

Chapter three

Work and heat

Objective

- understand clearly the definitions of both work and heat
- To construct a model for heat and work and use the result in our analysis of energy transfers and changes
- Identifying the forms of work
- To understand Heat transfer by conduction, convection (flow), and radiation
- To evaluate energy transfer on or from the system in the form of work and heat

3.1 Work

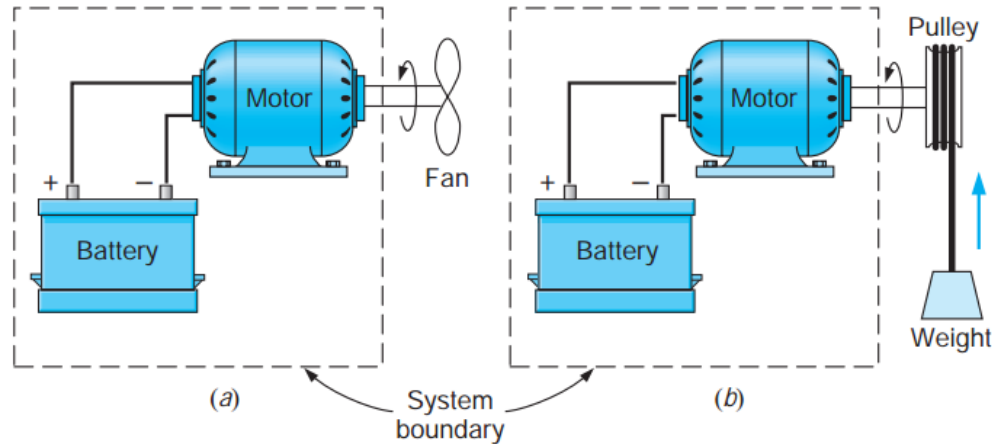
- Work is usually defined as a force F acting through a displacement x , where the displacement is in the direction of the force. That is,

$$W = \int_1^2 F dx$$

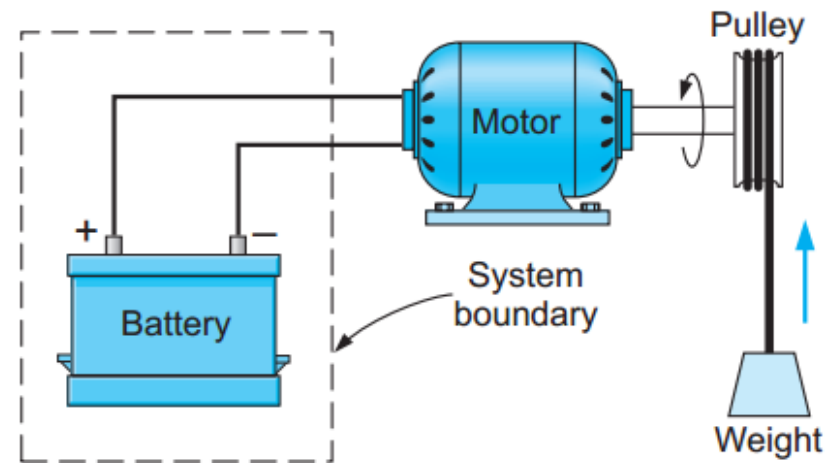
- This is a very useful relationship because it enables us to find the work required to raise a weight, to stretch a wire, or to move a charged particle through a magnetic field.
- Work is done by a system: the sole effect on the surroundings (everything external to the system) it could be the raising of a weight.
- Work done *by* a system is considered positive and work done *on* a system is considered negative. The symbol W designates the work done by a system.

Cont..

- Example of work crossing the boundary of a system.



- Example of work crossing the boundary of a system because of an electric current flow across the system boundary.



Cont..

- Kinetic energy, potential energy, and internal energy, are forms of energy that are properties of a system.
- *Work* is a form of energy, but it is energy in transit.
- Work is not a property of a system. Work is a process done by or on a system, but a system contains no work.
- This distinction between the forms of energy that are properties of a system and the forms of energy that are transferred to and from a system is important to the understanding of energy transfer systems.

