CHAPTER ONE

INTRODUCTION TO OPEN CHANNEL FLOW

WHAT IS OPEN CHANNEL?

Open channel:- It may be defined as a passage in which liquid flows with its upper surface exposed to the atmosphere. e.g.:- curvets, spillways, and similar human made structures

Differences b/n the flow in pipes & open channel flow

Open channel flow

- Is exposed to atmospheric pressure.
- The cross-sectional area of the flow is variable. (that depends on many parameters of the flow)
- The force causing motion is gravity.

Pipe Flow

- Is closed channel
- The top surface is covered by solid boundary
- It is not exposed to atmospheric Pressure.

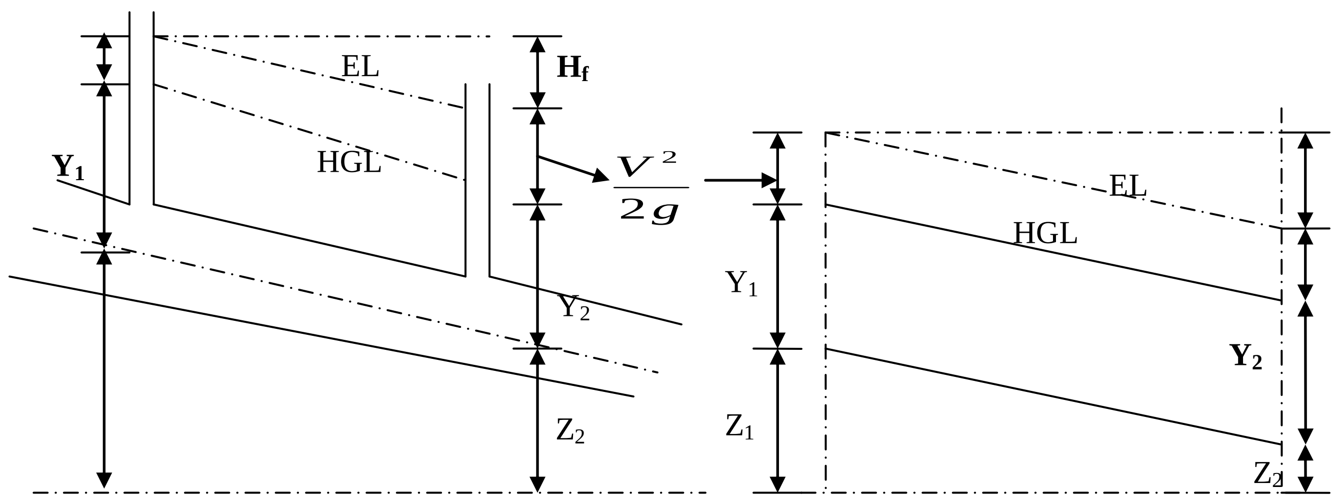

Fig 1(a) Pipe flow

Where HGL - Hydraulic grade line (coincide with water surface)

EGL - Energy grade line

H*f* - head loss due to friction

$V^2/2g$ - velocity head

Types of channels

- **Natural channels**: These channels naturally exist without the influence of human beings. E.g. Rivers, streams, tidal estuaries, aqueducts.

- **Artificial channels**: Such channels are formed by man's activity for various purposes. E.g. irrigation channel, navigation channel, sewerage channel, culverts, power canal…… etc.
- ***Prismatic channel***: - channels with constant shape and slope.
- ***Non-prismatic channels***: - channels with varying shape and slope.
- ***Open channel***:-A channel without any cover at the top.
- ***Closed channel***:-The channel having a cover at the top.

ACTIVITY1.1
What is open channel?
What are the different types of channel? Give example in each case.

1.1 Types of flow in open channel

The flow in a channel classified in to the following type, depending on the change in the depth of flow with respect to space and time.

a) Steady flow & Unsteady flow
b) Uniform flow &Non uniform flow
c) Steady uniform flow &Un steady uniform flow
d) . Unsteady uniform flow

Time as criteria

Steady flow & Unsteady flow

When the flow characteristic (such as depth of flow, flow velocity and the flow rate at any cross section) do not change with respect to time, the flow in a channel is to be steady.

Mathematically, $\frac{\delta V}{\delta t} = 0$, $\frac{\delta p}{\delta t} = 0$ and $\frac{\delta y}{\delta t} = 0$

The flow is said to be un steady flow when the flow parameter vary with time.

Mathematically, $\frac{\delta V}{\delta t} \neq 0$, $\frac{\delta p}{\delta t} \neq 0$ and $\frac{\delta y}{\delta t} \neq 0$

Space as a criterion

Uniform flow & Non uniform flow

Flow in a channel is said to be uniform if the depth, slope, cross-section and velocity remain constant over a given length of the channel.

Mathematically, $\frac{\delta V}{\delta s} = 0$, and $\frac{\delta y}{\delta s} = 0$

Flow in channel is said to be non- uniform (varied)when the channel depth varies continuously from one section to another.

