CHAPTER 3

FLOW THROUGH PIPES

Introduction

Pipes were introduced in the earliest days of the practice of hydraulics. Their common place use today makes it of great importance that the laws governing the flow in them should be fully understood.

Water is conveyed from its source, normally in pressure pipelines, to water treatment plants where it enters the distribution system & finally arrives at the consumer. In addition oil, gas, irrigation water, sewerage can be conveyed by pipeline system.

Some loss of energy is inevitable in the flow of any real fluid. In the case of flow in a horizontal uniform pipeline, this is evidenced by the fall of pressure in the direction of flow. Predicting the energy loss per unit length is essential to efficient pipeline design.

The prime concern in the analysis of real flows is to account for the effect of friction. The effect of friction is to decrease the pressure, causing a pressure '**loss**' compared to the ideal, frictionless flow case. The loss will be divided into ***major losses*** (due to friction in fully developed flow in constant area portions of the system) & ***minor losses*** (due to flow through valves, elbow fittings & frictional effects in other non-constant –area portions of the system).

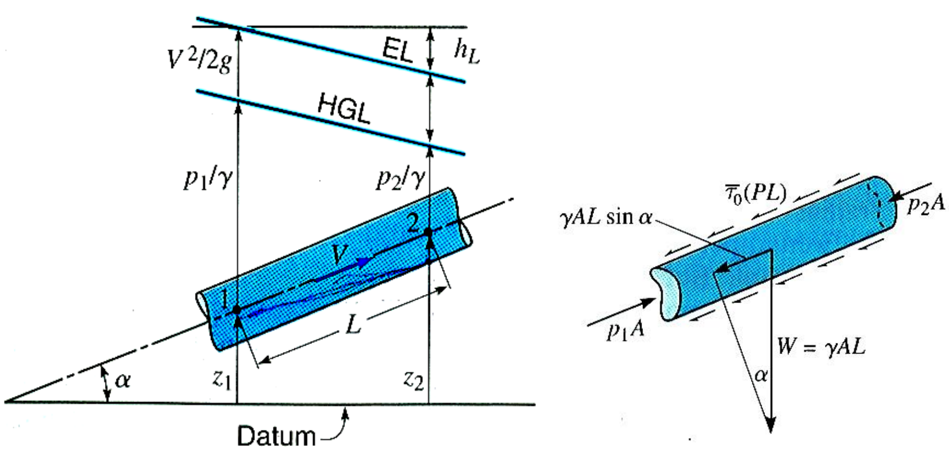


Figure 3.1 Flow in the pipes (circular pipe)

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L \quad \text{...3.1}$$

h_L = Head loss (major + minor)

Activity 3.1: Define the term pipe flow?

3.1 Major Losses (Head loss in conduits of constant cross-section)

Referring to Figure 3.1 and for equilibrium in steady flow, the summation of forces acting on any fluid element must be equal to zero, i.e. $\sum F = 0$,

$$p_1 A - p_2 A + W\sin\alpha - \overline{\tau_o}(pL) = 0$$

$$\sin\alpha = \frac{(z_1 - z_2)}{L}$$

$\bar{\tau}_o$ - average shear stress (average shear force per unit area) at the conduit wall, is defined by:

$$\bar{\tau}_o = \frac{1}{P}\int_0^P \tau_o dP \quad \text{.. (3.2)}$$

τ_o - is the ***local shear stress***[1] acting over a small incremental portion *dP* of the wetted perimeter.

$$p_1 A - p_2 A - \gamma A L\frac{(z_2 - z_1)}{L} - \bar{\tau}_o PL = 0$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - (z_2 - z_1) - \bar{\tau}_o\frac{PL}{\gamma A} = 0$$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + \bar{\tau}_o\frac{PL}{\gamma A} \quad \text{.. (3.3)}$$

Form the above equations (5.1) and (5.3)

$$h_L = \bar{\tau}_o\frac{PL}{\gamma A} = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

$$h_L = \bar{\tau}_o\frac{L}{R_h\gamma} \quad \text{... (3.4)}$$

This equation is applicable to any shape of uniform cross-sections, regardless of whether the flow is *laminar or turbulent*. For smooth-walled conduits, where wall roughness may be neglected, it may be assumed that the average shear stress $\overline{\tau_o}$ is a function of ρ, μ, ν& some characteristic linear dimension, which will here be taken as hydraulic radius R. Thus:

$\overline{\tau_o}$ = ϕ(ρ, μ, ν, R)

By dimensional analysis:

[1] The local shear stress varies from point to point around the perimeter of all conduits (irrespective of whether the wall is smooth or rough), except for the case of a circular pipe flowing full where the shear stress at the wall is the same at all points of the perimeter.

$$\bar{\tau}_o = \rho V^2 \phi\left(\frac{R_h V \rho}{\mu}\right) = \rho V^2 \phi(\mathrm{Re})$$ and let ϕ (Re) = ½ C_f (dimensionless term)

$$\bar{\tau}_o = C_f \rho \frac{V^2}{2} \qquad (3.5)$$

From equation (3.4): $$h_L = C_f \frac{L}{R_h}\frac{V^2}{2g} \qquad (3.6)$$

(Applied for any shape of smooth walled conduits).

For ***circular conduits (pipe)*** flowing full, R= ¼ D, Therefore,

$$h_L = C_f 4\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g} \qquad (3.7)$$

Where, $$f = 4C_f = 8\phi(\mathrm{Re}) \qquad (3.8)$$

Equation (3.7) is applicable for both smooth-walled and rough walled conduits. It is known as ***pipe – friction equation***, and commonly referred to as the ***Darcy-Weisbach*** equation. Friction factor, *f*, is dimensionless and is also some function of Reynolds number. The exact form of $\phi(\mathrm{Re})$ and numerical values for C_f and *f* must be determined by experiments or other means.

For ***laminar flow*** (Recall chapter three)

$$f = 64\frac{v}{DV} = \frac{64}{\mathrm{Re}} \qquad (for\ laminar\ flow) \qquad (3.9))$$

Head loss: $$h_f = \left(\frac{64}{\mathrm{Re}}\right)\frac{L}{D}\frac{V^2}{2g} \qquad (3.10)$$

Experimental Investigation on friction losses in Turbulent flow

In fully developed turbulent flow, the pressure drop, Δp, due to friction in a horizontal constant area pipe depends upon the diameter, D, the pipe length, L, the pipe roughness, ∈, the average velocity, $\bar{V}$, the fluid density, ρ, and the fluid viscosity, μ.

By dimensional analysis $$\Delta p = \phi(V, D, \rho, \mu, \varepsilon)$$

$$\frac{\Delta P}{\rho V^2} = \phi\left(\frac{\mu}{\rho v D}, \frac{L}{D}, \frac{\varepsilon}{D}\right)$$

$$\frac{h_L}{V^2/g}=\frac{L}{D}\phi\left(\mathrm{Re},\frac{\varepsilon}{D}\right)$$

$$\frac{h_L}{V^2/2g}=\frac{L}{D}\phi_1\left(\mathrm{Re},\frac{\varepsilon}{D}\right)$$

$$\therefore\ f=\phi_1\left(\mathrm{Re},\frac{\varepsilon}{D}\right) \text{ .... (3.11)}$$

- **Blasius** had concluded that there were two types of pipe friction in turbulent flow. The first is the smooth pipes where the viscosity effects predominate so that the friction factor is dependent solely on the Reynolds number (f=ϕ (Re). He deduced the following expression for the friction in smooth pipes:

$$f=\frac{0.316}{\sqrt[4]{\mathrm{Re}}} \text{ .... (3.12)}$$

The second type was relevant to rough pipes where the viscosity & roughness effects influence the flow & the friction factor (f) is dependent both on the Reynolds number & a parameter of relative roughness ($^{\varepsilon}/_{D}$).**L.F Moody** prepared a chart for determining friction factor for rough pipes experimentally by plotting f versus Re curve for each value of $\frac{\varepsilon}{D}$. (**See Moody Chart**)

The moody chart, the various flows it represents, may be divided into four zones: the ***laminar flow zone***; a ***critical zone*** where values are uncertain because the flow might be either laminar or turbulent; a ***transition zone***. Where f is a function of both Reynolds number and relative pipe roughness; and a zone of ***complete turbulence*** (fully rough pipe flow), where the value of f is independent of Reynolds number and depends solely upon the relative roughness.

