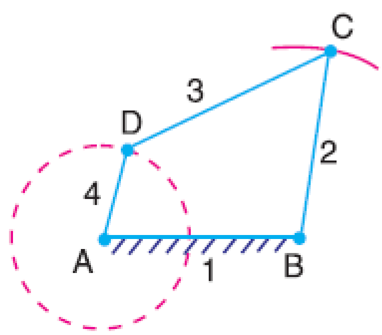
Chapter One

Introduction to Mechanisms of Machines

✓ Four Bar Chain or Quadric Cycle Chain

We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained. The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig below. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths. According to **Grashof 's law** for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links. A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful.



Four bar chain

In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof 's law. Such a link is known as **crank** or **driver**. In Fig. 5.18, AD (link 4) is a crank. The link BC (link 2) which makes a partial rotation or oscillates is known as **lever** or **rocker** or **follower** and the link CD (link 3) which connects the crank and lever is called **connecting rod** or **coupler.** The fixed link AB (link 1) is known as **frame** of the mechanism. When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

Inversions of Four Bar Chain

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view:

1. Beam engine (crank and lever mechanism).

A part of the mechanism of a beam engine (also known as cranks and lever mechanism) which consists of four links is shown in Fig. 5.19. In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

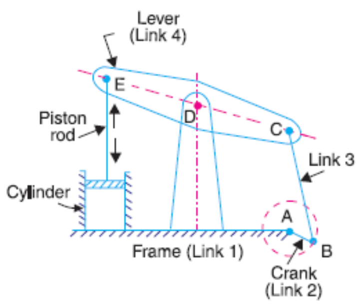


Fig. 5.19. Beam engine.

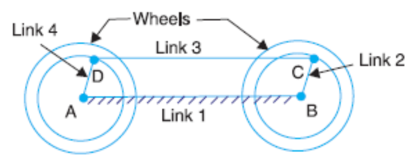


Fig. 5.20. Coupling rod of a locomotive.

2. *Coupling rod of a locomotive* (*Double crank mechanism*). The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in Fig. 5.20. In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant center to center distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

3. *Watt's indicator mechanism* (*Double lever mechanism*). A *Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in Fig. 5.21. The four links are: fixed link at *A*, link *AC*, link *CE* and link *BFD*. It may be noted that *BF* and *FD* form one link because these two parts have no relative motion between them. The links *CE* and *BFD* act as levers.

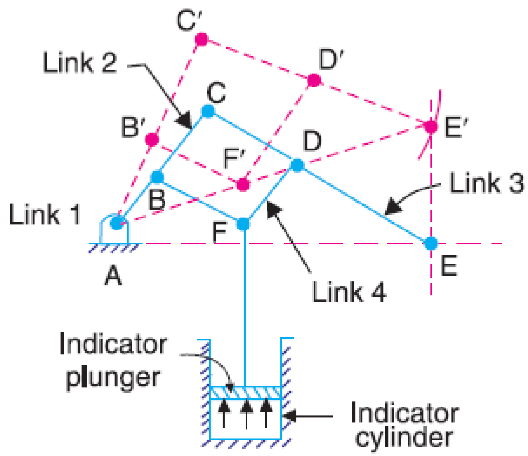


Fig. 5.21. Watt's indicator mechanism.

The displacement of the link *BFD* is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point *E* at the end of the link *CE* traces out approximately a straight line. The initial position of the mechanism is shown in Fig. 5.21 by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.

Dynamics of linkages

Velocity in Mechanisms (Relative Velocity Method)

The relative velocity method for determining the velocity of different points in the mechanism. The study of velocity analysis is very important for determining the acceleration of points in the mechanisms.

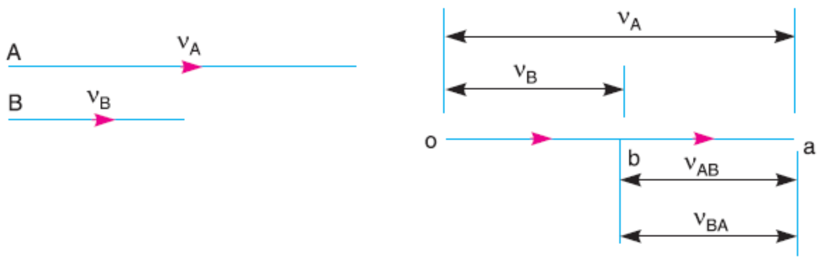
Relative Velocity of Two Bodies Moving in Straight Lines

Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 7.1 (*a*) and 7.2 (*a*) respectively. Consider two bodies *A* and *B* moving along parallel lines in the same direction with absolute velocities vA and vB such that vA> vB , as shown in Fig. 7.1 (*a*). The relative velocity of *A* with respect to *B*,

v_{AB} = Vector difference of v_A and v_B = $v_A - v_B$ ………………………....**(*i*)**

From Fig. 7.1 (*b*), the relative velocity of *A* with respect to *B* (*i.e.* v_{AB}) may be written in the vector form as follows:

$$\overline{ba} = \overline{oa} - \overline{ob}$$



(*a*) (*b*)

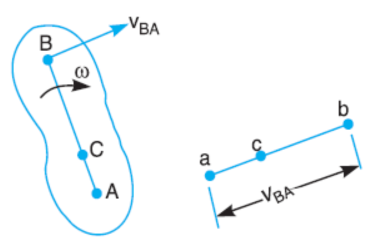
Similarly, the relative velocity of *B* with respect to *A*,

v_{BA} = Vector difference of *v*B and *v*A = *v*B – *v*A ……………………..**(*ii*)**

or $ab = ob - oa$

Motion of a Link

Consider two points *A* and *B* on a rigid link *AB*, as shown in Fig. 7.3 (*a*). Let one of the extremities (*B*) of the link move relative to *A*, in a clockwise direction. Since the distance from *A* to *B* remains the same, therefore there can be no relative motion between *A* and *B*, along the line *AB*. It is thus obvious, that the relative motion of *B* with respect to *A* must be perpendicular to *AB*.



(*a*) (*b*)

Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.

The relative velocity of B with respect to A (i.e. v_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in Fig. 7.3 (b).

Let ω = Angular velocity of the link *A B* about *A*.

We know that the velocity of the point *B* with respect to *A*,

$$v_{BA} = \overline{ab} = \omega . AB \qquad ...(i)$$

Similarly, the velocity of any point *C* on *A B* with respect to *A*,

$$v_{CA} = \overline{ac} = \omega . AC \qquad ...(ii)$$

From equations (*i*) and (*ii*),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \qquad ...(iii)$$

Thus, we see from equation ***(iii)***, that the point *c* on the vector *ab* divides it in the same ratio as *C* divides the link *AB*.

Note: The relative velocity of *A* with respect to *B* is represented by *ba*, although *A* may be a fixed point. The motion between *A* and *B* is only relative. Moreover, it is immaterial whether the link moves about *A* in a clockwise direction or about *B* in a clockwise direction.

Velocity of a Point on a Link by Relative Velocity Method

The relative velocity method is based upon the relative velocity of the various points of the link as discussed in Art. 7.3 Consider two points *A* and *B* on a link as shown in Fig. 7.4 (*a*). Let the absolute velocity of the point *A* *i.e.* v_A is known in magnitude and direction and the absolute velocity of the point *B* *i.e.* v_B is known in direction only. Then the velocity of *B* may be determined by drawing the velocity diagram as shown in Fig. 7.4 (*b*). The velocity diagram is drawn as follows:

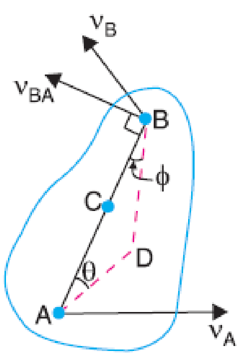
1. Take some convenient point *o*, known as the pole.

2. Through *o*, draw *oa* parallel and equal to v_A, to some suitable scale.

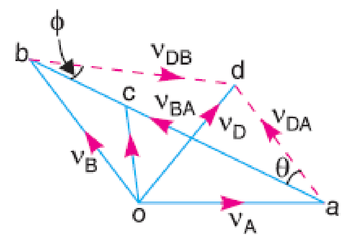
3. Through *a*, draw a line perpendicular to *AB* of Fig. 7.4 (*a*). This line will represent the velocity of *B* with respect to *A*, *i.e.* v_{BA}.

4. Through *o*, draw a line parallel to *v*B intersecting the line of v_{BA} at *b*.

5. Measure *ob*, which gives the required velocity of point *B* (v_B), to the scale.



(*a*) Motion of points on a link.



(*b*) Velocity diagram.

Fig. 7.4

Notes :

1. The vector *ab* which represents the velocity of *B* with respect to *A* (v_{BA}) is known as velocity of image of the link *AB*.