Mathematically, $\frac{\delta V}{\delta s} \neq 0$, and $\frac{\delta y}{\delta s} \neq 0$

Time and space as a criteria

Steady uniform flow: - The depth of flow does not change during time interval and space under consideration.

Unsteady uniform flow: - This is a flow in which the depth is varying time but not with space.

Unsteady non uniform flow: - Is the flow in which the depth is varying with space and time.

ACTIVITY 1.2
Explain briefly the following:
1. Steady and Un steady flow
2. Uniform and non uniform flow
3. State the condition under which uniform and non uniform flows are produced.

1.2 Geometric elements of open channel section

Geometric elements are properties of a channel section that can be defined entirely by the geometry of the section and the depth of flow. The most used geometric properties include:

1. **Depth of flow(y)**: it the vertical distance from the lowest point of the channel to the free surface.
2. **Top width (T):** it is the width of channel section at free surface.
3. **Stage (h):** is the elevation or vertical distance of the free surface above a datum.
4. **Wetted perimeter (p)**: it is the length of the channel boundary which is in contact with water.
5. **Wetted area (A)**: is the cross-sectional area of the flow normal to the direction of flow.
6. **Hydraulic radius(hydraulic mean depth)(R) :** it is the ratio of wetted area to its wetted perimeter

$$R = \frac{A}{P}$$

7. **Hydraulic depth(D):** the ratio of wetted area to the top width,

$$D = \frac{A}{T}$$

8. **Section factor (Z):** is the product of the wetted area and the two-third power of the hydraulic radius

$$Z = A\sqrt{D} = A\sqrt{\frac{A}{T}} = \left(\frac{A^3}{T}\right)^{\frac{1}{2}} = A R^{\frac{2}{3}}$$

9. Conveyance (K) :

$$Q = VA \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$Q = A \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$= A R^{\frac{2}{3}} \frac{1}{n} S^{\frac{1}{2}}$$

$$= K S^{\frac{1}{2}}$$ S= bed slope

$$K = \frac{1}{n} A R^{\frac{2}{3}}$$ n= Mannings constant

$$= CA\sqrt{R}$$ c= Chezy's constant

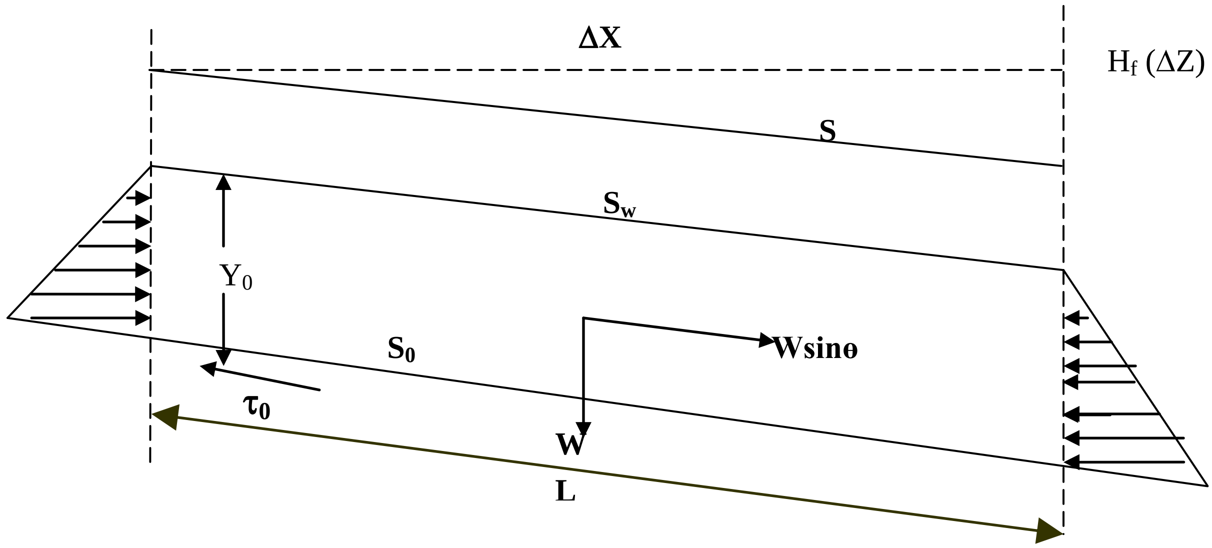

Fig. 1.2

Where S_0- bed slope of channel

S_w- Water surface slope

S- Slope of EGL

W – Weight of water

τ_0 – Shear force

L- Length of channel

Uniform flow is the result of exact balance between the gravity and friction force

$W\sin\theta = \bar{\tau}_o .P.L$……………………………(1)

$\gamma A\ L \sin\theta = \bar{\tau}_o .P.L$

But $\sin\theta = h_f/L = S$, solving for $\bar{\tau}_o$,

$\bar{\tau}_o = \gamma \frac{A}{P}.S = \gamma R.S$ ………………………………… (2)

Where γ- unit weight of the water

The shear stress is assumed proportional to the square of the mean velocity,

or $\tau_o = kV^2$………………………………..……..(3)

Therefore, $kv^2 = \gamma RS$

$V^2 = \frac{\gamma}{k} RS$,

Let $\frac{\gamma}{k} = C^2$ -constant (b/c γ&k- are constant)

$V = C\sqrt{RS}$.……………………………………………….... (4)

This is the Chezy –formula

C= chezy coefficient (chezy's resistance factor)

V= Average velocity of flow

Manning Formula

V= $\frac{1}{n} R^{2/3} S_0^{1/2}$ ……………………………………………(5)

- The best as well as most widely used formula for uniformly for uniform flow.

n- is the roughness coefficient.