There is no sharp line of demarcation between the transition zone and the zone of complete turbulence. The dashed line that separates the two zone was suggested by R. J. S. Pigott; the equation of this line is **R**e= $3500/(\varepsilon/D)$. On the other hand side of the equation of this line is corresponding to the curve and not the grid.

The Colebrook has developed the formula:

$$\frac{1}{\sqrt{f}}=-0.809\ln\left(\frac{\in/D}{3.7}+\frac{2.523}{\mathrm{Re}\sqrt{f}}\right) \text{ .... (3.13)}$$

A simplified form of this equation is provided with restriction placed on it:

$$f = \frac{1.325}{\left[\ell n\left(\frac{\in}{3.7D} + \frac{5.74}{R^{0.9}}\right]^2} \Rightarrow \begin{Bmatrix} 10^{-6} \le \frac{\in}{D} \le 10^{-12} \\ 5000 \le \text{Re} \le 10^8 \end{Bmatrix} \quad \text{.................................} (3.14)$$

(For Rough pipes)

∴ Head loss in pipes is given by:

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$ (For all pipes rough, smooth, laminar, & turbulent)

Activity 3.2: Define the term: Major energy losses in a pipe.

Example 3.1 ; in a pipe of diameter 350 mm and length 75 m water is flowing at a velocity of 2.8 m/s. find the head loss due to friction using :

i. Darcy weisbach formulae
ii. Chezy`s formula for when C=55.

Assume kinematic viscosity of the water as 0.012 stokes.
Solution: D= 350 mm=0.35 m , L= 75 m , V=2.8 m/s , C=55
V= 0.012 stokes= 0.012*10-^{4} m^2/s
Head loss due to friction, hf:
i. Darcy weisbach formulae;

$$hf = \frac{4flv^2}{2Dg}$$

where f=coefficient of friction (a function of Reynolds number)

$$Re = \frac{V*d}{V}$$

$hf = \frac{2.8*0.35}{0.012*10^{-4}}$ =8.167* 10^5

f= $\frac{0.0791}{(Re)^{\frac{1}{4}}}$ = $\frac{0.0791}{(8.167*10^5)^{\frac{1}{4}}}$ =0.00263

Therefore,

$$hf = \frac{4*0.00263*75*2.8^2}{2*0.33*9.81}$$ = 0.9 m (Ans)

ii. Chezy's formulae:

V= $C\sqrt{mi}$

Where C= 55, m=$\frac{A}{P}$ = $\frac{\frac{\pi}{4}*D2}{\pi*D}$ =$\frac{D}{4}$=$\frac{0.35}{4}$ =0.0875 m

Therefore 2.8=55*$\sqrt{0.087*i}$

i=0.0296

but , i=$\frac{hf}{L}$=0.0296

hf =i*L= 0.0296*75=2.22m

hf= 2.22m

Example 3.2 : a water flow through a pipe of diameter 300 mm with a volume of 5 m/s.if the coefficient of fraction given by f=0.015+$\frac{0.08}{Re^{0.3}}$, where Re is the Reynolds number , find the head loss due to friction for length of 10 m, take kinematic viscosity of water as 0.01 stokes.

Solution: Diameter of the pipe, D=300mm=0.3, V=5 m/s , l=10 m, viscosity of water ,v= 0.01 stokes=0.01*10^{-4} m^2/s, (take , 1 stoke=1 $cm^2/s=10^{-4}m^2/s$)

Head loss due to friction hf;

Coefficient of friction, f=0.015+$\frac{0.08}{Re^{0.3}}$,

But the Reynolds number , Re $=\frac{pVD}{\mu}$ = $\frac{VD}{v}$ =$\frac{5*0.3}{0.01*10^{-4}}$=1.58*$10^6$

Therefore, f=0.015+$\frac{0.08}{(1.58*10^6)^{0.3}}$ =0.0161,

Therefore head loss due to friction, $hf = \frac{4flv^2}{2Dg}$ =$\frac{4*0.0161*10*5^2}{2*0.3*9.81}$

, hf= 2.735 m (Ans)

3.2 Minor losses in the pipes

Loss due to the local disturbances of the flow conduits such as changes in cross-section; bend, elbows, valves, joints, etc are called ***minor losses***. In case of a very long pipe, these losses may be insignificant in comparison with the fluid friction in the length considered.

Whenever, the velocity of a flowing stream is altered either *in direction* or *in magnitude* in turbulent flow, ***eddy currents*** are set up and a loss of energy in excess of the pipe friction in that same length is created[2]. Head losses in decelerating (i.e., diverging) flow is much larger than that in accelerating (i.e., converging) flow.

The most common minor losses can be represented in one of two ways. It may be expressed as $kv^2/2g$, where the ***loss coefficient*** *k* must be determined for each case. Or it may be expressed as an equivalent length of a straight pipe, usually in terms of the number of pipe diameters, N. Since,

$$k\frac{V^2}{2g} = \frac{f(ND)}{D}\frac{V^2}{2g}$$, it follows that $k = Nf$.

i. ***Loss of head at entrance***

A poorly designed inlet to a pipe can cause an appreciable head loss. Referring to Figure 3.2 it may be seen that, a cross section with maximum velocity and minimum pressure at B. This minimum flow area is known as the ***vena contracta***.

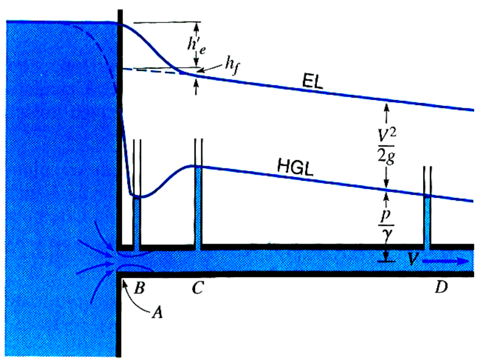


Figure 3.2 Condition at entrance

It is seen that the loss of energy at entrance is distributed along the length AC, a distance of several diameters. The increased turbulence and vortex motion in this portion of the pipe cause the friction loss to be much greater than in a corresponding length where the flow is normal, as it is shown by the drop of the total-energy line. Of this total loss, a small portion h_f would be due to the normal pipe friction (See Figure 3.2). Hence, the difference between this and that total, or $h_e^{'}$ is the true value of the extra loss caused at entrance.

The loss of head at entrance may be expressed as

$$h_e^{'} = k_e\frac{V^2}{2g} \quad \text{.. (3.15)}$$

Where V is the mean velocity in the pipe, and k_e is the loss coefficient

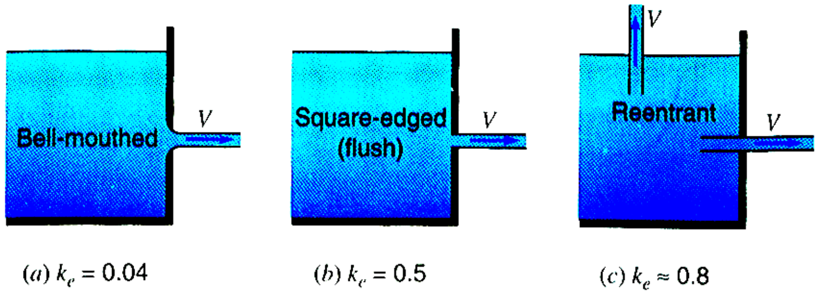


Figure 3.3 Entrance Loss Coefficients

ii. ***Loss of head at submerged discharges: (leave of pipe),*** (**$h_{d'}$**)

When the fluid with a velocity V is discharged from the end of a pipe in to a large reservoir, (v = 0), the entire kinetic energy of the coming flow is dissipated.