2. The absolute velocity of any point *C* on *AB* may be determined by dividing vector *ab* at *c* in the same ratio as *C* divides *AB* in Fig. 7.4 (*a*). In other words

$$\frac{ac}{ab} = \frac{AC}{AB}$$

Join *oc*. The *vector *oc* represents the absolute velocity of point *C* (vC) and the vector *ac* represents the velocity of *C* with respect to *A i.e.* v_{CA}.

3. The absolute velocity of any other point *D* outside *AB*, as shown in Fig. 7.4 (*a*), may also be obtained by completing the velocity triangle *abd* and similar to triangle *ABD*, as shown in Fig. 7.4 (*b*).

4. The angular velocity of the link *AB* may be found by dividing the relative velocity of *B* with respect to *A* (*i.e.* v_{BA}) to the length of the link *AB*. Mathematically, angular velocity of the link *AB*,

Cam follower

A **cam** is a rotating machine element which gives reciprocating or oscillating motion to another element known as **follower.** The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc.

Classification of Followers

The followers may be classified as discussed below:

1. According to the surface in contact. The followers, according to the surface in contact, are as follows:

(a) Knife edge follower. When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig. 20.1 (a). The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface). It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

(*b*) *Roller follower*. When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. 20.1 (*b*). Since the rolling motion takes place between the contacting surfaces (*i.e.* the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

(*c*) *Flat faced or mushroom follower*. When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. 20.1 (*c*). It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers. The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axis of the follower, as shown in Fig. 20.1 (*f*) so that when the cam rotates, the follower also rotates about its own axis. The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

Note : When the flat faced follower is circular, it is then called a mushroom follower.

(*d*) *Spherical faced follower*. When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 20.1 (*d*). It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced. In order to minimise these stresses, the flat end of the follower is machined to a spherical shape.

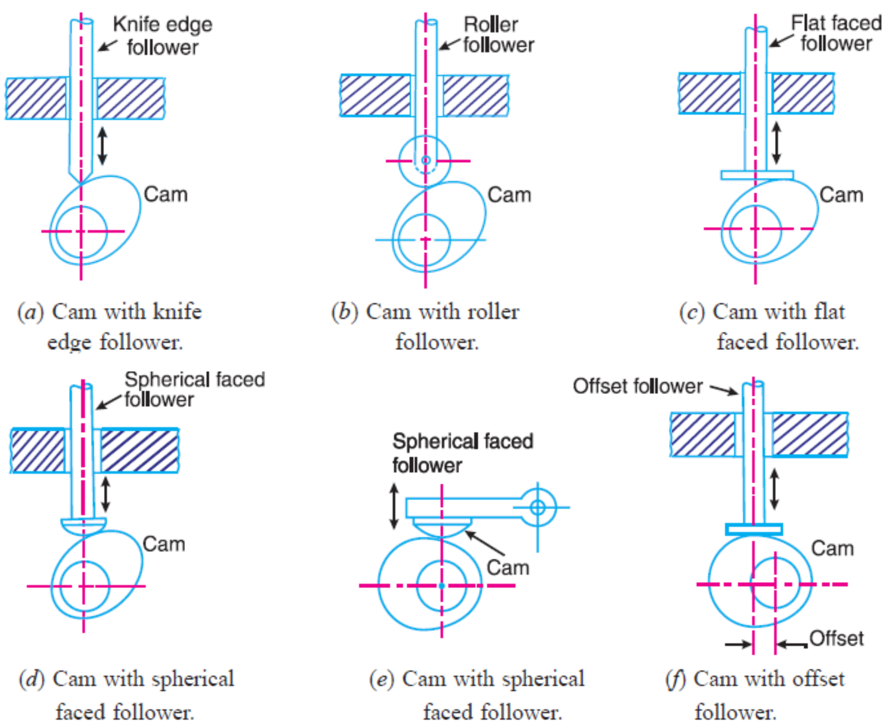


Fig. 20.1. Classification of followers.

2. *According to the motion of the follower*. The followers, according to its motion, are of the Following two types:

(*a*) *Reciprocating or translating follower*. When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown in Fig. 20.1 (*a*) to (*d*) are all reciprocating or translating followers.

(*b*) *Oscillating or rotating follower*. When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower. The follower, as shown in Fig 20.1 (*e*), is an oscillating or rotating follower.