3.2 Units For Work

- As already noted, work done *by* a system, such as that done by a gas expanding against a piston, is positive, and work done *on* a system, such as that done by a piston compressing a gas, is negative. Thus, positive work means that energy leaves the system, and negative work means that energy is added to the system.
- unit for work in SI units is called the joule (J). that is, the product of a unit force (one newton) acting through a unit distance (one meter).

$$1 \text{ J} = 1 \text{ N m}$$

- Power is the time rate of doing work and is designated by the symbol “ P ” :

$$P = dW/dt$$

- The unit for power is joule per second, which is a watt (W):

$$1 \text{ W} = 1 \text{ J/s}$$

- A familiar unit for power in English units is the horsepower (hp), where

$$1 \text{ hp} = 550 \text{ ft lbf/s}$$

3.3 other form of work

Electrical Work

- In an electric field, electrons in a wire move under the effect of electromotive forces, doing work.
- When N coulombs of electrical charge move through a potential difference V , the electrical work done is

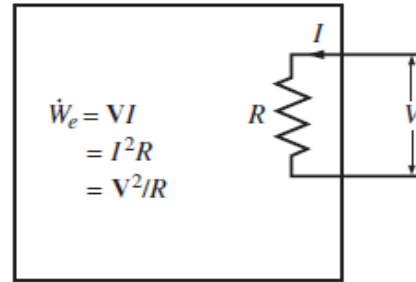
$$W_e = VN$$

- which can also be expressed in the rate form as:

$$\dot{W}_e = VI \quad (\text{W})$$

Cont...

- Where \dot{W}_e is the electrical power and I is the number of electrical charges flowing per unit time, that is, the current.



- In general, both V and I vary with time, and the electrical work done during a time interval Δt is expressed as

$$W_e = VI \Delta t \quad (\text{kJ})$$

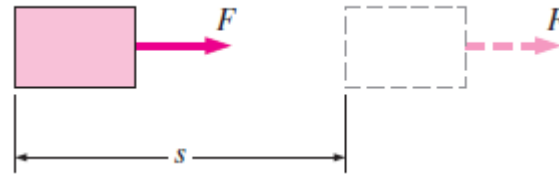
- When both V and I remain constant during the time interval t , it reduces to:

$$W_e = \int_1^2 VI dt \quad (\text{kJ})$$

cont..

Mechanical Work

- There are several different ways of doing work, each in some way related to a force acting through a distance.



- In elementary mechanics, the work done by a constant force F on a body displaced a distance s in the direction of the force is given by:

$$W = Fs \quad (\text{kJ})$$

- If the force F is not constant, the work done is obtained by adding (i.e. integrating) the differential amounts of work,

$$W = \int_1^2 F ds \quad (\text{kJ})$$

Cont..

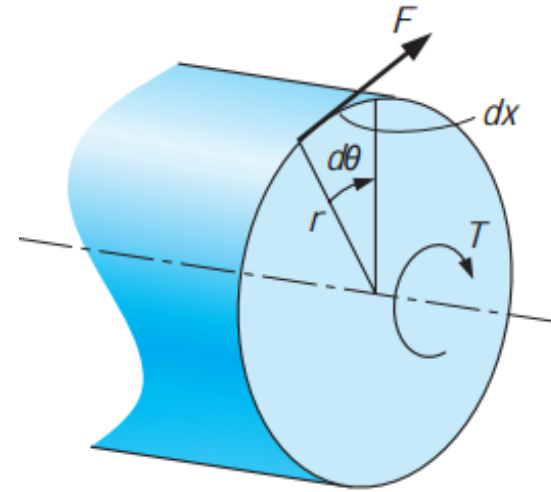
- **1. Shaft Work**

- the work crossing the boundary of the system associated with a rotating shaft.

$$\delta W = F dx = Fr d\theta = T d\theta$$

- that is, force acting through a distance dx or a torque ($T = Fr$) acting through an angle of rotation, as shown in Figure below.

- Force acting at radius r gives a torque $T = Fr$



- Now the power becomes

$$P = \frac{dw}{dt} = F \frac{dx}{dt} = FV = Fr \frac{d\theta}{dt} = T\omega$$

- that is, force times rate of displacement (velocity) or torque times angular velocity.
- It is often convenient to speak of the work per unit mass of the system, often termed *specific work*.