This may be shown by writing an energy equation between (a) and (b) in Figure 3.4 Taking the datum plane through (a) and recognizing that the pressure head of the fluid at (a) is *y*, its depth below the surface, $H_a = y + 0 + V^2/2g$ and $H_c = 0 + y + 0$. Therefore,

$$h_d^{'} = H_a - H_c = \frac{V^2}{2g} \qquad (3.16)$$

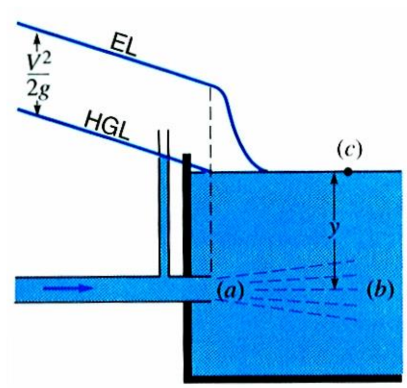


Figure 3.4 Submerged Discharge Loss

iii. ***Loss due to contraction*** (h_c)

a) Sudden contraction

There is a marked drop in pressure due to increase in velocity and to the loss of energy in turbulence. The loss of head for sudden contraction may be represented by

$$h_c^{'} = k_C \frac{V_2^2}{2g} \qquad (3.17)$$

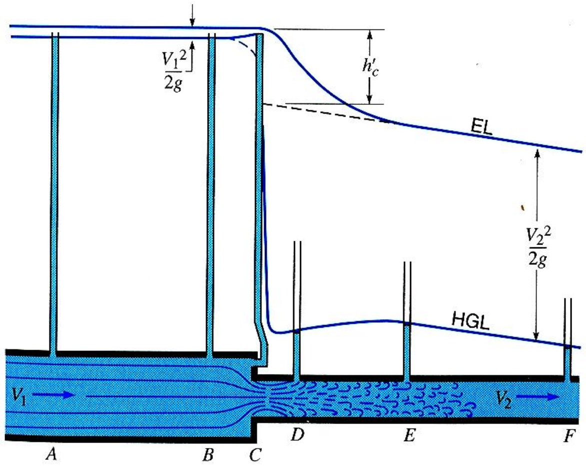


Figure 3.5 Loss due to sudden contraction

Table 3.1 Loss coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K_C	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

b) Gradual contraction

In order to reduces high losses, abrupt changes of cross section should be avoided. This is accomplished by changing from one diameter to the other by means of a smoothly curved transition or by employing the frustum of a cone. With a smoothly curved transition a loss coefficient k_c as small as 0.05 is possible. For conical reducers, a minimum k_c of about 0.10 is obtained, with a total cone angle of 20-40^0. Smaller or larger total cone angle results in higher values of k_c.

A nozzle at the end of a pipe line is a special case of gradual contraction. The head loss through a nozzle at the end of a pipeline is given by equation (3.17), where k_c is the nozzle loss coefficient whose value commonly ranges from 0.04 to 0.20 and v_j is the jet velocity. The head loss through a nozzle cannot be regarded as a minor loss because the jet velocity head is usually quite large.

iv. *Loss due to Expansion* (h_e)

a) Sudden Expansion

Both the figures in Figure 3.6, drawn to scale from test measurements for the same diameter ratios and the same velocities, and show that the loss due to sudden expansion is greater than the loss due to a corresponding contraction. This is so because of the inherent instability of flow in an expansion where the diverging paths of the flow tend to encourage the formation of eddies within the flow. Moreover, separation of the flow from the wall of the conduit induces pockets of eddying turbulence outside the flow region. In converging flow, there is a dampening effect on eddy formation, and the conversion from pressure energy to kinetic energy is quite efficient.

After the flow enters expanded pipe, there is excessive turbulence and formation of eddies which causes loss of energy. The loss due to sudden enlargement in a pipe line system can be calculated with the application of energy and momentum equations by neglecting the small shear force exerted on the walls of between sections 1 and 2 (***figure3.6***) for steady incompressible turbulent flow.

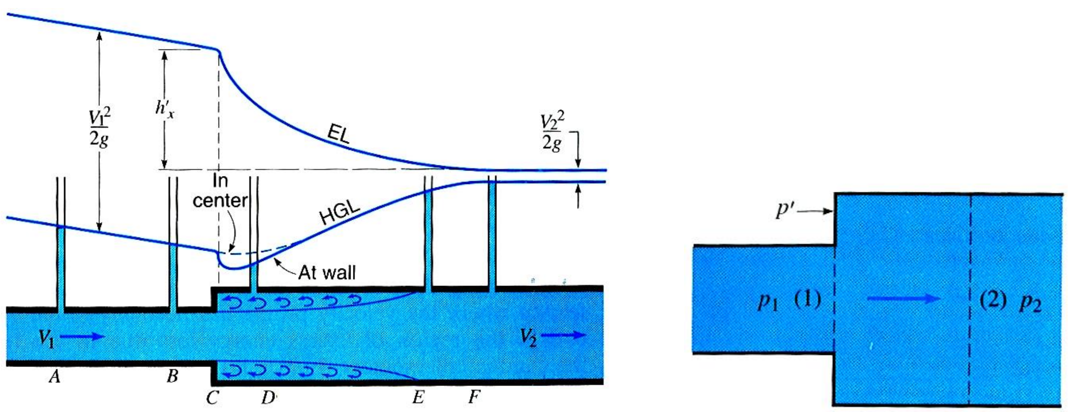


Figure 3.6 Loss due to sudden enlargement

Rate of momentum between section (1) & (2)

$$p_1A_2 - p_2A_2 = \frac{\gamma}{g}(A_2V_2^2 - A_1V_1^2)$$

Energy relation between section (1) & (2)

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + h_e$$

Solving for $\frac{p_1 - p_2}{\gamma}$ in each equation and equating the results

$$\frac{v_2^2 - v_2v_1}{g} = \frac{v_2^2 - v_1^2}{2g} + h_e$$

And noting that from continuity equation $A_1V_1 = A_2V_2$ and that

$A_1V_1^2 = (A_1V_1)\ V1 = (A_2V_2)V_1$

Substituting in the above equation

$$\therefore \quad h_x^{'} = \frac{(V_1 - V_2)^2}{2g} = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 \frac{V_1^2}{2g} = \left(\frac{D_2^2}{D_1^2} - 1\right)^2 \frac{V_2^2}{2g} \quad \text{.................................. (3.18)}$$

Example 3.3: At a sudden enlargement of water main from 240 mm to 480mm diameter, the hydraulic gradient rises 10mm. determine the rate of flow.

Solution: D1= 240 mm=0.24, D2=480 mm=0.48m

-rise of hydraulic gradient, i.e.

$(\frac{P_2}{w}+Z_2)-(\frac{P1}{w}+Z_1)$ =10mm=0.01

[The term $(\frac{P}{W}+Z)$ prescribes hydraulic gradient rate of flow, Q

Applying Bernoulli's equation to small and large pipe section (1-1) and (2-2)

$\frac{P_1}{w}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{w}+\frac{V_2^2}{2g}+Z_2+h_e$

But $h_e=\frac{(v1-v2)^2}{2g}$

From continuity equations, we have

$A_1V_1=A_2V_2$

Therefore, $V_1=\frac{A_2V_2}{A_1}=\frac{\frac{\pi}{4}*D_2^2}{\frac{\pi}{4}*D_1^2}*V_2=\left(\frac{D_2}{D_1}\right)^2*V_2$

Or $V_1=\left(\frac{0.48}{0.24}\right)^2*V_2= 4V_2$

$h_e=\frac{(4v2-v2)^2}{2g}=\frac{9V_2^2}{2g}$

Now substituting the value of h_e and V_1 in equation (1), we have

$\frac{P_1}{w}+\frac{(4V_1)^2}{2g}+Z_1=\frac{P_2}{w}+\frac{V_2^2}{2g}+Z_2+\frac{9V_2^2}{2g}$

$\frac{16V_2^2}{2g}-\frac{V_1^2}{2g}-\frac{9V_2^2}{2g}=(\frac{P_2}{w}+Z_2)-(\frac{P_1}{w}+Z_1)$ or

$\frac{6V_2^2}{2g}$=0.01 or

$\sqrt{\frac{0.01*2*9.81}{6}}$=0.181m/s

b) Gradual Expansion

To minimize the loss accompanying a reduction in velocity a diffuser may be used. Diffuser is a curved outline, or it may be a frustum of cone. In figure (3.7) the head loss will be some function of the angle of divergence and also of the ratio of two areas, the length of the diffuser being determined by these two variables.