3. *According to the path of motion of the follower*. The followers, according to its path of motion, are of the following two types:

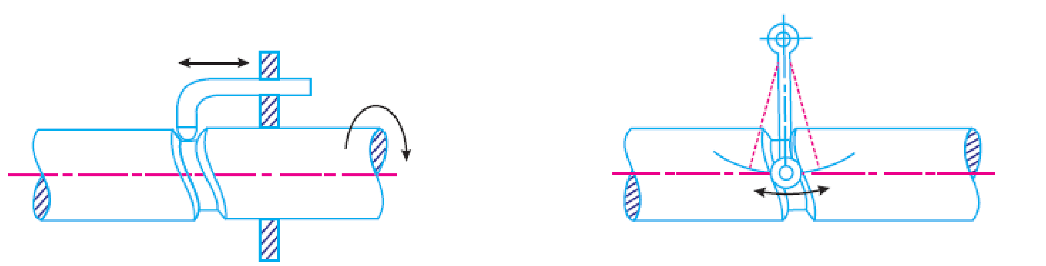
(*a*) *Radial follower*. When the motion of the follower is along an axis passing through the center of the cam, it is known as radial follower. The followers, as shown in Fig. 20.1 (*a*) to (*e*), are all radial followers.

(*b*) *Off-set follower*. When the motion of the follower is along an axis away from the axis of the cam center, it is called off-set follower. The follower, as shown in Fig. 20.1 (*f*), is an off-set follower.

Note: In all cases, the follower must be constrained to follow the cam. This may be done by springs, gravity or hydraulic means. In some types of cams, the follower may ride in a groove.

Classification of Cams

Though the cams may be classified in many ways, yet the following two types are important from the subject point of view



(*a*) Cylindrical cam with reciprocating follower.

(*b*) Cylindrical cam with oscillating follower.

Fig. 20.2. Cylindrical cam.

1. *Radial or disc cam*. In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in Fig. 20.1 are all radial cams.

2. *Cylindrical cam*. In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. 20.2 (*a*) and (*b*) respectively.

Note: In actual practice, radial cams are widely used. Therefore our discussion will be only confined to radial cams.

Terms Used in Radial Cams

Fig. 20.3 shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. *Base circle.* It is the smallest circle that can be drawn to the cam profile.

2. *Trace point.* It is a reference point on the follower and is used to generate the *pitch curve*. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the center of the roller represents the trace point.

3. *Pressure angle.* It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. *Pitch point.* It is a point on the pitch curve having the maximum pressure angle.

5. *Pitch circle.* It is a circle drawn from the center of the cam through the pitch points.

6. *Pitch curve.* It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

7. *Prime circle.* It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

8. *Lift or stroke.* It is the maximum travel of the follower from its lowest position to the topmost position.

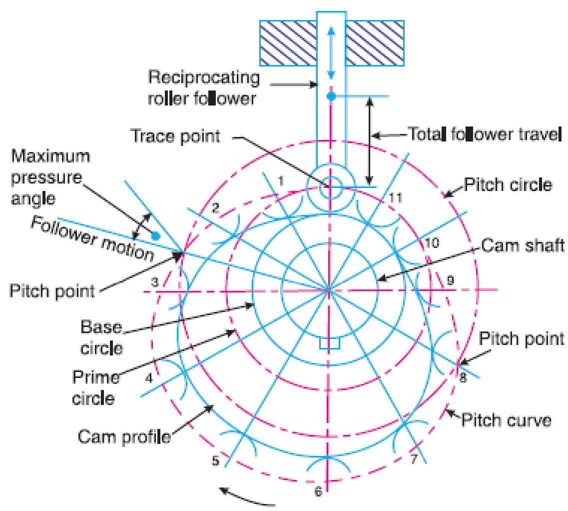


Fig. 20.3. Terms used in radial cams.

Motion of the Follower

The follower, during its travel, may have one of the following motions.

1. Uniform velocity,

2. Simple harmonic motion,

3. Uniform acceleration and retardation, and

4. Cycloidal motion.

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 20.6 (*a*), (*b*) and (*c*) respectively. The displacement diagram is drawn as follows:

1. Draw a semi-circle on the follower stroke as diameter.

2. Divide the semi-circle into any number of even equal parts (say eight).

3. Divide the angular displacements of the cam during out stroke and return stroke into the same number of equal parts.

4. The displacement diagram is obtained by projecting the points as shown in Fig. 20.6 (*a*).

The velocity and acceleration diagrams are shown in Fig. 20.6 (*b*) and (*c*) respectively. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve. We see from Fig. 20.6 (*b*) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.