A relation between the Chezy's C and Manning's n may be obtained by comparing eqn (4) & (5)

$C = \frac{R^{\frac{1}{6}}}{n}$ …………………………………………..(6)

- The value of n ranges from 0.009 (for smooth straight surfaces) to 0.22 (for very dense flood plain forests).

What is hydraulic efficiency channel (most economical channel) means?

- A channel section is said to be efficient (economical) if it gives the maximum discharge for the given shape, area and roughness.

1.3 Most economical channel section

Most economical rectangular channel section

Let B and Y be the base width and depth of flow respectively

A=BY………………………….(i)

P=B+2Y……………………….(ii)

From eqn. (i), B=A/Y

Substituting in (ii) P=A/Y+2Y………….(iii)

For maximum Q, P- is minimum.

$$\frac{dp}{dY} = 0 \Rightarrow \frac{d}{dY}(A/Y = 2Y) = 0$$

$$\Rightarrow -A/Y = 2 = 0$$

$$\Rightarrow A = 2Y^2 = B * Y$$

So, B=2Y (or Y=B/2)

Thus the rectangular channel is most efficient and economical when the depth of water is one half of the width of the channel and the discharge flow will be maximum.

EXAMPLE -1

1 .A rectangular channel is to be dug in the rocky portion of a soil. Find its most economical cross-section if its to convey 12 m^3/s of water with an average velocity of 3 m/s. Take chezy constant C=50

Given

Q=12 m^3/s

V=3 m/s

C=50

Solution

The geometric relations for optimum discharge through a rectangular channel are

$$B = 2Y \quad and \quad R = \frac{Y}{2}$$

Then $\boldsymbol{area\ A = B \times Y = 2Y^2}$

When B,Y and R are base width, depth of flow and hydraulic radius respectively

Now $Q = A \times V$ $\quad or\ 12 = 2Y^2 \times 3$

From this equation solve for depth of flow

$$Y = 1.414m$$

Therefore base width of flow $B = 2Y = 2 \times 1.414 = 2.828m$

Hydraulic radius, $R = \frac{Y}{2} = \frac{1.414}{2} = 0.707m$

Also $V = C\sqrt{RS}$ chezy formula

$$S = \frac{V^2}{RC^2} = \frac{3^2}{0.707 \times 50^2} = \frac{1}{196}$$

Hence $B = 2.828, Y = 1.414, S = \frac{1}{196}$

Most economical trapezoidal channel section

$$T = B + 2mY$$
$$A = BY + mY^2 \quad \dots\dots\dots\dots\dots \quad B = \frac{A}{Y} - mY$$
$$P = B + 2Y\sqrt{1 + m^2}$$

$$P = \frac{A}{Y} - mY + 2Y\sqrt{1+m^2}$$

But for most economic section

$$\frac{\partial P}{\partial Y} = 0$$

$$\frac{\partial P}{\partial Y} = \frac{\partial}{\partial Y}\left(\frac{A}{Y} - mY + 2Y\sqrt{1+m^2}\right) = 0$$

$$A = \left(2\sqrt{1+m^2} - m\right)Y^2 \;\; but \;\; A = (B+mY)Y$$

$$(B+mY)Y = \left(2\sqrt{1+m^2} - m\right)Y^2$$

$$B = 2Y(\sqrt{1+m^2} - m)$$

$$R = \frac{A}{p} = \frac{(B+mY)Y}{(B+2Y\sqrt{1+m^2}}$$

$$R = \frac{(2\sqrt{1+m^2} - m)Y^2}{2Y\sqrt{1+m^2} - 2mY + 2Y\sqrt{1+m^2}}$$

$$R = \frac{Y}{2}$$

EXAMPLE-2

An irrigation channel of trapezoidal section has side slope, m=2 and carries a discharge of $15m^3/s$ on a longitudinal slope of 1 in 5000. The channel is to be lined for which the value of friction coefficient in Manning's formula is n=0.012. Find the dimension of the most economic section of the channel.