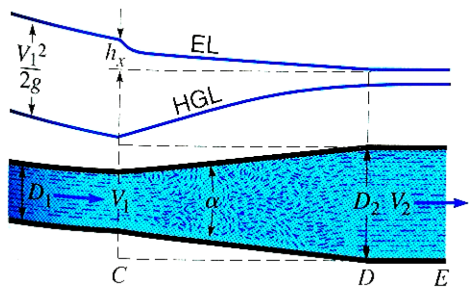


Figure 3.7 Loss due to gradual enlargement

In flow through a diffuser, the total loss may be considered as made up of two components. One is the ordinary pipe-friction loss, which may be represented by

$$h_{\ell e} = \int \frac{f}{D} \cdot \frac{V^2}{2g} dL \,.$$

In order to integrate, it is necessary to express the variables *f,* D, and V as functions of L. For our present purpose, it is sufficient, however, merely to note that the friction loss increases with the length of the cone. Hence, for given values of D_1 and D_2, the larger the angle of the cone, the less its length and the less the pipe friction.

The other is turbulence loss due to divergence. Turbulence loss increase with the degree of divergence, if the rate of divergence is great enough then there may be a separation at the wall and eddies flowing backward along the walls.

The total loss for gradual expansion pipe is the sum of these two losses, marked k'. It has been seen that the loss due to a sudden enlargement is very nearly represented by $(V_1 - V_2)^2 / 2g$. The loss due to a gradual enlargement is expressed as

$$h' = k' \frac{(V_1 - V_2)^2}{2g} \quad \text{..(3.19)}$$

.

Where K' loss coefficient which is a function of cone angleα

Table 3.2 Loss coefficients for gradual expansion

K'	0.4	0.6	0.95	1.1	1.18	1.09	1.0	1.0
α	20^0	30^0	40^0	50^0	60^0	90^0	120^0	180

v. ***Loss in pipe fittings***

The loss of head in pipefittings is expressed as $h_f = k_f \frac{V^2}{2g}$ where v is the velocity in a pipe of the nominal size of the fitting. Typical values are given below.
Table 3.3 Values of "K_f" based on the type of fittings.

Table 3.3 values of k_f loss for pipe fittings

Fitting	K
Globe valve, wide open	10
Angle valve, wide open	5
Close –return bend	2.2
T-through side outlet	1.8
Short-radius elbow	0.9
Medium radius elbow	0.75
Long radius elbow	0.60
Gate valve, wide open	0.19
Half open	2.06
Pump foot value	5.60
Standard branch flow	1.80

vi. ***Losses in bend & Elbow***

In flow around a bend or elbow, because of centrifugal effects, there is an increase in pressure along the outer wall and a decrease in pressure along the inner wall. Most of the loss of head in a sharp bend may be eliminated by the use of a vaned elbow. The vane tends to impede the formation of the secondary flow that would otherwise occur.

The head loss produced by a bend or elbow is:

$$h_b = k_b . \frac{V^2}{2g} \quad \text{..............................(3.19i)}$$

k_b - depends on the ratio of curvature r to pipe diameter D.

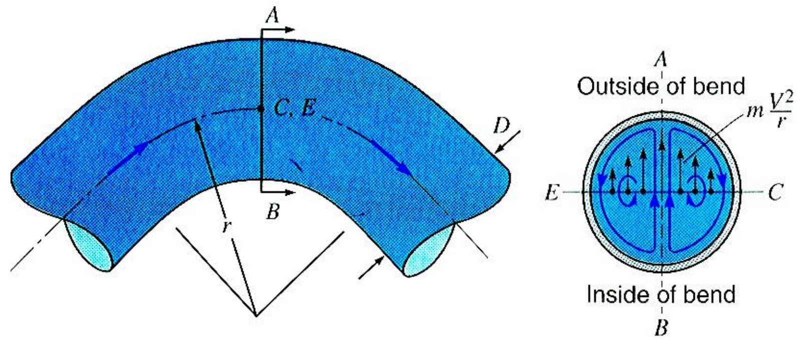


Figure 3.8 secondary flows in bend

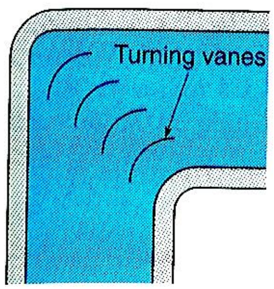


Figure 3.9 Vaned elbow

Activity 3.3: a) Define the term: Minor losses in a pipe.

b) Discuss on all types of losses that can be considered as minor losses.

c) Derive the formula for calculating loss of head due to :

(I) Sudden enlargement , and

(II) Sudden contraction.

Solution of single-pipe flow problems

The fundamental fluid mechanics associated with frictional loss of energy in single pipe flow, caused by both the wall roughness of the pipes and by pipe fittings that disturb the flow (minor losses).

It is generally conceded that for pipes of length greater than 1000 diameters, the error incurred by neglecting minor losses is less than that inherent in selecting a value for the friction factor$\left(f, n, \text{ or } C_{HW}\right)$.

When minor losses are negligible, as they often are, pipe flow problems may be solved by the methods, which are available are **Hazen-Williams** equation, the Manning equation or the Darcy-Weisbach equation. The **Darcy-Weisbach** equation is to be preferred, since it will provide greater accuracy because its application utilizes the basic parameters that influence pipe friction, namely, Reynolds Number **Re** and the relative roughness$\left(\in/D\right)$. To get good results with the Hazen-Williams and **Manning's** equations, the user must selected proper values for C_{HW} and ***n***, respectively.

The total head losses between two points is the sum of the pipe friction loss plus the minor losses, or

$$h_L = h_{Lf} + \sum h' \text{ ……………………………… (3.20)}$$

Where h_L = total head loss h_{Lf} = major head loss $\sum h'$ = total minor losses

In problem where *f* is given, equation (3.18) still has only one unknown, namely, h_L or V or Q or D. In most cases, this equation is explicit in the unknown, and so it is easy to solve. However, for sizing problems, the resulting equation in D is of the fifth degree, requiring trial and error or an equation solver.

The universal turbulent flow equation for use in an equation solver, including minor losses, eliminating h_{Lf} and equation (3.19) with the help of equation (3.7) and (3.11), and by replacing V by $4Q/(\pi D^2)$. Expressing minor losses $\sum h'$ in terms of $\sum kV^2/2g$,

$$\sqrt{\frac{L/D}{\frac{\pi^2 g D^4 h_L}{8Q^2} - \sum k}} = -2\log\left(\frac{\in/D}{3.7} + \frac{2.51\pi\nu}{4Q}\sqrt{\frac{LD}{\frac{\pi^2 g D^4 h_L}{8Q^2} - \sum k}}\right) \text{ ………………… (3.21)}$$

An important reminder when using these equations is to use Reynolds equation to check the Reynolds number and confirm that the flow is turbulent. If **Re** < 2000 the flow is laminar and the problem must instead be solved with equation (3.16).

Activity 3.4: Why the Darcy-Weisbach equation is preferable than the other equations which are used to solve pipe flow problems?

3.3 Pipeline system

3.3.1 Pipes in Series

When two pipes of different sizes or roughness are so connected that the fluid flows through one pipe& then through the other, they are said to be connected in series. A typical series pipe problem, in which head H may be wanted for a given discharge or the discharge wanted for a given H, is illustrated in figure 3.10 and the continuity equations establish the following two simple relations that must be satisfied.

$$Q = Q_1 = Q_2 = Q_3 = \dots = Q_n.$$
$$h_L = h_{L1} + h_{L2} + h_{L3} + \dots$$