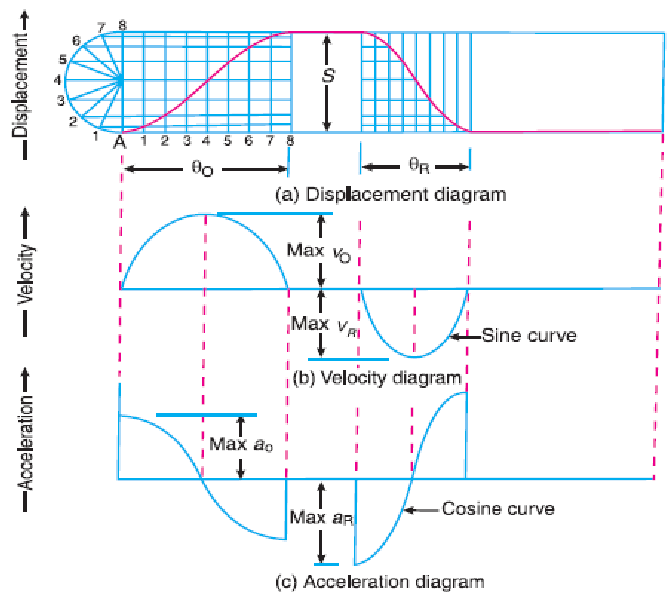


Fig. 20.6. Displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion.

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

∴Time required for the out stroke of the follower in seconds,

$t_O = \theta_O / \omega$

Consider a point *P* moving at a uniform speed ω_P radians per sec round the circumference of a circle with the stroke *S* as diameter, as shown in Fig. 20.7. The point *P′* (which is the projection of a point *P* on the diameter) executes a simple harmonic motion as the point *P* rotates. The motion of the follower is similar to that of point *P′*.

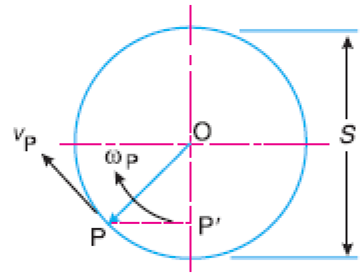


Fig. 20.7. Motion of a point.

∴ Peripheral speed of the point *P′*,

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$

and maximum velocity of the follower on the outstroke,

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega . S}{2\theta_O}$$

We know that the centripetal acceleration of the point *P*,

$$a_P = \frac{(v_P)^2}{OP} = \left(\frac{\pi \omega . S}{2\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

∴ Maximum acceleration of the follower on the outstroke,

$$a_O = a_P = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \omega . S}{2\theta_R}$$

and maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2}$$

Example 20.2. *A cam is to be designed for a knife edge follower with the following data* :

1. *Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.*

2. *Dwell for the next 30°.*

3. *During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.*

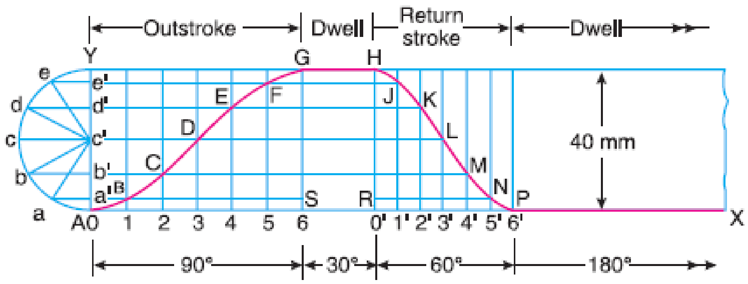
4. *Dwell during the remaining 180°.*

Draw the profile of the cam when

(a) *the line of stroke of the follower passes through the axis of the cam shaft, and*

(b) *the line of stroke is offset 20 mm from the axis of the cam shaft.*

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.



First of all, the displacement diagram, as shown in Fig 20.13, is drawn as discussed in the following steps :

1. Draw horizontal line AX = 360° to some suitable scale. On this line, mark AS = 90° to represent out stroke ; SR = 30° to represent dwell ; RP = 60° to represent return stroke and PX = 180° to represent dwell.

2. Draw vertical line AY = 40 mm to represent the cam lift or stroke of the follower and complete the rectangle as shown in Fig. 20.13.

3. Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.

4. Since the follower moves with simple harmonic motion, therefore draw a semicircle with AY as diameter and divide into six equal parts.

5. From points a, b, c ... etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and 0′ ,1′ , 2′ ...etc. at B, C, D ... M, N, P.

6. Join the points A, B, C ... etc. with a smooth curve as shown in Fig. 20.13. This is the required displacement diagram.

(a) Profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft

The profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft, as shown in Fig. 20.14, is drawn in the similar way as is discussed in Example 20.1

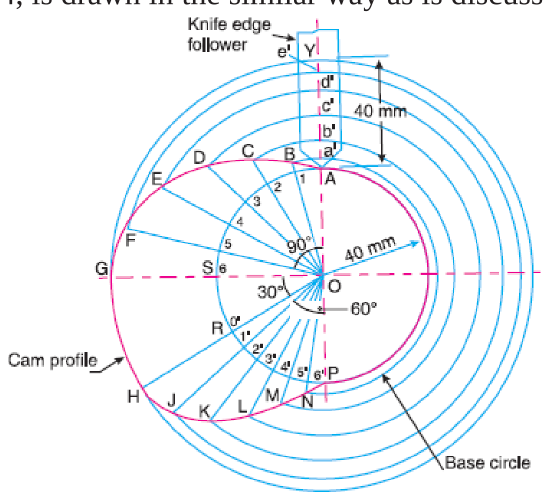


Fig. 20.14

(b) Profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft

The profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft, as shown

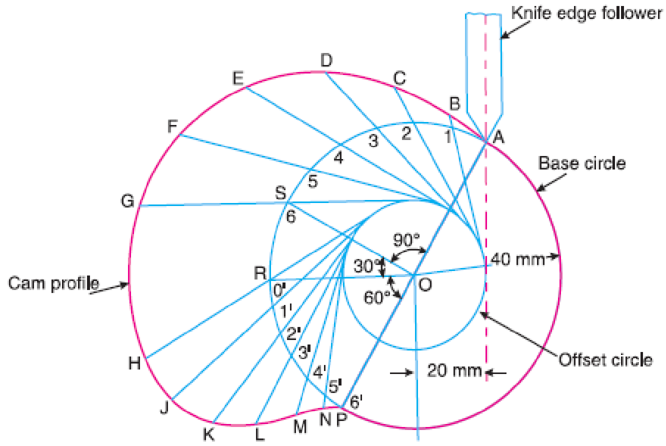


Fig. 20.15

Maximum velocity of the follower during its ascent and descent

We know that angular velocity of the cam

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

We also know that the maximum velocity of the follower during its ascent,

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s } \textbf{Ans.}$$

and maximum velocity of the follower during its descent,

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s } \textbf{Ans.}$$

Maximum acceleration of the follower during its ascent and descent

We know that the maximum acceleration of the follower during its ascent,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2 \textbf{ Ans.}$$

and maximum acceleration of the follower during its descent,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2 \textbf{ Ans.}$$

Gear Trains

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called **gear train** or **train of toothed wheels.** The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

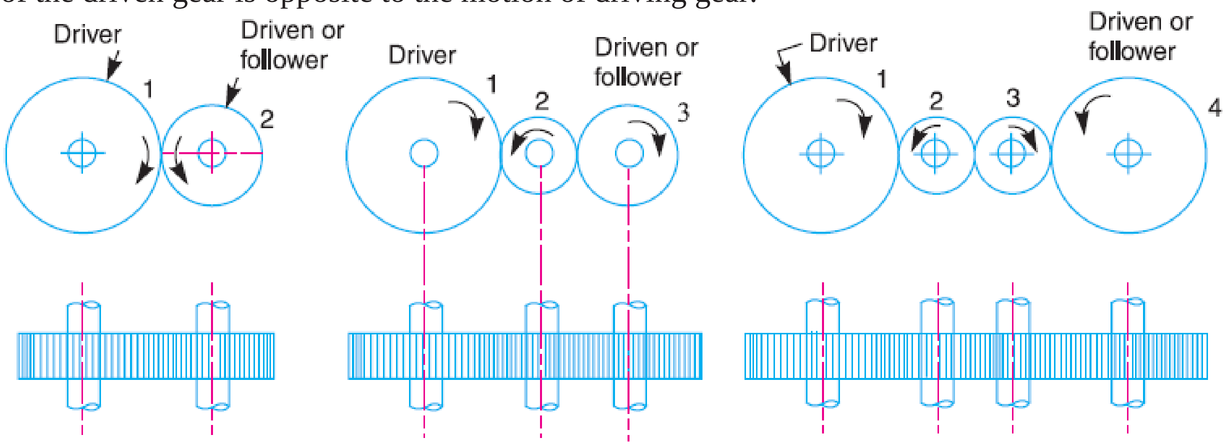
Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train, **2.** Compound gear train, **3.** Reverted gear train, and **4.** Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