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Side slope m=2
Discharge $Q=15m^3/s$
Longitudinal slope S=1:5000
Manning´s coefficient n=0.012

SOLUTION

$$A = (B+2Y)Y$$

$$P = B + 2\sqrt{(2Y)^2 + Y^2} = B + 2\sqrt{5}Y$$

$$B = \frac{A}{Y} - 2Y$$

$$P = \frac{A}{Y} - 2Y + 2\sqrt{5}Y$$

$$\frac{\partial P}{\partial Y} = -\frac{A}{Y^2} - 2 + 2\sqrt{5} = 0$$

$$A = 2.47Y^2$$

$$P = B + 2\sqrt{5}Y \;\; but \; B = \frac{A}{Y} - 2Y$$

$$= \frac{2.47Y^2}{Y} - 2Y + 2\sqrt{5}Y = 4.94Y$$

$Q = \frac{A}{n} R^{2/3} S^{1/2}$

$15\, m^3/_s = \frac{2.47Y^2}{0.012} \times (0.5Y)^{2/3} \times (\frac{1}{5000})^{1/2}$

$Y = 2.198m$

$B = \frac{A}{Y} - 2Y = 1.04m$

ACTIVITY1.3
What do you mean by most economical section of an open channel? How is it determined?
What are the conditions for the rectangular channel of best section?
Show that the hydraulic mean depth of a trapezoidal

1.4 Specific energy

What is specific energy?

- Specific energy is the energy per unit weight of flowing liquid above the channel bottom.

For any cross section, shape, the specific energy (E) at a particular section is defined as the energy head to the channel bed as datum. Thus,

$$E = Y + \alpha \frac{V^2}{2g} \quad \text{..(1)}$$

(α- is kinetic energy correction factor ≅1)

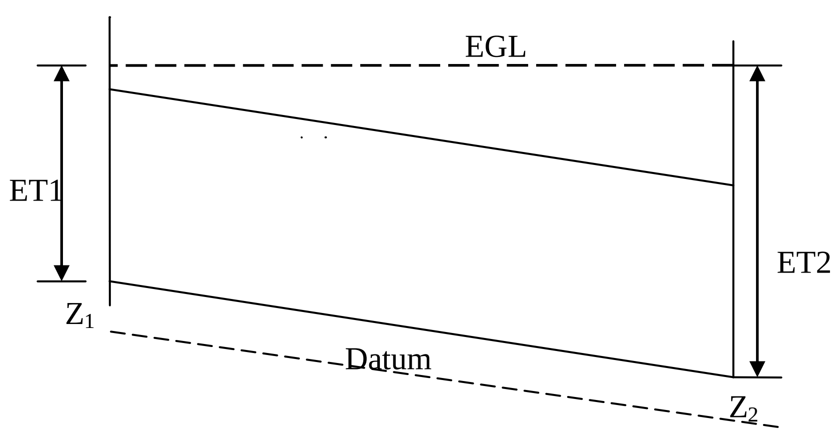


Fig 1.3 Specific Energy at a particular section

For a rectangular channel, the value of flow per unit width is Q/B=q, and average velocity

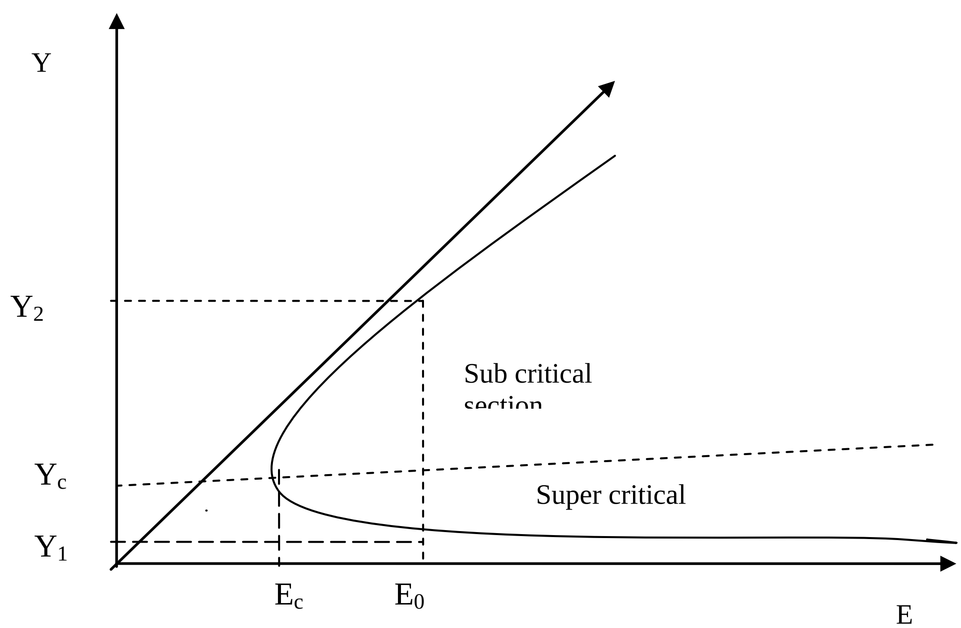
$$V = {Q}/{A} = \frac{qB}{BY} = \frac{q}{Y}$$

Therefore eqn (1) becomes:

$$E = y + \frac{\left({q}/{y}\right)^2}{2g} = y = \frac{q^2}{2gy^2} \text{ ……………………………………} (2)$$

$$(E - y)Y^2 = \frac{q^2}{2g} \text{ (For the case of constant q)………………………… (3)}$$

A plot of E Vs Y is a hyperbola like with asymptotes (E-Y) =0 i.e. E=Y and y=0. Such a curve is known as specific energy diagram.