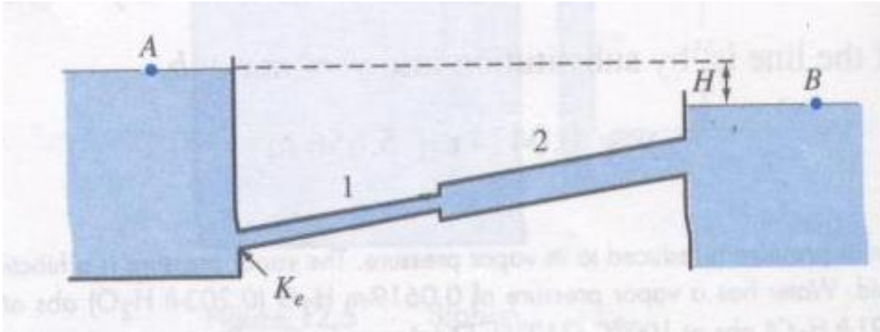


Figure 3.10 Pipes Connected in Series

Applying the energy equation from A to B, including all losses, gives:

$$\frac{P_A}{\gamma}+Z_A+\frac{V_A^{\ 2}}{2g}=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+hi+h_{f1}+he'+h_{f2}+h_d.$$

$$H+0+0=0+0+0+ki\frac{V_1^2}{2g}+f_1\frac{L_1}{D_1}\frac{V_1^{\ 2}}{2g}+\frac{(V_1-V_2)^2}{2g}+f_2\frac{L_2}{D_2}\frac{V_2^{\ 2}}{2g}+\frac{V_2^{\ 2}}{2g}$$

$$From\ continuty\ equation.:\qquad V_1D_1^{\ 2}=V_2D_2^{\ 2}$$

$$H=\frac{V_1^2}{2g}\left\{ki+f_1\frac{L_1}{D_1}+\left[1-\left({D_1}/{D_2}\right)^2\right]^2+f_2\frac{L_2}{D_2}\left(\frac{D_1}{D_2}\right)^4+\left(\frac{D_1}{D_2}\right)^4\right\}$$

Example 3.4: three pipes of diameters 300, 200 and 400 mms and length of 450m, 255mm, and 315 m respectively are connected in series. and the difference in water surface levels in two tanks is 18m. Determine the rate of flow of water if coefficients of friction are 0.0075, 0.0078 and 0.0072 respectively considering:

i. Minor losses also and
ii. Neglecting Minor losses

Solution: pipe 1: L_1=450 m D_1=300mm=0.3 m f_1=0.0075

L_2=255m D_2=200mm=0.2 m f_2=0.0078

L_3=315 mm D_3=400 mm=0.4 f_3=0.0072

i. Considering Minor losses: let V1, V2 and V3 be the velocity 1st, 2nd and 3rd pipes respectively. From continuity consideration, we have

$$A_1V_1=A_2V_2=A_3V_3$$

$$V_2=\frac{A_1V_1}{A_2}=\frac{\frac{\pi}{4}*D_1^2}{\frac{\pi}{4}*D_2^2}*V_1=\left(\frac{D_1}{D_2}\right)^2*V_1=\left(\frac{0.3}{0.2}\right)^2*V_1$$

$$V_2=2.25V_1$$

and $V_3 = \frac{A_1 V_1}{A_3} = \frac{\frac{\pi}{4} * D_1^2}{\frac{\pi}{4} * D_3^2} * V_1 = \left(\frac{D_1}{D_3}\right)^2 * V_1 = \left(\frac{0.3}{0.4}\right)^2 * V_1$

$V_3 = 0.5625 V_1$

We know that hf = $\frac{0.5V_1^2}{2g} + \frac{4f1l1v1^2}{2D1g} + \frac{0.5V_2^2}{2g} + \frac{4f2l2v2^2}{2D2g} + \frac{(v2-v3)^2}{2g} + \frac{4f3l3v3^2}{2D3g} + \frac{V_3^2}{2g}$

$18 = \frac{0.5V_1^2}{2g} + \frac{4*0.0075*450*v1^2}{2*0.3*g} + \frac{0.5*(2.25V)_1^2}{2g} + \frac{4*0.0078*255*(2.25V)^2}{2*0.2*g} + \frac{(2.25v1-0.5625v1)^2}{2g} +$

$\frac{4*0.0072*315*(0.5625V)_1^2}{2*0.4*g} + \frac{(0.5625V1)^2}{2g}$

$18 = \frac{V_1^2}{2g}$+(0.5+45+2.53+201.4+2.847+7.176+0.316)

$= 259.77\frac{V_1^2}{2g}$ or

V1= $\sqrt{\frac{18*2*9.81}{259.77}}$ =1.116 m/s

3.3.2 Equivalent pipes

Series pipes can be solved by the method of equivalent lengths. Two pipe systems are said to be equivalent when the same head loss produces the same discharge in both systems. From Equation (3.7)

$hf_1 = f_1 \frac{L_1}{D_1^5} \frac{8Q_1^2}{\Pi^2 g}$ for a second pipe hf$_2$ = $\frac{f_2 L_2}{D_2^5} \frac{8Q_2^2}{\Pi^2 g}$

For two pipes to be equivalent,

$h_{f1} = h_{f2}, \quad Q_1 = Q_2$

$\therefore \frac{f_1 L_1}{D_1^5} = \frac{f_2 L_2}{D_2^5}$

$L_2 = L_1 \frac{f_1}{f_2}\left(\frac{D_2}{D_1}\right)^5$ ....(3.24)

Activity 3.5: What is an equivalent pipe?

3.3.4 Pipes in Parallel

A combination of two or more pipes connected as in **figure** 3.11 so that the flow is divided among the pipes and then is joined again, is a parallel – pipe system. In series pipe system the same fluid flows through all the pipes and the head losses are cumulative, but in parallel pipe – system the head losses are the same in each of the lines the discharge are cumulative.

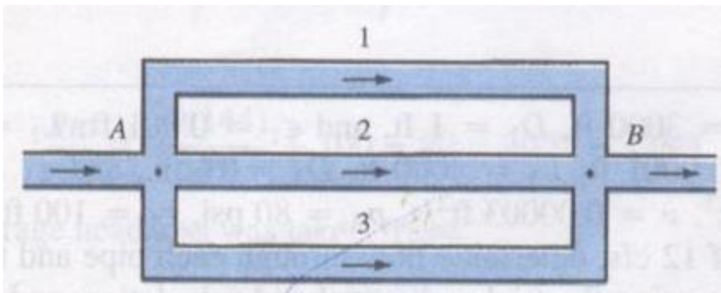


Fig **3.11 Parallel Pipes system**

$h_{f1} = h_{f2} = h_{f3} = \frac{P_A}{\gamma} + Z_A - \left(\frac{P_B}{\gamma} + Z_B\right)$

$Q = Q_1 + Q_2 + Q_3$

Two types of problems occur:

1) If the head loss b/n A & B is given, Q is determined.
2) If the total flow Q is given, then the head loss & distribution of flow are determined.

Size of pipes, properties, and roughness are assumed to be known. Since this type of problem is more complex, as neither the head loss nor the discharge for any one pipe is known. The procedure is:

1) Assume discharge Q'_1 through pipe 1,
2) Solve for $h'f_1$, using assumed discharge,
3) Using $h'f_1$, find Q'_2 & Q'_3
4) With the three discharges for a common head loss, now assume that the given Q is split up among the pipes in the same proportion as Q'_1, Q'_2 & Q'_3, Thus,

$$Q_1 = \frac{Q_1'}{\sum Q'}Q, \qquad Q_2 = \frac{Q_2'}{\sum Q'}Q, \qquad Q_3 = \frac{Q_3'}{\sum Q'}Q$$

5) Check the correctness of these discharges by computing hf_1, hf_2, & hf_3 for the computed Q_1 , Q_2 & Q_3

→ $Q - Q_1 - Q_2 - Q_3 = 0$

Activity 3.5: what are the differences between pipes in series and pipes in parallel?

3.4 Branching pipes

Let us consider three pipes connected to three reservoirs as in fig. below & connected together or branching at the common junction point J. We shall assume that all the pipes are sufficiently long that minor losses & velocity heads may be neglected. The continuity & energy eqn. require that the flow entering the junction equal the flow leaving it& that the pressure head at J (with open piezometer tube water at elevation P) be common to all pipes.