Cont...

- For a specified constant torque, the work done during n revolutions is determined as follows:

$$W_{\text{sh}} = Fs = \left(\frac{T}{r}\right)(2\pi rn) = 2\pi nT \quad (\text{kJ})$$

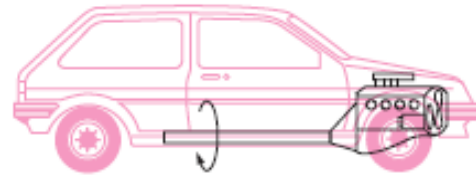
- The power transmitted through the shaft is the shaft work done per unit time, which can be expressed as:-

$$\dot{W}_{\text{sh}} = 2\pi \dot{n}T \quad (\text{kW})$$

- where \dot{n} is the number of revolutions per unit time.

Cont...

- **Example:** Determine the power transmitted through the shaft of a car when the torque applied is $200 \text{ N} \cdot \text{m}$ and the shaft rotates at a rate of 4000 revolutions per minute (rpm).

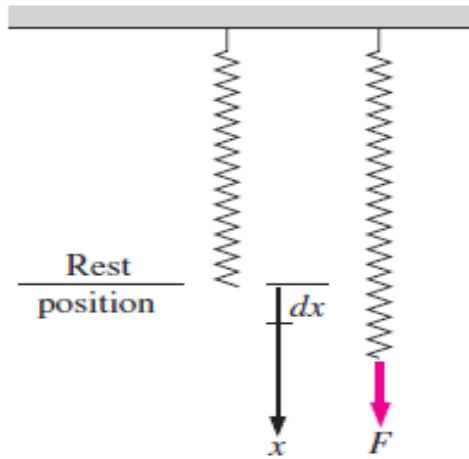


$$\dot{n} = 4000 \text{ rpm}$$
$$T = 200 \text{ N} \cdot \text{m}$$

Cont...

2. Spring Work

- It is common knowledge that when a force is applied on a spring, the length of the spring changes.



- When the length of the spring changes by a differential amount dx under the influence of a force F , the work done is:

$$\delta W_{\text{spring}} = F dx$$

Cont...

- To determine the total spring work, we need to know a functional relationship between F and x .
- For linear elastic springs, the displacement x is *proportional* to the force applied. That is,

$$F = kx \quad (\text{kN})$$

- where k is the spring constant and has the unit kN/m . The displacement x is measured from the undisturbed position of the spring (that is, $x=0$ when $F=0$). Substituting the above equations and integrating yield:-

$$W_{\text{spring}} = \frac{1}{2}k(x_2^2 - x_1^2) \quad (\text{kJ})$$

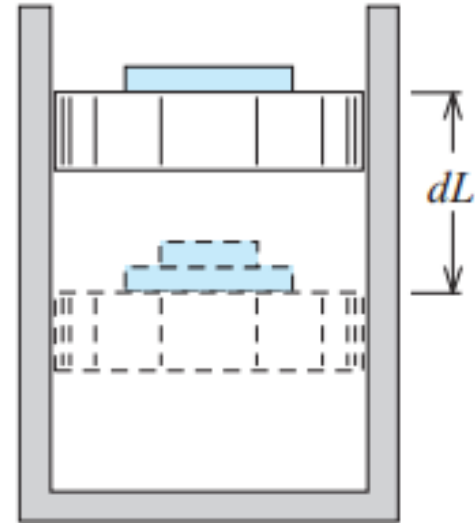
- where x_1 and x_2 are the initial and the final displacements of the spring respectively, measured from the undisturbed position of the spring.

3.4 Work Done At The Moving Boundary Of A Simple Compressible System

- We have already noted that there are a variety of ways in which work can be done on or by a system.
- These include work done by a rotating shaft, electrical work, and work done by the movement of the system boundary, such as the work done in moving the piston in a cylinder.
- In this section we will consider in some detail the work done at the moving boundary of a simple compressible system during a quasi-equilibrium process.

Cont..

