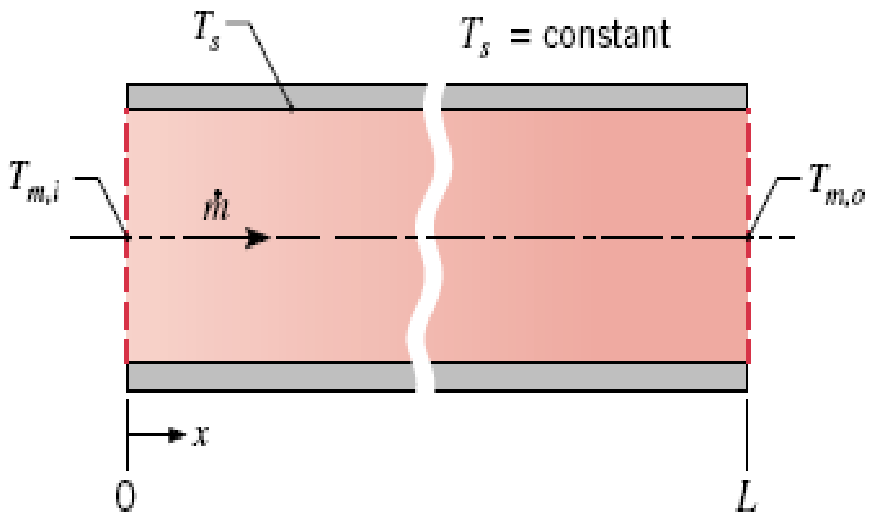


$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \text{constant}$$

- Integrating from *x*= 0 to some axial position-*x*, we obtain the *mean temperature distribution, Tm*(*x*)

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x \qquad [q_s'' = \text{constant}]$$

Thermal Condition: Constant Surface Temperature, *Ts*



If we had integrated from $x = 0$ to some axial position, we obtain the *mean temperature distribution, Tm*(x)

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right) \qquad [T_s = \text{constant}]$$

- Defining ΔT as (*Ts - Tm*),

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\, \Delta T$$

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m}c_p}\int_0^L h\, dx$$

$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\left(\frac{1}{L}\int_0^L h\, dx\right)$$

$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\bar{h}_L \qquad [T_s = \text{constant}]$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right) \qquad [T_s = \text{constant}]$$

Where $\bar{h}$ is now the average value of h from the tube inlet to x.

- Determination of an expression for the total heat transfer rate *q*conv. is complicated by the exponential nature of the temperature decrease.

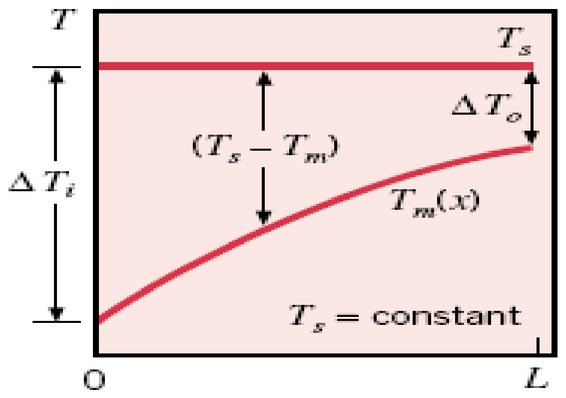
- Inserting mcp in the above equation we will obtain

$$q_{conv} = \dot{m}c_p[(T_s - T_{m,i}) - (T_s - T_{m,o})] = \dot{m}c_p(\Delta T_i - \Delta T_o)$$

$$q_{conv} = \bar{h}A_s\Delta T_{lm} \qquad [T_s = \text{constant}]$$

where A_s is the tube surface area ($A_s = P \cdot L$) and ΔT_{lm} is the *log mean temperature difference (LMTD)*

$$\Delta T_{lm} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}$$



Convection Correlations for Tubes: Fully Developed Region

- In this section we present correlations for estimating the coefficients for *fully developed laminar* and *turbulent* flows in *circular* and *noncircular tubes.*

Laminar Flow

- The problem of laminar flow (Re*D* <2300) in tubes has been treated theoretically, and the results can be used to determine the convection coefficients.
- For flow in a *circular* tube characterized by *uniform surface heat flux* and *laminar, fully developed conditions,* the *Nusselt number is a constant,* independent of Re*D*, Pr, and axial location.