There being no pumps, the elevation of p must lay b/n the surfaces of reservoirs A& C. If p is level with the surface of reservoir B then water must flow in to B & $Q_1 = Q_2 + Q_3$

If P is below the surface of reservoir B then the flow must be out of B & $Q_1 + Q_2 = Q_3$

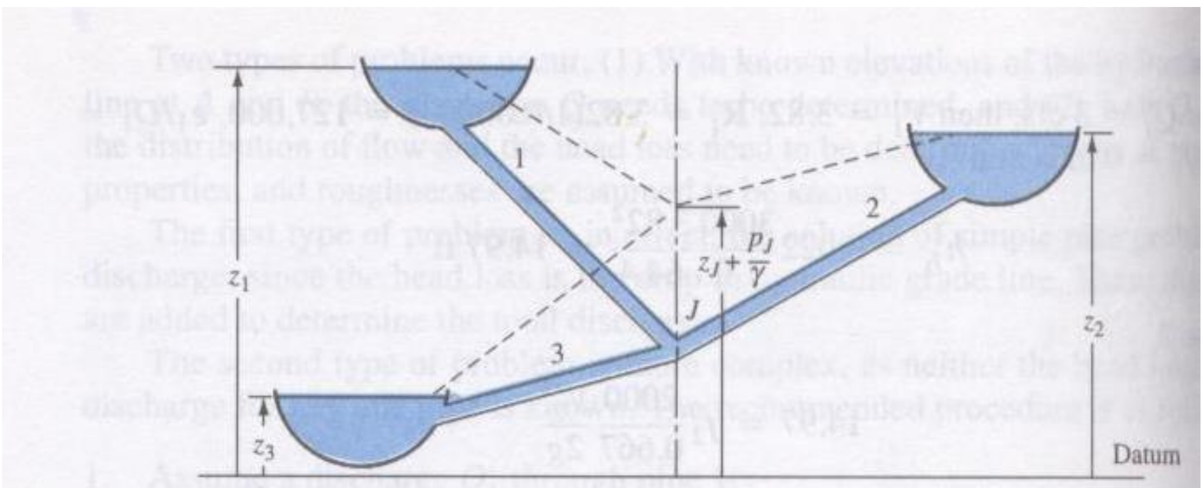


Fig 3.12 Three interconnected reservoirs

Therefore, for the situation of the above figure, we have the following governing conditions:

1) $Q_1 = Q_2 + Q_3$ or $Q_1 + Q_2 = Q_3$
2) Elevation of p is common to all.

a. Length, diameter, & friction factors are required.
b. The flow is steady & minor losses neglected
c. Three basic equations to solve these problems are:-
    i. Continuity equations
    ii. Bernoulli's equation
    iii. Darcy- Weisbech equation

- Total rate of in flow at junction = total rate of out flow (continuity equation)

❖ Pipe 1 Pipe 2 Pipe 3

D_1, L_1, V_1, Q_1 h_{f1} D_2, L_2, V_2, Q_2, h_{f2} D_3, L_3, V_3, Q_3, h_{f3}

Elevation, Z_1, Reserv, A Z_2, Reserv, B Z_3, Reserv. C

Junction of elevation

Zj, pressure head $\frac{p_j}{\gamma}$ = total head at junction = $\frac{p_j}{\gamma}$ + Zj

❖ Applying Bernoulli's eqn b/n the junction point & each of reservoirs,

if $\frac{p_j}{\gamma}$ + Zj > Z_2 & Z_3

=> $Z_1 = (\frac{p_j}{\gamma} + Zj) + hf_1$ ---------------------- (*) (1)

$Z_2 + h_{f2} = (\frac{p_j}{\gamma} + Zj)$ -------------------------- (**) (2)

$Z_3 + h_{f3} = (\frac{p_j}{\gamma} + Zj)$ --------------------------- (***) (3)

=> If the head of reservoir A is greater than head at junction, the flow is in to the junction from A & out of the junction to B&C

=> Q1 = Q2 + Q3 -------------------------- * (4)

$\frac{\pi}{4}D_1^2$ V_1 = $\frac{\pi}{4}D_2^2$ V_2 + $\frac{\pi}{4}$ D_3^2 V_3 ------------ (5)

=> $D_1^2 V_1 = D_2^2 V_2 + D_3^2$ V3 ---------------------- (6)

❖ There are three types of problem fouling of branching pipes :-

Case 1: Given all pipes data (L, D, f, Z_1, Z_2 & Q_1), find Z_3, Q_2 & Q_3?

=> Solution: first hf_1 can be calculated directly ($h_{f1} = f_1 \frac{L_1}{D_1}\frac{v_1^2}{2g}$)

Then ($\frac{p_j}{\gamma}$ + Zj) piezometric head at junction can be determine

⇨ From eqn (2) h_{f2} & Q_2 can be determined

- ⇨ Q_3 can be detrained from eqn (4) continuity eqn
- ⇨ Then from eqn (3) h_{f3} and finally Z_3 can be determined

Case 2: Given a pipe data, the surface elevation of two reservoirs (A& C) and the flow the second pipe, Q_2, find Z_2 and Q_1, Q_3?

- ⇨ From eqn (1) & iii) $(h_{f1} + h_{f3}) = (Z_1 - Z_3)$ $(h_{f1} + h_{f3})$ is known & $(Q_1 - Q_3)$ or $(Q_3 - Q_1)$ is known.
- ⇨ Assume trail values of h_{f1} & h_{f3} & from these compute the discharge $Q_1 + Q_3$ & compare with $(Q_1 - Q_3)$
- ⇨ Repeat the procedure until the two values are equal.
- ⇨ From then, piezometric head at junction can be determined
- ⇨ From h_{f2} & $(\frac{p_j}{\gamma} + Zj)$ → Z_2 can be determined.

Case:3 Given a pipe lengths , diameters, and the elevation of all the three reservoirs , find Q_1 Q_2, Q_3,

→ In this case, the direction of the flow is **not known** clearly.
→ Assume the elevation of B (Z_2) is equal to the piezometric head (Z_p) & (i.e. no flow into and out of pipe 2)
→ From Zp the head losses h_{f1} & h_{f3} determined, and then Q_1 & Q_3 can be obtained
→ If $Q_1 > Q_3$, then Zp must be increased to satisfy continuity eqn at J, causing water to flow into reservoir B, and we will have $Q_1 = Q_2 + Q_3$
→ If Q1<Q3, then Zp must be lowered, causing water to flow out of reservoir B, & we will have $Q_1 + Q_2 = Q_3$

Activity 3.6: Discuss on the three types (cases) of problem falling of branching pipes.

Example 3.5: (flow through the branched pipes):the water level in the two reservoir A and B are 104.5m and 100m respectively above the datum. A pipe joints each to a common points D, where a pressure is 98.1KN/m^2 gauge and height is 83.5 m above the datum. Another pipe connects D to another tank C. what will be the height of water level in C assuming the same value diameters of the pipes AD, BD and CD are 300 mm, 450 mm 600 mm respectively and their length are 240m, 270m, 300m respectively

Solution: for pipe AD: D_{AD}=300mm=0.3m, L_{AD} =240m

For pipe BD: D_{BD}=450 mm=0.45m L_{BD}= 270

For pipe CD : D_{CD}=600mm= 0.6 m L_{CD}= 300m

Friction coefficient for each pipe, f=0.0075

Pressure at D, P_D= 93.1 KN/m^2 height of water level in tank C: the

Pressure head at D=$\frac{P_D}{w}$=$\frac{93.1}{9.81}$=10 mm of water

Therefore the piezomertic head at D=83.5+10=93.5m

Figure hear---

Head loss between A and D =104.5-93.5=11 m

Head loss between B and D = 100-93.5=6.5 m

Using Darcy weisbach formulae equation we get

For pipe AD: 11=$\frac{4f V_{AD}^2 L_{AD}}{2gD_{AD}}$ = $\frac{4*0.0075*240V_{AD}^2}{0.3*2*9.81}$

Or V_{AD}^2=$\frac{11*0.3*2*9.81}{4*0.0075*240}$

V_{AD}=3m/s

For pipe:BD: 6.5=$\frac{4f V_{BD}^2 L_{BD}}{2gD_{BD}}$= $\frac{4*0.0075*270V_{BD}^2}{0.45*2*9.81}$

V_{BD}^2= $\frac{6.5*0.45*2*9.81}{4*0.0075*270}$ or

V_{BD}= 2.66m/s

From continuity terms, we get

Q_{AD}+Q_{BD}=Q_{CD}

Or Q_{CD}=$\frac{\pi}{4}$*D_{AD}^2*V_{AD}+$\frac{\pi}{4}$*D_{BD}^2*V_{BD}

Q_{CD}=$\frac{\pi}{4}$*$(0.3)^2$*3+$\frac{\pi}{4}$*$(0.45)^2$*2.66=0.635m^3/s

Therefore velocity of pipe CD, V_{CD}=$\frac{Q_{CD}}{\frac{\pi}{4}*D_{CD}^2}$=$\frac{0.635}{\frac{\pi}{4}*D_{CD}^2}$=2.24 m/s

Head loss in pipe CD=$\frac{4f V_{CD}^2 L_{CD}}{2gD_{CD}}$=$\frac{4*0.0075*300*2.24^2}{0.6*2*9.81}$=3.84 m

Therefore water level in tank C= 93.5-3.84=89.66 m (Ans)

3.5 Pipe Networks

A group of interconnected pipes forming several loops or circuits as shown in **fig 3.13** is called a network of pipes. Such network of pipes is commonly used for municipal water distribution systems in cities. The main problem in pipe network is to determine the distribution of flow through the various pipes of the network such that all the conditions of flow are satisfied and all the circuits are then balanced.