- Consider as a system the gas contained in a cylinder and piston, Remove one of the small weights from the piston, which will cause the piston to move upward a distance dL .
 - We can consider this quasi-equilibrium process and calculate the amount of work W done by the system during this process.
 - The total force on the piston is PA , where P is the pressure of the gas and A is the area of the piston. Therefore, the work δW is
- Example of work done at the moving boundary of a system in a quasi-equilibrium process



$$\delta W = PA dL$$

Cont..

- But, $A dL = dV$ the change in volume of the gas. Therefore,

$$\delta W = P dV$$

- The work done at the moving boundary during a given quasi-equilibrium process can be found by integrating the above equation. However, this integration can be performed only if we know the relationship between P and V during this process.
- This relationship may be expressed as an equation, or it may be shown as a graph.
- Let us consider a graphical solution first.
- example a compression process occurs during the compression of air in a cylinder as shown below.

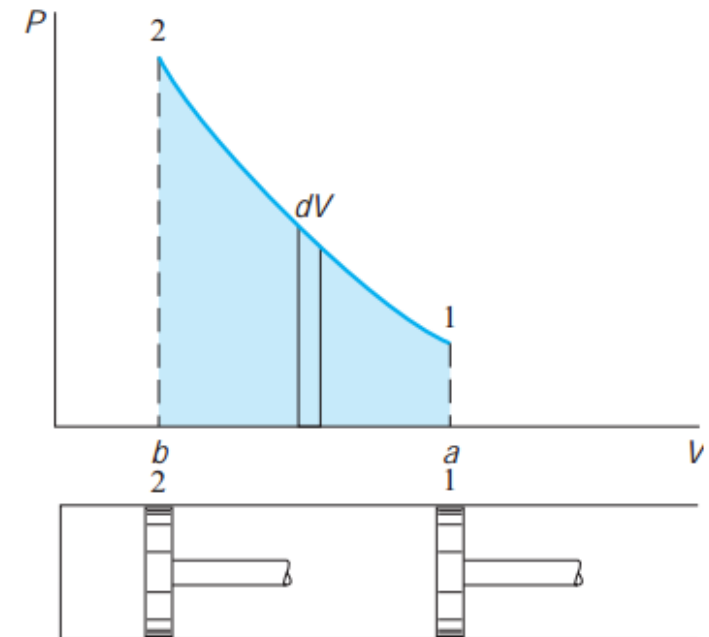
Cont..

- At the beginning of the process the piston is at position 1, and the pressure is relatively low. This state is represented on a pressure–volume diagram .
- At the end of the process the piston is in position 2, and the corresponding state of the gas is shown at point 2 on the P – V diagram.
- Let us assume that this compression was a quasi-equilibrium process and that during the process the system passed through the states shown by the line connecting states 1 and 2 on the P – V diagram.
- The assumption of a quasi-equilibrium process is essential here because each point on line 1–2 represents a definite state, and these states correspond to the actual state of the system only if the deviation from equilibrium is infinitesimal.

- The work done on the air during this compression process can be found by integrating dw .

$${}_1W_2 = \int_1^2 \delta W = \int_1^2 P dV$$

- Use of a P – V diagram to show work done at the moving boundary of a system in a quasi-equilibrium process.



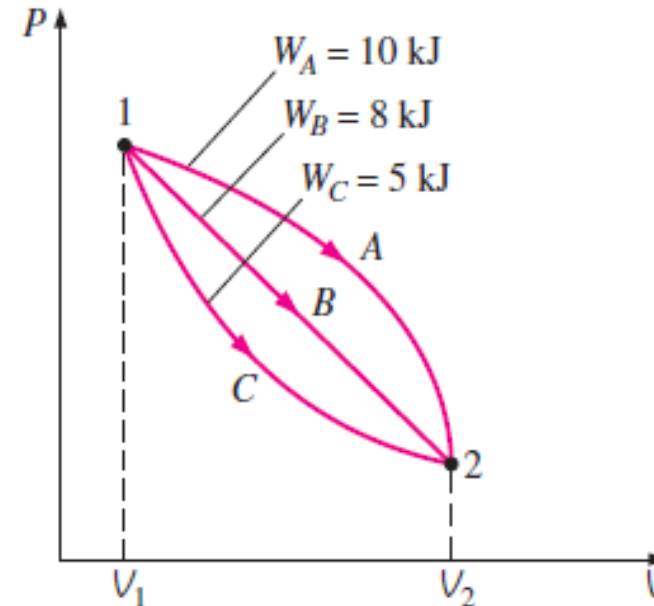
Cont..