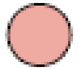
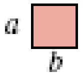
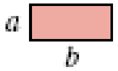
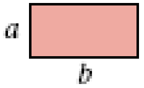
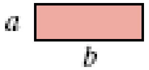
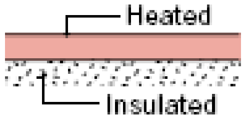
$$Nu_D = \frac{hD}{k} = 4.36 \qquad [q''_s = \text{constant}]$$

- When the thermal surface condition is characterized by a *constant surface temperature,* the results are of similar form, but with a smaller value for the Nusselt number

- In using these equations to determine *h*, the thermal conductivity should be evaluated at *Tm*.

$$\mathrm{Nu}_D = \frac{hD}{k} = 3.66 \qquad [T_s = \text{constant}]$$

Table Nusselt Numbers for Fully Developed Laminar Flow in Noncircular Tubes for Constant T_s and q''_s Surface Thermal Conditions[a]

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ Constant q''_s	Constant T_s
	—	4.36	3.66
	1.0	3.61	2.98
	1.43	3.73	3.08
	2.0	4.12	3.39
	3.0	4.79	3.96
	4.0	5.33	4.44
	8.0	6.49	5.60
	∞	8.23	7.54
 Heated Insulated	∞	5.39	4.86
	—	3.11	2.47

[a]The characteristic length is the hydraulic diameter, D_h.

- For applications involving convection transport in *noncircular tubes,* to at least a first approximation, the foregoing correlations can be applied by using the ***hydraulic diameter*** as the characteristic length

$$D_h \equiv \frac{4A_c}{P}$$

- Where *Ac* and *P* are the … tional area and the *wetted perimeter,* resp … this diameter that should be used in calc … nolds and Nusselt numbers.

Turbulent Flow

- A commonly used expression for computing the *local* Nusselt number for *fully developed* (hydrodynamically and thermally) *turbulent* flow in a smooth *circular tube* is the *Dittus-Boelter correlation* of the form

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^n \qquad \begin{bmatrix} 0.6 \lesssim Pr \lesssim 160 \\ Re_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10 \end{bmatrix}$$

- where *n*=0.4 for heating (*Ts*>*Tm*) and 0.3 for cooling (*Ts*<*Tm*). These correlations have been confirmed experimentally for the range of conditions shown in the brackets. The correlations can be used for small to moderate temperature differences, (*Ts*-*Tm*) with all properties evaluated at Tm.

- For flows characterized by large property variations, the *Sieder-Tate correlation* is recommended

$$Nu_D = 0.027\, Re_D^{4/5}\, Pr^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad \left[\begin{array}{l} 0.7 \le Pr \le 16{,}700 \\ Re_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10 \end{array}\right]$$

where all properties except *s* are evaluated at T_m.

Table Summary of Forced Convection Heat Transfer Correlations for Internal Flow in Smooth Circular Tubes[c]

Flow/Surface Thermal Conditions	Correlation[a,b]		Restrictions on Applicability
Laminar, fully developed, $(x_{fd}/D) > 0.05\,Re_D Pr$			
Constant q_s''	$Nu_D = 4.36$	(17.61)	$Pr \geq 0.6$, $Re_D \leq 2300$
Constant T_s	$Nu_D = 3.66$	(17.62)	$Pr \geq 0.6$, $Re_D \leq 2300$
Turbulent, fully developed, $(x_{fd}/D) > 10$			
Constant q_s'' or T_s (*Dittus-Boelter*)	$Nu_D = 0.023\,Re_D^{4/5}\,Pr^n$	(17.64)	$0.6 \leq Pr \leq 160$, $Re_D \geq 10{,}000$ $n = 0.4$ for $T_s > T_m$ and $n = 0.$ for $T_s < T_m$
Constant q_s'' or T_s (*Sieder-Tate*)	$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$	(17.65)	$0.7 \leq Pr \leq 16{,}700$, $Re_D \geq 10{,}0$

[a]Thermophysical properties in Eqs. 17.61, 17.62, and 17.64 are based upon the mean temperature, T_m. If the correlations are used to estimate the *average* Nusselt number over the entire tube length, the properties should be based upon the average of the mean temperatures, $\overline{T}_m = (T_{m,i} + T_{m,o})/$