Specific Energy diagram

For a particular q, we see there are two possible values of Y for a given value of E. These are known as Alternative depths (for e.g. Y_1 & Y_2 on fig. above)

- The two alternative depths represent two totally different flow regimes slow & deep on the upper limp of the curve (sub critical flow) & fast and shallow on the lower limb of the curve.(super critical flow)

What is critical depth?

- Depth of flow at which specific energy is minimum is called critical depth.

  The velocity of flow at critical depth is known as critical velocity.

For example, a relation for critical depth in a wide rectangular channel can be found by differentiation E of eqn.2 with respect to Y to find the value of Y for which E is a minimum.

$$\frac{dE}{dY} = 1 - \frac{q^2}{gy^3} \text{ .... (4)}$$

And when E is a minimum Y=Yc and $dE/dy = 0$, so that

$$0 = 1 - \frac{q^2}{gY_c^{\ 3}} \Rightarrow q^2 = gy_c^{\ 3} \text{ .... (5)}$$

Substituting q= vy = V_C*Yc, gives

$$V_c^{\ 2} = gy_c$$

$$\Rightarrow V_c = \sqrt{gy_c} = \frac{q}{y_c} \text{ .... (6)}$$

It may be expressed as:

$$y_c = \frac{V_{c^2}}{g} = \left(\frac{q^2}{g}\right)^{1/3} \text{ .... (7)}$$

From eqn (7) $\frac{V_c^{\ 2}}{2g} = \frac{y_c}{2}$, hence,

$$E_c = E_{\min} = y_c + \frac{V_c^{\ 2}}{2g} = y_c + \frac{1}{2} y_c = \frac{3}{2} y_c \text{ .... (8)}$$

And $y_c = \frac{2}{3} E_{\min}$ ....(9)

From eqn. (7): $q_{\max} = \sqrt{gy_c^{\ 3}}$ ....(10)

For non rectangular cross section the specific energy eqn.

$$E = y + \frac{Q^2}{2gA^2} \quad \ldots\ldots (11)$$

[V=Q/A]

To find the critical depth,

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} \quad \ldots\ldots (12)$$

From fig 1.3 (b) dA = dy*T (at Yc, T= Tc)

Therefore the above equation becomes:

$$\frac{Q_{\max}{}^2 T_c}{gA_c{}^3} = 1 \quad \ldots\ldots (13)$$

The critical depth must satisfy this equation

From eqn. (13) $Q^2 = \frac{gA_c{}^3}{T_c}$ and substitute in eqn. (11) then

$$E_c = y_c + \frac{A_c}{2T_c} \quad \ldots\ldots (14)$$

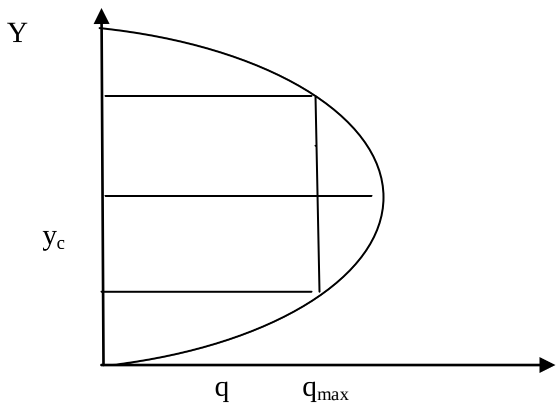
eqn.(13) can be solved by trial & error for irregular section by plotting $f(y) = \frac{Q^2 T}{gA^3}$ and

critical depth occurs for the value of y which makes f(y)=1

What are sub critical, critical, and super critical flow?

- Sub critical flow:-when the depth of flow in a channel is greater than the critical depth(Yc) in this case Fr <1
- Critical flow is one in which specific energy is minimum. A few corresponding to critical depth also known as critical flow.
- Super critical flow:-when the depth of flow in a channel is less than critical depth (Yc) in this case Fr>1.

If specific energy curve for Q- constant is redraw alongside a second curve of depth against discharge for constant E, will show the variation of discharge with depth.



For a given constant discharge fig

i) The specific energy curve has a minimum value Ec at point C with a corresponding depth Yc known as critical depth.

ii) For any other value of E there are two possible depth of flow known as alternative depth one of which is termed sub critical (y>Yc) and the other supercritical (Y<Yc).

a) For a given constant specific energy (fig.1.5(b))

i) the depth discharge curve shows that discharge is a maximum at the critical depth

ii) For all other discharges there are two possible depth of flow (sub- & super critical) for any particular value of E,

From eqn. (13) above if we substitute

Q= AV (continuity equation), we get

$$\frac{Q^2T}{gA^3} = 1$$

$$\frac{A^2V^2T}{gA^3} = 1 \Leftrightarrow \frac{V^2T}{gA} = 1$$

but A/T = D (Hydraulic depth), then [D=Y for rectangular section)

$$\frac{V^2}{gy} = 1 \Rightarrow V = \sqrt{gy} \text{ ....(*)}$$

$\frac{V}{\sqrt{gy}} = 1 \Rightarrow$ Froude number at critical state.

$$F = \frac{V}{\sqrt{gy}} \text{ ....(**)}$$

Thus, i) F= 1⇒critical flow

ii) F< 1⇒ sub critical flow Type equation here.

iii) F>1 ⇒Super critical

ACTIVITY1.4
What is specific energy and specific energy curve?
What do you understand by critical depth of an open channel when the flow in it is not uniform?