- from the P - V diagram that the work done during this process, is represented by the area under curve 1-2, area $a-1-2-b-a$.
- In this example the volume decreased, and area $a-1-2-b-a$ represents work done on the system.
- If the process had proceeded from state 2 to state 1 along the same path, the same area would represent work done by the system.
- Further consideration of a P - V diagram, as shown Figure below, leads to another important conclusion.
- It is possible to go from state 1 to state 2 along many different quasi-equilibrium paths, such as A , B , or C .
- Since the area under each curve represents the work for each process, the amount of work done during each process not only is a function of the end states of the process but also depends on the path followed in going from one state to another

Cont...

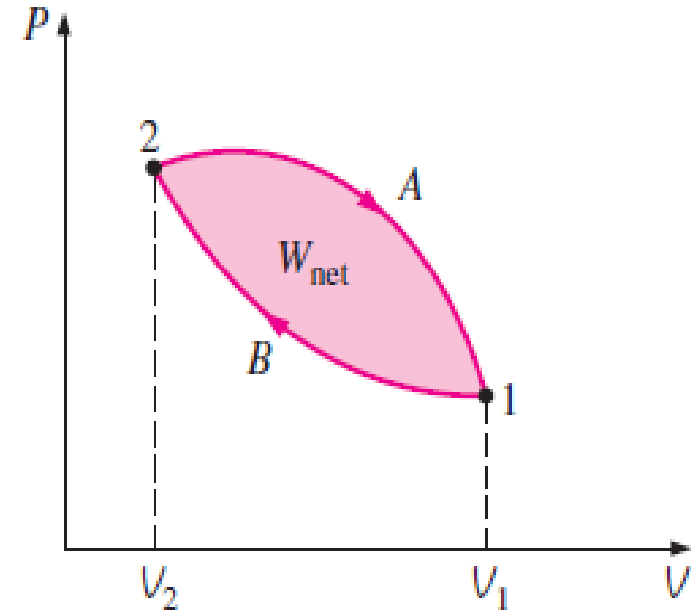
- A gas can follow several different paths as it expands from state 1 to state 2.
- In general, each path will have a different area underneath it, and since this area represents the magnitude of the work, the work done will be different for each process.
- This is expected, since work is a path function (i.e., it depends on the path followed as well as the end states).

The boundary work done during a process depends on the path followed as well as the end states.



Cont...

- If work were not a path function, no cyclic devices (car engines, power plants) could operate as work-producing devices.
- The work produced by these devices during one part of the cycle would have to be consumed during another part, and there would be no net work output.
- The cycle shown in the figure produces a net work output because the work done by the system during the expansion process (area under path A) is greater than the work done on the system during the compression part of the cycle (area under path B), and the difference between these two is the net work done during the cycle (the colored area).



Cont...

- In evaluating the integral of dW , we should always keep in mind that we wish to determine the area under the curve in figure above. In connection with this point, we identify the following two classes of problems:
 - I. The relationship between P and V is given in terms of experimental data or in graphical form. Therefore, we may evaluate the integral of dW by graphical or numerical integration.
 - II. The relationship between P and V makes it possible to fit an analytical relationship between them. We may then integrate directly.

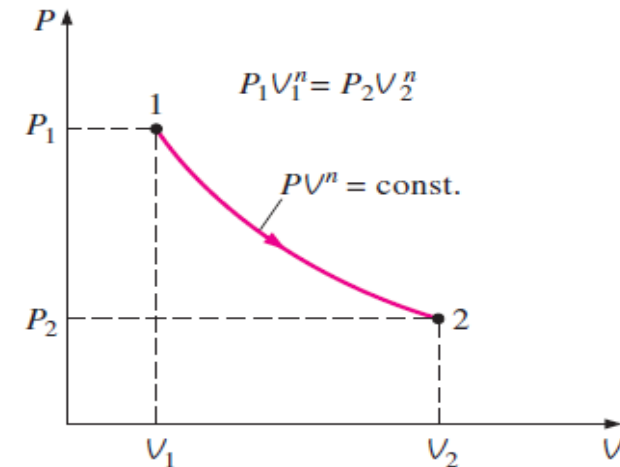
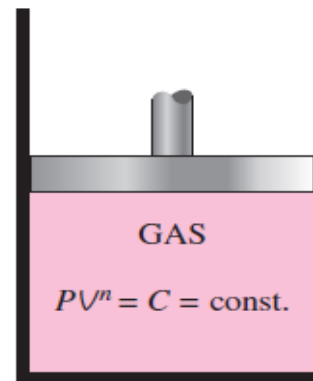
Cont..