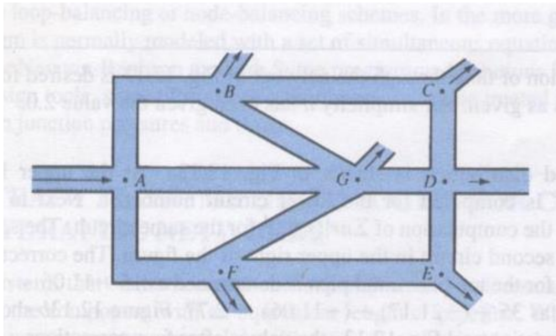


Fig 3.13 Pipe Network

The following conditions of basic relation of continuity and energy should be satisfied in network of pipes:

1) The flow in to any *junction* must equal the flow out of it (*continuity principle*).
2) In any loop, the loss of head due to flow in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction. ($\sum h_f = 0$)
3) The Darcy-Weisbach eqn of pipe-friction laws must be satisfied, (i.e. proper relation b/n the head loss and discharge must be maintained for each pipe). Minor losses may be neglected if the pipe lengths are large. However, if the minor losses are large, they may be taken into account by considering them in terms of the head loss due to friction in equivalent pipe lengths. According to Darcy-Weisbach eqn the loss of head h_f through any pipe discharging at the rate Q can be expressed as:

$$h_f = rQ^n$$

Where r is proportionality factor, which can be determined for each pipe knowing the friction factors *f,* the length *L* and the diameter *D* of the pipe.

$$r = \frac{fL}{2g(\pi/4)^2 D^5} = \frac{fL}{12.1D^5}$$

, and *n* is an exponent having a numerical value ranging from 1.72 to 2.

The pipe network problems are in general complicated and can't be solved analytically. As such, methods of successive approximations are used.

The **Hardy-Cross Method** is one of the commonly used methods that is used for solving flows in a pipe network.

Steps:

1. Assume a most suitable distribution of flow that satisfies continuity equation at each junction.
2. With the assumed values of Q, compute the head losses for each pipe using $h_f = rQ^n$ equation.
3. Consider different loops and compute the net head loss around each circuit considering the head loss in clockwise flows as positive and in anti-clockwise flows as negative. For a correct

distribution of flow, the net head loss around each loop should be equal to zero, so that the circuit will be balanced. However, in most of the cases, for the assumed distribution of flow the head loss around the circuit will not be equal to zero. The assumed flows are then corrected by introducing a correction ΔQ for the flows, until the circuit is balanced. The value of the correction ΔQ to be applied to the assumed flows of the circuit may be obtained as follows:

For any pipe if Q_0 is the assumed discharge and Q is the correct discharge, then,

$Q = Q_0 + \Delta Q$

and the head loss for the pipe is

$$h_f = rQ^n = r(Q_o + \Delta Q)^n$$

Thus, for a complete circuit,

$$\sum h_f = \sum rQ^n = \sum r(Q_o + \Delta Q)^n = 0$$

By expanding the terms in the brackets by binomial theorem

$$\sum rQ^n = \sum r[(Q_o{}^n + nQ_o{}^{n-1}\Delta Q + ...] = 0$$

For small ΔQ compared with Q_0, all the series after the second can be dropped. Therefore,

$$\sum rQ^n = \sum rQ_o{}^n + \sum rnQ_o{}^{n-1}\Delta Q = 0$$

$$\sum rQ_0|Q|^{n-1} + \Delta Q \sum rn|Q|^{n-1} = 0$$

For each loop, solve for ΔQ in the networks as:

$$\Delta Q = \frac{-\sum rQ_0|Q_0|^{n-1}}{\sum rn|Q_0|^{n-1}} = \frac{-\sum rQ_0^n}{\sum rn|Q_0|^{n-1}} = \frac{-\sum hf}{\frac{\sum nhf}{Q_0}}$$

This is the correction to the assumed discharge (Q0).

4. Corrections are now applied to each pipe & to all loopy. ***For pipes common to two loops or circuits, a correction from both the loops will be required to be applied***. Clockwise direction is considered as positive & anticlockwise as negative direction.
5. With the corrected flows in all the pipes, a second trial calculation is made for all the loops and process is repeated until the corrections ΔQ become negligible.

Activity 3.7: what are the conditions that must be satisfied in pipe net work?

Example 3.6

1) The following figure shows three reservoirs connected by pipes. Each pipe is 300 mm in diameter and 1500 m long. Assuming coefficient of friction for f=0.01. Find the discharge in each pipe,

Solution: Diameter in each pipe, D_1=D_2=D_3= 300 mm=0.3 m length of each pipe,

L_1=L_2=L_3= 1500 m coefficient of friction for each pipe, f=0.01

Discharge in each pipe:

To find over the direction of flow in pipe 2, let us assume that no flow occurs in pipe 2.that is, the piezometric level is 30m.

Therefore, head loss in pipe 1, hf=70m- 30m= 40m

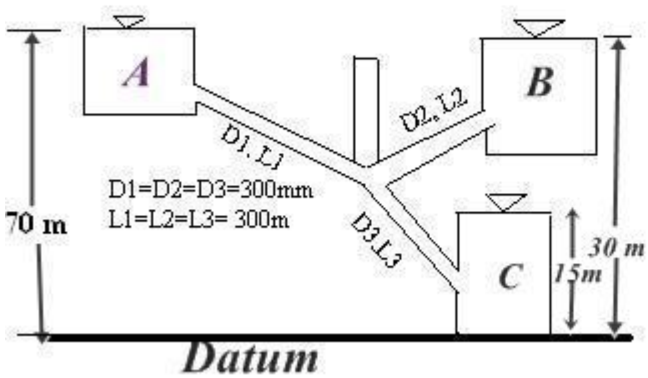


Figure : Branching Pipes

Also hf= $\frac{4f1l1v1^2}{2D1g}$

40= $\frac{4*0.01*1500v1^2}{2*0.3*9.81}$

V_1^2= $\frac{40*0.3*2*9.81}{4*0.01*1500}$ = 3.924

V_1= 1.921 m/s

Or Discharge through the pipe 1,

Q_1= π/4*0.3^2* 1.981= $A_1 V_1$=π/4*0.3^2* 1.981= 0.14 m^3/s

Again, head loss in pipe 3, hf_3=30-15=15m

But hf_3= $\frac{4f3l3v3^2}{2D3g}$

15= $\frac{4*0.01*1500*v3^2}{0.3*2*9.81}$

V_3^2=$\frac{15*0.3*2*9.81}{4*0.01*1500}$= 1.471

Or V_3=1.213 m/s

Therefore the Discharge through the pipe 3,

Q_3=$A_3 V_3$= π/4*0.3^2* 1.213=0.0857 m^3/s

Since Q_1>Q_3 the direction of flow is from J to B.