Examples

1. For constant specific energy of $1.8NM/_{s}$, calculate the maximum discharge that may occur in a rectangular channel 5m width.

Given

$$specific\ energy = 1.8NM/_{s}$$
$$width\ of\ channel = 5m$$

Solution

For constant specific energy discharge is maximum

$$Y_c = \frac{2}{3}E_c = \frac{2}{3} \times 1.8NM/_{s} = 1.2m$$
$$Y_c = (\frac{q^2}{g})^{1/3}, q^2 = Y_c^{\ 3} \times g$$
$$q^2 = (1.2m)^3 \times 9.81\, m/_{s^2} = 16.95 \, \frac{m^3/_s}{m}$$
$$q = 4.12m^2/_{s}$$
$$Q = B \times q = 5m \times 4.12\, m^2/_{s} = 20.59\, m^3/_{s}$$

2. Most efficient rectangular channel, which is laid on a bottom slope of 0.0064, is to carry $20m^3/s$ of water. Determine the width of the channel when the flow is in critical condition. Take n=0.015.

Given

$$Discharge\ Q = 20\, m^3/_{s}$$
$$Bottom\ slope = 0.0064$$
$$Manning\ coefficient = 0.015$$

Solution

$$For\ most\ effcient\ rectangular\ channel \quad B = 2Y, Y = \frac{B}{2}$$
$$Q = \frac{A}{n} R^{2/3} S^{1/2}$$
$$\boldsymbol{A = B \times Y = B \times \frac{B}{2} = \frac{B^2}{2}}$$
$$\boldsymbol{R = \frac{Y}{2} = \frac{B}{4}}$$

$$Q = \frac{B^2}{2n} \times \left(\frac{B}{4}\right)^{2/3} \times S^{1/2}$$

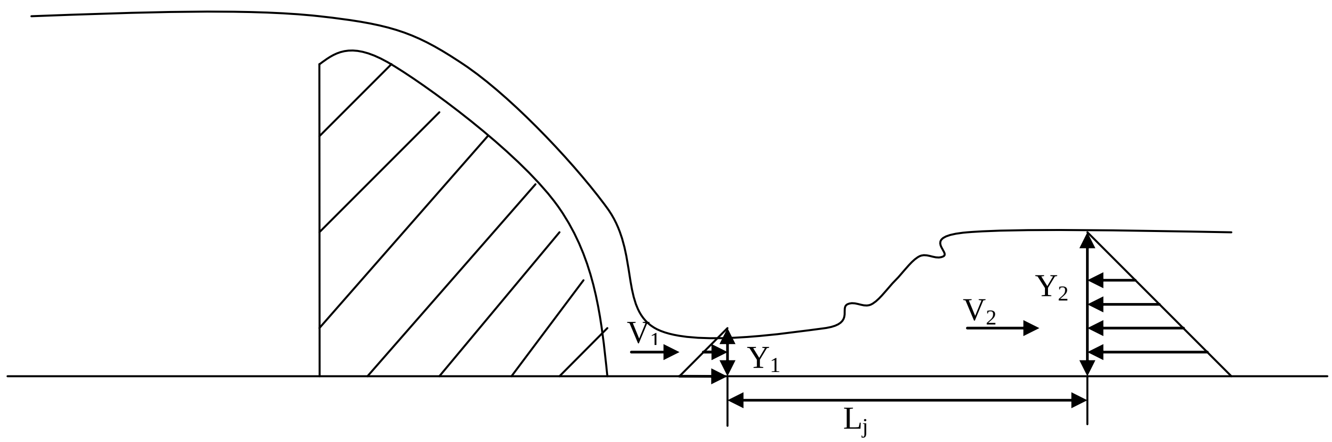
$$20 = \frac{B^{8/3}}{5.04 \times 0.015} \times 0.0064^{1/2}$$

$$B = 16.9om$$

1.5 Hydraulic jump

What is hydraulic jump?

- A flow phenomenon which occurs when supercritical flow has its velocity reduced to sub critical. There is sudden rise in water level at the point where hydraulic jump occurs e.g (Rapidly varied flow).