Simple Gear Train

When there is only one gear on each shaft, as shown in Fig. 13.1, it is known as ***simple gear train.*** The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. 13.1 (*a*). Since the gear 1 drives the gear 2, therefore gear 1 is called the ***driver*** and the gear 2 is called the ***driven*** or ***follower.*** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.



Simple gear train.

Let $N1$ = Speed of gear 1(or driver) in r.p.m.,

$N2$ = Speed of gear 2 (or driven or follower) in r.p.m.,

$T1$ = Number of teeth on gear 1, and

$T2$ = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or **2.** By providing one or more intermediate gears. A little consideration will show that the former method (*i.e.* providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (*i.e.* providing one or more intermediate gear) is very convenient and economical. It may be noted that when the number of intermediate gears are ***odd,*** the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in Fig. 13.1 (*b*). But if the number of intermediate gears are ***even,*** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. 13.1 (*c*). Now consider a simple train of gears with one intermediate gear as shown in Fig. 13.1 (*b*).

Let

$N1$ = Speed of driver in r.p.m.,

$N2$ = Speed of intermediate gear in r.p.m.

$N3$ = Speed of driven or follower in r.p.m.,
$T1$ = Number of teeth on driver,
$T2$ = Number of teeth on intermediate gear, and
$T3$ = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2}$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations ***(i)*** and ***(ii)*.**

$$\therefore \qquad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \qquad \text{or} \qquad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

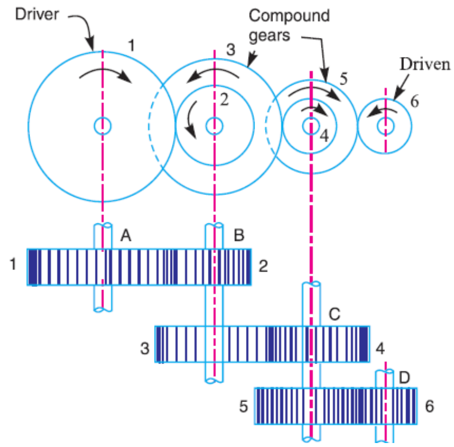
Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears*,** as they do not affect the speed ratio or train value of the system. The idle gears are used for the following two purposes:

1. To connect gears where a large center distance is required, and
2. To obtain the desired direction of motion of the driven gear (*i.e*. clockwise or anticlockwise).

Compound Gear Train

When there are more than one gear on a shaft, as shown in Fig. 13.2, it is called a ***compound train of gear*.** We have seen in Art. 13.3 that the idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.below



Compound gear train

In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft *A*, gears 2 and 3 are compound gears which are mounted on shaft *B*. The gears 4 and 5 are also compound gears which are mounted on shaft *C* and the gear 6 is the driven gear mounted on shaft *D*.

Let $N1$ = Speed of driving gear 1,

$T1$ = Number of teeth on driving gear 1,

$N2$, $N3$ ..., $N6$ = Speed of respective gears in r.p.m., and

$T2$, $T3$..., $T6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{...}(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{...}(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \text{...}(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

* Since gears 2 and 3 are mounted on one shaft *B*, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft *C*, therefore $N_4 = N_5$.

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}} = \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

and

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}} = \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Note: The gears which mesh must have the same circular pitch or module. Thus gears 1 and 2 must have the same module as they mesh together. Similarly gears 3 and 4, and gears 5 and 6 must have the same module.

Design of Spur Gears

Sometimes, the spur gears (*i.e.* driver and driven) are to be designed for the given velocity ratio and distance between the centers of their shafts.

Let x = Distance between the centers of two shafts,

$N1$ = Speed of the driver,

$T1$ = Number of teeth on the driver,

$d1$ = Pitch circle diameter of the driver,

$N2$, $T2$ and $d2$ = Corresponding values for the driven or follower, and

pc = Circular pitch.