Polytropic Process

- During actual expansion and compression processes of gases, pressure and volume are often related by:

$$PV^n = C,$$

- where n and C are constants.
- A process of this kind is called a **polytropic process** .



Cont...

- Below we develop a general expression for the work done during a polytropic process.
- The pressure for a polytropic process can be expressed as:-

$$P = CV^{-n}$$

- Substituting this

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-n} dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n + 1} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

Since $C = P_1 V_1^n = P_2 V_2^n$.

- For an ideal gas ($PV = mRT$), *this equation can* also be written as:-

$$W_b = \frac{mR(T_2 - T_1)}{1 - n} \quad n \neq 1 \quad (\text{kJ})$$

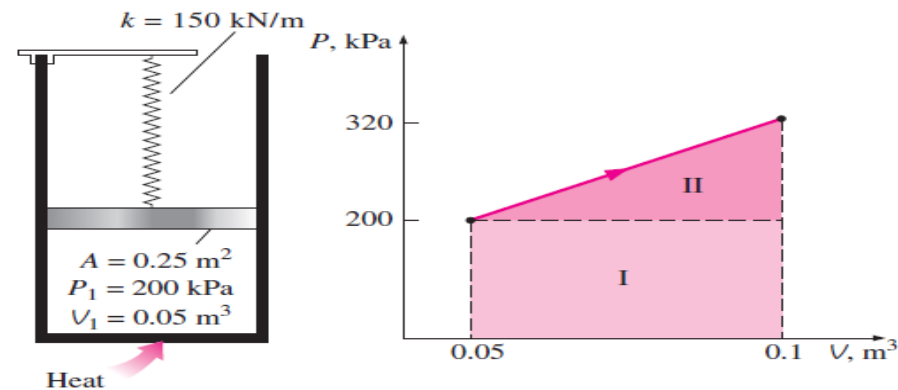
Cont...

- For the special case of $n=1$ the boundary work becomes:-

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-1} dV = PV \ln\left(\frac{V_2}{V_1}\right)$$

Example

- A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa . At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2 , determine (a) the final pressure inside the cylinder, (b) the total work done by the gas, and (c) the fraction of this work done against the spring to compress it.



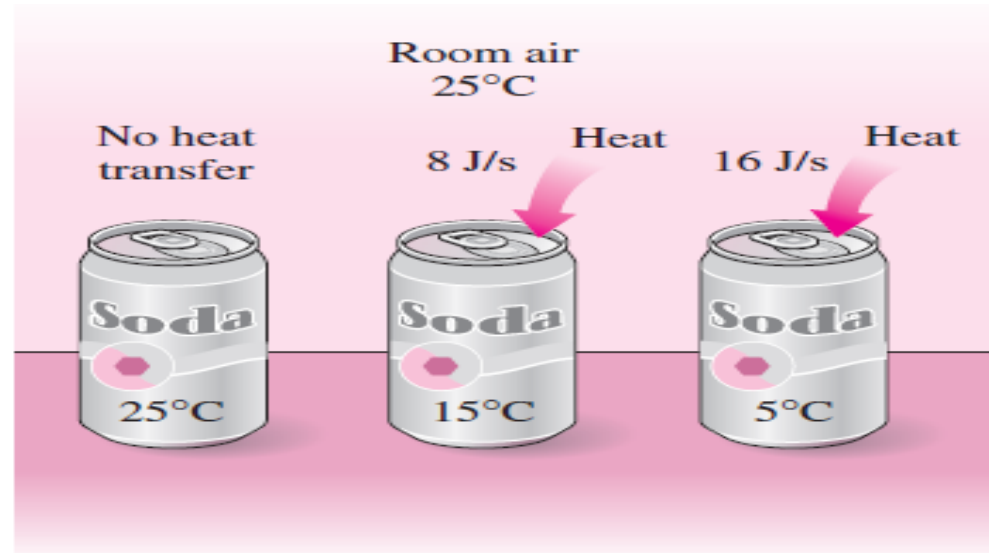
Cont...