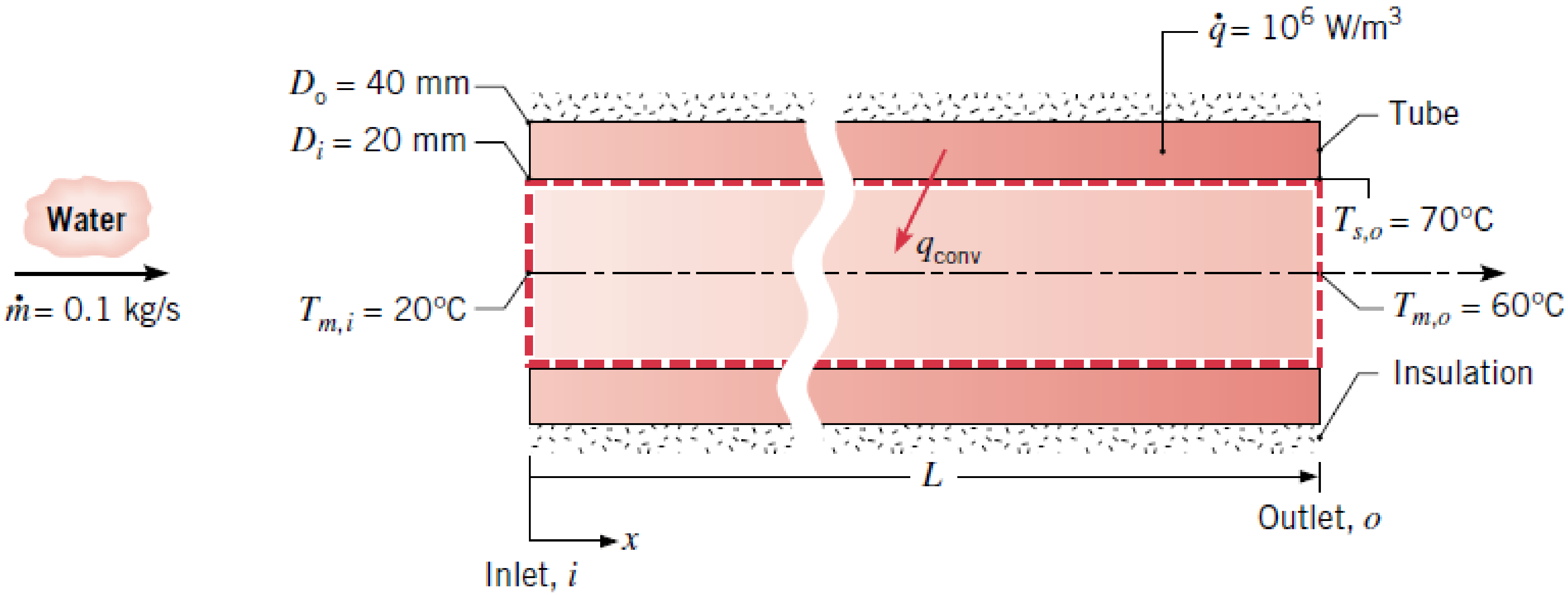
[b]Thermophysical properties in Eq. 17.65 should be evaluated at T_m or $\overline{T}_m$, except for μ_s, which is evaluated at the tube wall temperature T_s or $\overline{T}$

A system for heating water from an inlet temperature of $T_{m,i} = 20°C$ to an outlet temperature of $T_{m,o} = 60°C$ involves passing the water through a tube having inner and outer diameters of 20 and 40 mm. The outer surface of the tube is well insulated, and electrical power dissipation within the wall provides for a uniform volumetric generation rate of $\dot{q} = 10^6$ W/m^3.

(a) For a water mass flow rate of $\dot{m} = 0.1$ kg/s, how long must the tube be to achieve the desired outlet temperature?

(b) Do fully developed hydrodynamic and thermal conditions exist in the flow?

(c) If the inner surface temperature of the tube is $T_s = 70°C$ at the outlet ($x = L$), what is the local convection heat transfer coefficient at the outlet?



Steam condensing on the outer surface of a thin-walled circular tube of 50-mm diameter and 6-m length maintains a uniform surface temperature of 100°C. Water flows through the tube at a rate of $\dot{m} = 0.25$ kg/s, and its inlet and outlet temperatures are $T_{m,i} = 15$°C and $T_{m,o} = 57$°C. What is the average convection coefficient associated with the water flow?