Hydraulic jump on horizontal bed following over a spillway

Where; V_1-velocity before jump

V_2 –velocity after jump

Y_1 –water depth before jump

Y_2 –water depth after jump

L_j –length of jump

Purposes of hydraulic jump:-

i) To increase the water level on the d/s of the hydraulic structures

ii) To reduce the net up lift force by increasing the downward force due to the increased depth of water,

iii) To increase the discharge from a sluice gate by increasing the effective head causing flow,

iv) For aeration of drinking water

v) For removing air pockets in a pipe line

vi) Reduce downstream erosion

vii) Very useful & effective for mixing fluids

- **Analysis of hydraulic jump**

Assumptions

a. The length of the hydraulic jump is small, consequently, the loss of head due to friction is negligible,

b. The channel is horizontal as it has a very small longitudinal slope. The weight component in the direction of flow is negligible.

c. The portion of channel in which the hydraulic jump occurs is taken as a control volume & it is assumed the just before & after the control volume, the flow is uniform & pressure distribution is hydrostatic.

Let us consider a small reach of a channel in which the hydraulic jump occurs.

The momentum of water passing through section (1) per unit time is given as:

$$\frac{p_1}{t} = \frac{rQV_1}{g} = \rho QV_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Momentum at section (2) per unit time is:

$$\frac{p_2}{t} = \frac{rQV_2}{g} = \rho QV_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Rate of change of momentum b/n section 1 & 2

$$\frac{\Delta P}{t} = \rho Q(V_2 - V_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

The net force in the direction of flow = F1-F2 ………………..(iv)

$F_1 = \gamma A_1 \overline{Y}_1$, $F_2 = \gamma A_2 \overline{Y}_2$

$\overline{Y}_1$ & $\overline{Y}_2$ are the center of pressure at section (1) & (2)

Therefore F_1-F_2 =ΔM =ρQ (V_2-V_1)

$$\gamma A_1 \overline{Y}_1 - \gamma A_2 \overline{Y}_2 = \frac{\gamma Q}{g}(V_2 - V_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)}$$

From continuity eqn. Q= A*V, ⟹ V= Q/A, so

$$\gamma A_1\bar{Y}_1 - \gamma A_2\bar{Y}_2 = \frac{\gamma Q}{g}\left(\frac{Q}{A_2} - \frac{Q}{A_1}\right)$$

$$A_1\bar{Y}_1 - A_2\bar{Y}_2 = \frac{Q^2}{g}\left(1/A_2 - 1/A_1\right)..(iv)$$

Rearranging this eqn.:

$$\underbrace{\left[\frac{Q^2}{gA_1} + A_1\bar{Y}_1\right]}_{M_1} = \underbrace{\left[\frac{Q^2}{gA_2} + A_2\bar{Y}_2\right]}_{M_2} = \text{Constant.} \ \ldots\ldots\ldots\ldots\ldots \ \text{(vii)}$$

M_1 and M_2 are the specific forces at section (1) & (2) indicates that these forces are equal before & after the jump.

Y_1= initial depth

Y_2 = sequent depth

Hydraulic jump in a rectangular channel

$A_1=By_1$
$A_2= By_2$] the section has uniform width (B)

$$\bar{Y}_1 = \frac{Y_1}{2}, \ \bar{Y}_2 = \frac{Y_2}{2}$$

Now from eqn. (Vii) above:

$$\frac{Q^2}{gBy_1} + By_1\left(\frac{y_1}{2}\right) = \frac{Q}{gBy_2} + By_2 * \left(y_2/2\right)$$

$$\frac{Q^2}{gBy_1} + \frac{By_1^2}{2} = \frac{Q^2}{Bgy_2} + \frac{By_2^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(viii)$$

Flow per unit width of q= Q/B ⟹ Q= qB, then eqn. (viii) becomes

$$\frac{q^2B^2}{Bgy_1} + \frac{By_1^2}{2} = \frac{q^2B^2}{Bgy_2} + \frac{By_2^2}{2}$$

$$\frac{q^2}{g}\left[\frac{1}{y_1} - \frac{1}{y_2}\right] = \frac{y_2^2 - y_1^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (.ix)$$

$$\frac{2q^2}{g} = y_1 y_2 \quad \frac{\left(y_2^2 - y_1^2\right)}{\left(y_2 - y_1\right)}$$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(x)$$

$$y_2 y_1^2 + y_1 y_2^2 - \frac{2q^2}{g} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(xi)$$

This is quadratic eqn. & the solution is given as

$$y_1 = \frac{-y_2}{2} + \sqrt{\left(\frac{y_2}{2}\right)^2 + \frac{2q^2}{gy_2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(xii)(a)$$

$$y_2 = \frac{-y_1}{2} + \sqrt{\left(y_1/2\right)^2 + \frac{2q^2}{gy_2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(b)$$

$$y_1 = y_2/2\,(-1 + \sqrt{1 + \frac{8q^2}{gy_2^3}}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(c)$$

$$y_2 = y_1/2\,(-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(xii)(d)$$

The ratio of conjugate depths;

$$\frac{y_1}{y_2} = \frac{1}{2}(-1+\sqrt{1+\frac{8q^2}{gy_2^3}}\text{...}(xii)(e)$$

$$\frac{y_2}{y_1} = \frac{1}{2}(-1+\sqrt{1+\frac{8q^2}{gy_1^3}}\text{...}(f)$$

$$F_1 = \frac{V_1}{\sqrt{gy_1}} =, F_2 = \frac{V_2}{\sqrt{gy_2}}\ \frac{q/y_2}{gy_2} = \frac{q}{\sqrt{gy_2^3}}$$

$$Therefore\ \frac{y_1}{y_2} = \frac{1}{2}(-1+\sqrt{1+8F_2^2})\text{..............................}(g)$$

$$\frac{y_2}{y_1} = \frac{1}{2}(-1+\sqrt{1+8F_1^2}\text{..}(h)$$

Energy dissipation in a Hydraulic Jump

The head loss $h_{l.f}$ caused by the jump is the drop in energy from section (1) to (2) or:

$h_{lf} = \Delta E = E_1 - E_2$

$$= \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)\text{......................................}(1)a$$

$$= \left(y_1 + \frac{q^2}{2gy_1^2}\right) - \left(y_2 + \frac{q^2}{2gy_2^2}\right)\text{......................................}(b)$$

From eqn. (x) substituting: $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$ in to this eqn. & by rearranging:

$$h_{lf} = \Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2}\text{...}(2)$$

Therefore power lost = γ Q h_{lf} (kw)…………………(3)

ACTIVITY1.5

What is mean by hydraulic jump in open channel and how it occurs?

Types of Hydraulic jump

Hydraulic jumps are classified according to the upstream Froude number and depth ratio.

F_1	Y_2/y_1	Classification
<1	1	Jump impossible
1-1.7	1-2	Undular jump (standing wave)
1.7-2.5	2-3.1	Weak jump
2.5-4.5	3.1-5.9	Oscillating jump
4.5-9.0	5.9-12	Steady jump (45-70% energy loss)
>9.0	>12	Strong or chopping jump (=85% energy loss)

Examples

A 3·6m wide rectangular channel conveys $9.0\ ^{m^3}/_s$ of water with a velocity of $6\ ^m/_s$.

a. Is there a condition for hydraulic jump occur? If so calculate the height, length and strength of the jump.

b. What is loss of energy?

Given

$$width\ of\ channel\ B = 3.6m$$

$$Discharge = 9.0m^3/_s$$

$$Velcity\ before\ jump = 6m/_s$$

Solution

a. $depth\ of\ water\ before\ jump = Y_1 = \frac{Q}{B \times V_1} = \frac{9m^3/_s}{3.6m \times 6m/_s} = 0.4167m$

$$Discharge\ per\ unit\ width\ q = \frac{Q}{B} = \frac{9\ m^3/_s}{3.6m} = 2.5\ ^{m^2}/_s$$

$$critical\ depth\ Y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.5^2}{9.81}\right)^{1/3} = 0.86m$$

$$since\ Y_1 < Y_c\ a\ jump\ would\ occur$$

$$Fr_1 = \frac{V_1}{\sqrt{g \times Y_1}} = \frac{6}{\sqrt{9.81 \times 0.4167}} = 2.967$$

$Depth\ of\ the\ jump\ down\ stream\ of\ the\ jump$

$$Y_2 = \frac{Y_1}{2}\left[\sqrt{1 + 8Fr_1{}^2} - 1\right]$$

$$Y_2 = \frac{0.4167}{2}\left[\sqrt{1 + 8 \times 2.967^2} - 1\right] = 1.5525m$$

$$height\ of\ the\ jump\ H_j = Y_2 - Y_1 = 1.5525m - 0.4167m = 1.1358m$$

$$Length\ of\ the\ jump\ L_j = 6(Y_2 - Y_1) = 6(1.1358m) = 6.8148m$$

$$strength\ of\ jump = \frac{Y_2}{Y_1} = \frac{1.5525m}{0.4167m} = 3.726$$

b. Loss of energy for rectangular channel $\Delta E = \frac{(Y_2 - Y_1)^3}{4Y_1Y_2}$

$$\Delta E = \frac{(1.5525m - 0.4167m)^3}{(4 \times 1.5525m \times 0.4167m)} = 0.57m$$

Exercises

1. A rectangular channel which is laid on a bottom slope of 0.0064 is to carry $20m^3/s$ of water. Determine the width of the channel when the flow is in critical condition. Take C=66

2.An irrigation canal of trapezoidal section having side slope 2 in 3 is to carry a flow of $10m^3/s$ on a longitudinal slope of 1 in 5000. The canal is lined for which the value of frictional coefficient in Manning's formula is n=0.012. Find the dimension of the most economical section

3. Determine the side slope of the most hydraulically efficient triangular section. . Show that the head loss in a hydraulic jump formed in a rectangular channel may be expressed as

$$\Delta E = (V_1 - V_2)^3 / [2g\,(V_1 + V_2)]$$

4. A rectangular channel there occurs a jump corresponding to Froude number (F=2.5). Determine the critical depth and head loss in terms of the initial depth y_1.

5. A trapezoidal channel having bottom width 10m and side slope 2:1(H:V) carries a discharge of $100m^3/s$. Find the depth conjugate to the initial depth of 1m before the jump. Also determine the loss of energy in the jump.