- Example:-

1. A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.
2. A piston–cylinder device initially contains 0.4 m³ of air at 100 kPa and 80°C. The air is now compressed to 0.1 m³ in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.(assume air is ideal gas)

3.5 Heat

- Heat is defined as the form of energy that is transferred across the boundary of a system at a given temperature to another system (or the surroundings) at a lower temperature by virtue of the temperature difference between the two systems.
- That is, heat is transferred from the system at the higher temperature to the system at the lower temperature, and the heat transfer occurs solely because of the temperature difference between the two systems.



Cont...

The most common units for heat are

- BTU (Btu) - British Thermal Unit - also known as a "heat unit" in United States
- Calorie
- Joule

BTU - British Thermal Unit

- The unit of heat in the imperial system - the BTU - is the amount of heat required to raise the temperature of one pound of water through 1°F (58.5°F - 59.5°F) at sea level (30 inches of mercury).
- 1 Btu (British thermal unit) = 1055.06 J = 2.931 10⁻⁴ kWh = 0.252 kcal = 778.16 ft.lbf =
- An item using one kilowatt-hour of electricity generates 3412 Btu.

Calorie

- A calorie is commonly defined as the amount of heat required to raise the temperature of one gram of water 1°C the kilogram calorie, large calorie, food calorie, Calorie (capital C) or just calorie (lowercase c) is the amount of energy required to raise the temperature of one kilogram of water by one degree Celsius
- 1 calorie (cal) = 1/860 international watt-hour (Wh)
- 1 kcal = 4186.8 J = 1.163 10⁻³ kWh = 3.088 ft lb_f = 3.9683 Btu = 1000 cal

Cont..

- Heat transferred *to* a system is considered positive, and heat transferred *from* a system is considered negative. The symbol Q represents heat.
- A process in which there is no heat transfer ($Q = 0$) is called an adiabatic process.
- heat, like work, is a path function That is, the amount of heat transferred when a system undergoes a change from state 1 to state 2 depends on the path that the system follows during the change of state.

$$\int_1^2 \delta Q = {}_1Q_2$$

- The rate at which heat is transferred to a system is designated by the symbol \dot{Q} :

$$\dot{Q} = \delta Q / dt$$

- It is also convenient to speak of the heat transfer per unit mass of the system, q , often termed *specific heat transfer*, which is defined as

$$q = Q / m$$

3.6 Heat Transfer Modes

- Heat transfer is the transport of energy due to a temperature difference between different amounts of matter. We know that an ice cube taken out of the freezer will melt when it is placed in a warmer environment such as a glass of liquid water or on a plate with room air around it.
- Heat is transferred by three mechanisms: conduction, convection, and radiation.
- **Conduction:** is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interaction between particles.

$$\dot{Q} = -kA \frac{dT}{dx}$$

- **Convection:** is the transfer of energy between a solid surface and the adjacent fluid that is in motion, and it involves the combined effects of conduction and fluid motion.

$$\dot{Q} = Ah \Delta T$$

- **Radiation:** is the transfer of energy due to the emission of electromagnetic waves (or photons).

$$\dot{Q} = \varepsilon\sigma AT_s^4$$

Cont..

- Example** - Consider the constant transfer of energy from a warm room at 20°C inside a house to the colder ambient temperature of -10°C through a single-pane window, as shown in Figure below. The temperature variation with distance from the outside glass surface is shown by an outside convection heat transfer layer, but no such layer is inside the room (as a simplification). The glass pane has a thickness of 5 mm (0.005 m) with a conductivity of 1.4 W/m K and a total surface area of 0.5 m^2 . The outside wind is blowing, so the convective heat transfer coefficient is $100 \text{ W/m}^2 \text{ K}$. With an outer glass surface temperature of 12.1°C , determine the rate of heat transfer in the glass and the convective layer.