Considering the flow from reservoir A and to B, we have (70-30) =head loss in pipe 1+ head loss in pipe 2

Or hf_1+hf_2 = $\frac{4f1l1v1^2}{2D1g}$ + $\frac{4f2l2v2^2}{2D2g}$

40= $\frac{4*0.01*1500*v1^2}{0.3*2*9.81}$ + $\frac{4*0.01*1500*v2^2}{0.3*2*9.81}$

40=10.2 ($V_1^2+V_2^2$)

$V_1^2+V_2^2=\frac{40}{10.2}$=3.92

$V_2=\sqrt{3.92-V_1^2}$

Similarly considering the flow from reservoir A to C, w have

70-15=hf_1+hf_3

55=10.2 ($V_1^2+V_3^2$)

($V_1^2+V_3^2$)= $\frac{55}{10.2}$=5.39

Or $V_3=\sqrt{5.39-V_1^2}$

From continuity consideration, we have

$Q_1=Q_1+Q_3$

$A_1V_1=A_2V_2+A_3V_3$

But $A_1=A_2=A_3$ (because $D_1=D_2=D_3$)

$V_1=V_2+V_3$

From equation (1), (2) and (3), we have

$V_1=\sqrt{3.92-V_1^2}+\sqrt{5.39-V_1^2}$

By trial and error we get , V_1=1.9 m/s

From equation (1) : $\sqrt{3.92-1.9^{\ 2}}$ =0.56 m/s

From (2) : $V_3=\sqrt{5.39-1.9^2}$ =1.54 m/s

Thus, Q_1= Π/4*0.3^2* 1.9=0.134 m^3/s (Ans)

Q_2= Π/4*0.3^2* 0.56= 0.0396 m^3/s (Ans)

Q_3= Π/4*0.3^2* 1.34 =0.0947 m^3/s (Ans)

2) The following figure shows a network in which Q and hf refers to discharges and pressure drops respectively .subscripts 1,2,3,4 and 5 designate respectively values in a pipe length AC,BC, CD, DA and AC. Subscripts A, B, C and D designate discharge entering or leaving the junction points A, B, c and D respectively.

By sticking to the valves given in the figure find the following dischargesQ_B, Q_2, Q_4, and Q_5 and pressure drops h_{f4}, h_{f5} and give this computed value at their respective places on a net sketch of the net work along with flow directions.

Solution: At junction, ∑Q=0

That is Discharge entering the junction =discharge leaving the junction

At junction D: $Q_D = Q_3 + Q_4$

100= 40+Q_4

Q_4=100-40=60

At junction A: $Q_4 = Q_A + Q_5 + Q_1$

60=20+30+Q_5

Q_5=60-20-30=10

At junction C: $Q_3 + Q_5 + Q_2 = Q_C$

40+10+Q_2= 30

Q_2=30-40-10=-20

At junction B: $Q_1 + Q_2 = Q_B$

30+20=Q_B

Q_B=50m**3**/s

For each elementary circuit, $\sum h_f$=0

Circuit A= B

$+h_{f1} - h_{f2} - h_{f5}$=0

60-40-h_{f5}=0

h_{f5}=20

Circuit A=B

$h_{f5} - h_{f3} + h_{f4}$

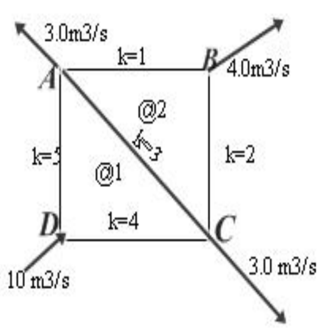
h_{f4}+20-120=

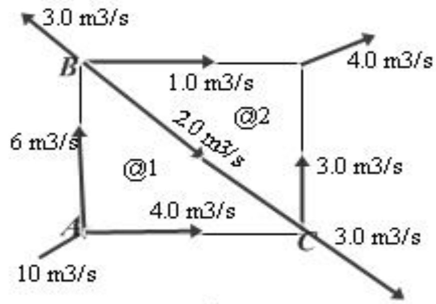
Therefore h_{f4}=100 m

3) Find the discharge in each pipe of the net work shown in the figure (a) the value of the value of the constant K corresponding to heading to the head loss equation H_f=KQ^2 are also shown in the figure.

Solution: for the first trial, the discharge as shown in figure (b) is assumed. The calculation for the correction ΔQ and the correction discharge are given in the table below.

It may be noted that if the ΔQ is positive, it is to be added to the assumed flows. Thus a clock wise flow will increase and a counter clock wise flow decrease in magnitude.



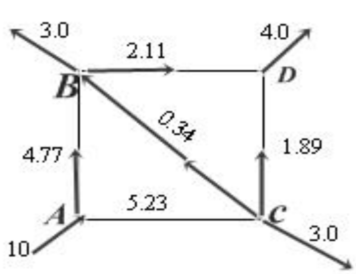


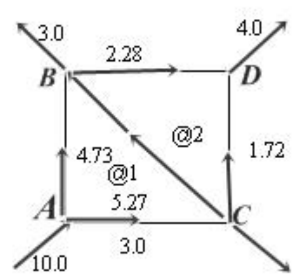
b

1. **Trial** for the first trial the values given in figure

Circuit	Pipe	K	Assumed Q_0^{-1}	$H_f=KQ_o^2$	$\sum H_f$	$\sum KQ_o^2$	$\sum 2KQ_o^2$	ΔQ0=((6)/(8)	Corrected Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	AB BC CA	5 3 4	+6.0 +2.0 -4.0	$5*(6.0)^2$=+180.0 $3*(2.0)^2$= 12.0 $-4*(4.0)^2$=-64.0	+128	2*5*6.0=60.0 2*3*2.0=12.0 2*4*4=32	104	-1.23	+4.77 -0.34 _5.23
(2)	BD DC CB	1 2 3	+1.0 -3.0 -2.0	$1*(4.0)^2$=1.0 $-2*(3.0)^2$=-18.0 $-3*(2.0)^2$=-12.0	-29.0	2*1.0*1.0=2.0 2*2*3.0=12.0 2*3*2.0=26.0	26.0	+1.11	+2.11 -1.89 +0.34

Corrected Discharge for AB=6.0-1.23=+4.77
Corrected Discharge for BC=2.0-1.23- (+1.11)=-0.34
Corrected Discharge for CA= -4.00-1.23=5.23





(b)

(a)

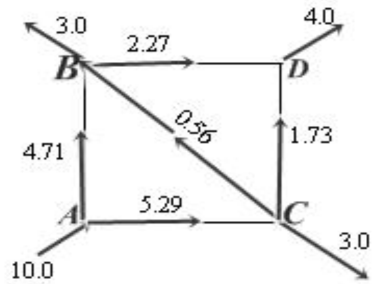


Figure 2 (C)

2nd Trial for the second trial the values given in fig 2 © are assumed and the process is repeated.

Circuit	Pipe	K	Assumed Q_0^{-1}	$H_f=KQ_o^2$	∑H_f	∑ KQ_o^2	∑ $2KQ_o^2$	ΔQ0=(6)/(8)	Corrected Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	AB BC CA	5 3 4	+4.77 -0.34 -5.23	$5*(4.77)^2$=+113.76 $3*(0.34)^2$= -12.0 $-4*(5.23)^2$=-109.41	+4	2*5*4.77=47.7 2*3*0.34=2.04 2*4*5.23=41.84	91.58	-0.04	+4.73 -0.55 -5.27
(2)	BD DC CB	1 2 3	+2.11 -1.89 =0.34	$1*(2.11)^2$=4.45 $-2*(1.89)^2$=-7.14 $-3*(0.34)^2$=-+0.34	-2.34	2*1.0*2.11=4.22 2*2*1.89=7.56 2*3*0.34=2.04	1.82	+0.17	+2.28 -1.72 +0.55

3rd Trial the values given in figure 2 (D)

Circuit	Pipe	K	Assumed Q_0^{-1}	$H_f=KQ_o^2$	∑H_f	∑ KQ_o^2	∑ $2KQ_o^2$	ΔQ0=(6)/(8)	Corrected Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	AB BC CA	5 3 4	+4.73 -0.55 -5.27	$5*(4.73)^2$=+111.86 $3*(0.55)^2$= -0.91 $-4*(5.27)^2$=-111.09	-0.14	2*5*4.73=47.30 2*3*0.55=3.30 2*4*5.27=42.16	91.76	-0.02	+4.71 -0.56 -5.29
(2)	BD DC CB	1 2 3	+2.28 -1.72 +0.55	$1*(2.28)^2$=+5.20 $-2*(1.72)^2$=-5.92 $-3*(0.55)^2$=-+0.91	+0.19	2*1.0*2.11=4.56 2*2*1.89=6.88 2*3*0.34=3.30	14.74	+0.01	+2.27 -1.73 +0.56