We know that the distance between the centers of two shafts,

$$x = \frac{d_1 + d_2}{2} \quad \text{...}(i)$$

and speed ratio or velocity ratio,

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1} \quad \text{...}(ii)$$

From the above equations, we can conveniently find out the values of d_1 and d_2 (or T_1 and T_2) and the circular pitch (pc). The values of $T1$ and $T2$, as obtained above, may or may not be whole numbers. But in a gear since the number of its teeth is always a whole number, therefore a slight alterations must be made in the values of x, $d1$ and $d2$, so that the number of teeth in the two gears may be a complete number.

Flywheel

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply. In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed; it simply reduces the fluctuation of speed. In other words, ***a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.***

Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed***. The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed***.

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

∴ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \qquad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \qquad \text{...(In terms of linear speeds)}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed.

Note. The reciprocal of the coefficient of fluctuation of speed is known as ***coefficient of steadiness*** and is denoted by *m*.

$$m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

Energy Stored in a Flywheel

A flywheel is shown in Fig. 16.5. We have discussed in Art. 16.5 that when a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in meters,

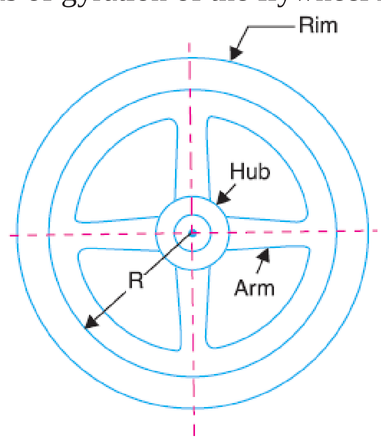


Fig. 16.5. Flywheel.

I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m^2 = $m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad/s,

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad/s $= \dfrac{\omega_1 + \omega_2}{2}$,

C_S = Coefficient of fluctuation of speed, $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \quad \text{(in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.}$$

$$= \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2 = \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} \times I(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \quad \text{...}(i)$$

$$\text{...}\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right)$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right) \quad \text{... (Multiplying and dividing by } \omega)$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \quad \text{... } (\because I = m.k^2) \quad \text{...}(ii)$$

$$= 2.E.C_S \text{ (in N–m or joules)} \quad \text{...}\left(\because E = \frac{1}{2} \times I.\omega^2\right) \quad \text{...}(iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore, substituting $k = R$, in equation **(*ii*)**, we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

where v = Mean linear velocity (*i.e.* at the mean radius) in m/s = $\omega.R$

Notes. 1. Since $\omega = 2\pi N/60$, therefore equation (*i*) may be written as

$$\Delta E = I \times \frac{2\pi N}{60}\left(\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right) = \frac{4\pi^2}{3600} \times I \times N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m.k^2.N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m.k^2.N^2.C_S \quad \text{...}\left(\because C_S = \frac{N_1 - N_2}{N}\right)$$

2. In the above expressions, only the mass moment of inertia of the flywheel rim (I) is considered and the mass moment of inertia of the hub and arms is neglected. This is due to the fact that the major portion of the mass of the flywheel is in the rim and a small portion is in the hub and arms. Also the hub and arms are nearer to the axis of rotation, therefore the mass moment of inertia of the hub and arms is small.

Example;- *The mass of flywheel of an engine is 6.5 tonnes and the radius of gyration is 1.8 metres. It is found from the turning moment diagram that the fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m., find the maximum and minimum speeds.*

Solution. Given : m = 6.5 t = 6500 kg ; k = 1.8 m ; ΔE = 56 kN-m = 56×10^3 N-m ; N = 120 r.p.m.

Let N_1 and N_2 = Maximum and minimum speeds respectively.

We know that fluctuation of energy (ΔE),

$$56 \times 10^3 = \frac{\pi^2}{900} \times m.k^2 \,.\, N (N_1 - N_2) = \frac{\pi^2}{900} \times 6500\,(1.8)^2\,120\,(N_1 - N_2)$$

$$= 27\,715\,(N_1 - N_2)$$

$$\therefore \quad N_1 - N_2 = 56 \times 10^3 / 27\,715 = 2 \text{ r.p.m.} \quad \text{...}(i)$$

We also know that mean speed (N),

$$120 = \frac{N_1 + N_2}{2} \text{ or } N_1 + N_2 = 120 \times 2 = 240 \text{ r.p.m.} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*),

N_1 = 121 r.p.m., and N_2 = 119 r.p.m. **Ans.**