CHAPTER THREE

Design for fatigue strength

Cyclic loadings

Cyclic loading is the application of repeated or fluctuating stresses, strains, or stress intensities to locations on structural components. The highest stress that a material can withstand for a given number of cycles without breaking called also endurance **strength**. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses.

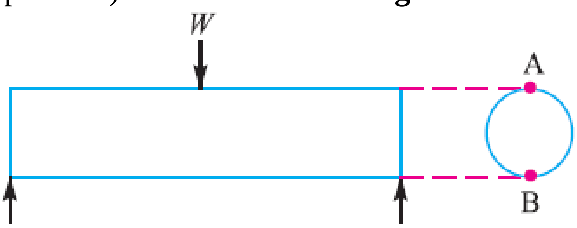
Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load W, as shown in Fig. 6.1. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (i.e. at point A) are under compressive stress and the lower fibres (i.e. at point B) are under tensile stress. After half a revolution, the point B occupies the position of point A and the point A occupies the position of point B. Thus the point B is now under compressive stress and the point A under tensile stress. The speed of variation of these stresses depends upon the speed of the beam. From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or **vice versa,** are known as **completely reversed** or **cyclic stresses.**

Notes: 1. The stresses which vary from a minimum value to a maximum value of the same nature, (i.e. tensile or compressive) are called **fluctuating stresses.**

2. The stresses which vary from zero to a certain maximum value are called **repeated stresses.**

3. The stresses which vary from a minimum value to a maximum value of the opposite nature (i.e. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called **alternating stresses.**



Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

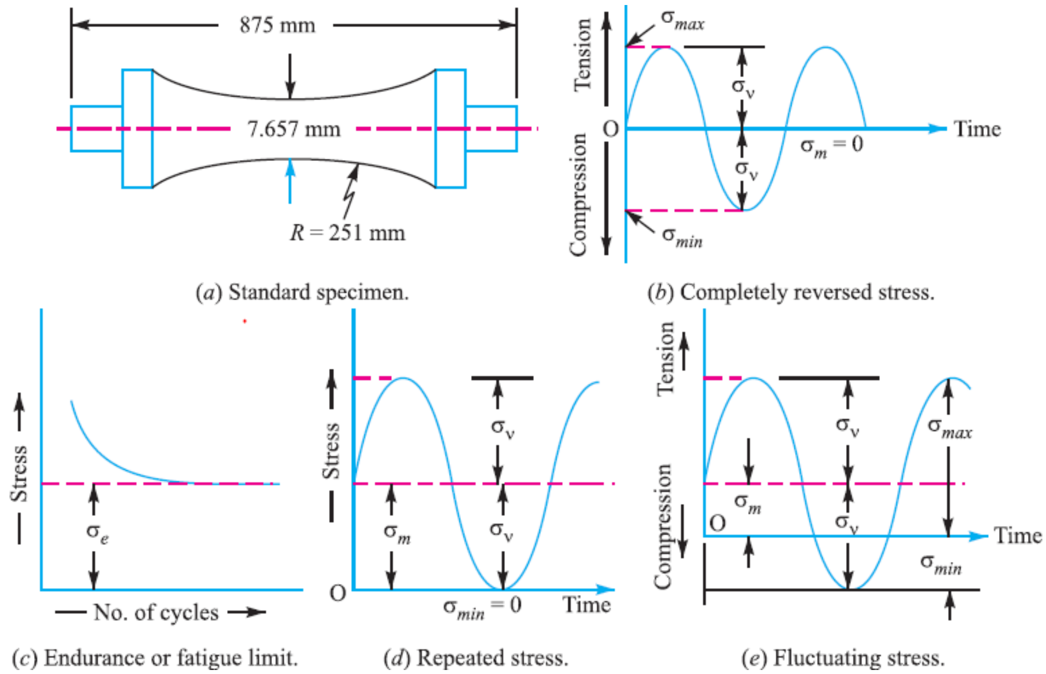


Fig. 6.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 6.2 (a), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 6.2 (b). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig.6.2 (c). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 6.2 (c), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as **endurance** or **fatigue limit** (σe). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 107 cycles). It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term **endurance strength** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions. We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 6.2 (b). In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress **verses** time diagram for fluctuating stress having values σmin and σmax is shown in Fig. 6.2 (e). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σv. The following relations are derived from Fig. 6.2 (e):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Note: For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma_{min} = 0$) in each cycle as shown in Fig. 6.2 (*d*).

$$\therefore \quad \sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \frac{\sigma_{max}}{\sigma_{min}}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

where σ'_e = Endurance limit for any stress range represented by R.
σ_e = Endurance limit for completely reversed stresses, and
R = Stress ratio.

Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e.K_b = \sigma_e$ ...($\because K_b = 1$)
Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e.K_a$ and
Endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e.K_s$

Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 6.3 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

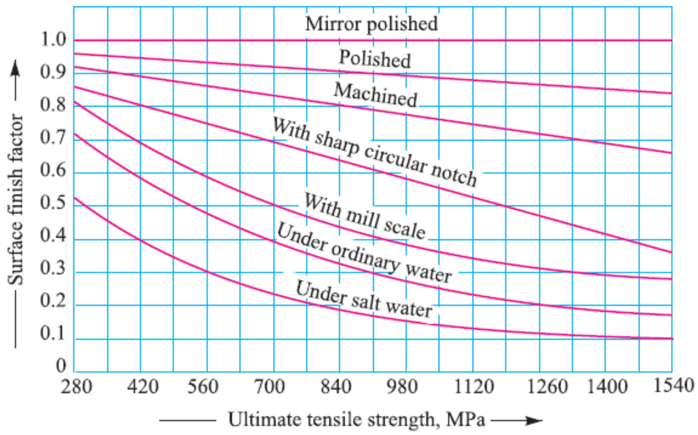


Fig. 6.3. Surface finish factor for various surface conditions.

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur} \quad ...(\because K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad ...\text{(For reversed axial load)}$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad ...\text{(For reversed torsional or shear load)}$$

Note : The surface finish factor for non-ferrous metals may be taken as unity.

Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig. 6.2 (a) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

∴ Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad ...\text{(Considering surface finish factor also)}$$

$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$

$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad ...\text{(For reversed axial load)}$$

$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad ...\text{(For reversed torsional or shear load)}$$

Notes:

1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (Ksur), size factor (Ksz) and load factors Kb, Ka and Ks, there are many other factors such as reliability factor (Kr), temperature factor (Kt), impact factor

(Ki) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions :

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σe) of a material subjected to fatigue loading is a function of ultimate tensile strength (σu). Fig. 6.4 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice :

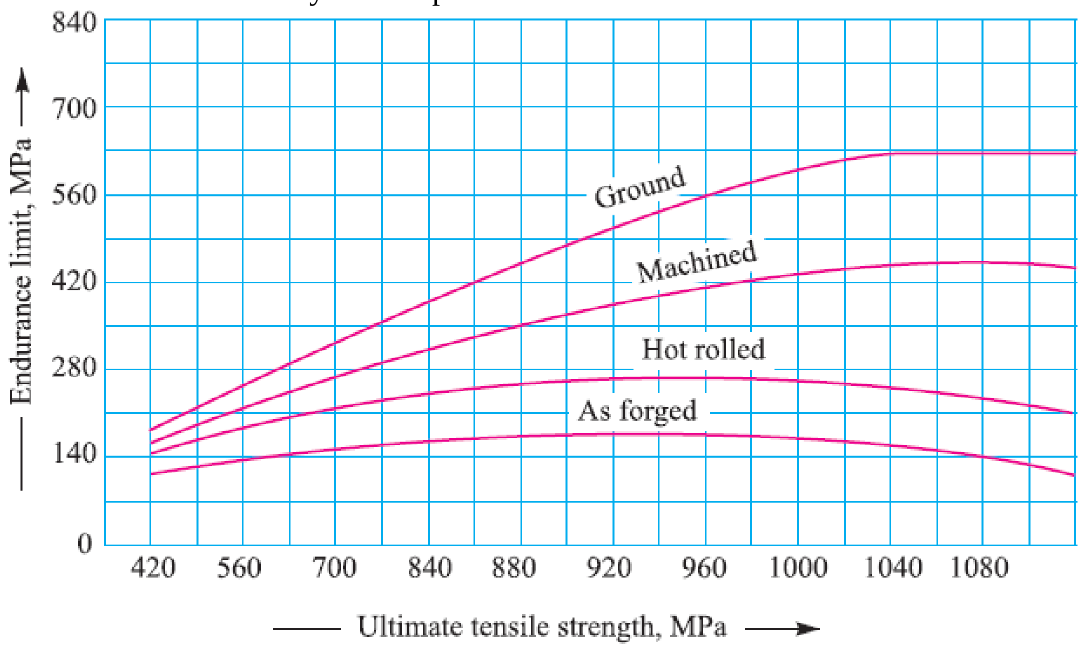


Fig. 6.4. Endurance limit of steel corresponding to ultimate tensile strength.

For steel, σe = 0.5 σu ;

For cast steel, σe = 0.4 σu ;

For cast iron, σe = 0.35 σu ;

For non-ferrous metals and alloys, σe = 0.3 σu

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for faliure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

Note: For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$

where σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Example Determine the design stress for a piston rod where the load is completely reversed. The surface of the rod is ground and the surface finish factor is 0.9. There is no stress concentration. The load is predictable and the factor of safety is 2.

Solution. Given : Ksur = 0.9 ; F.S. = 2 The piston rod is subjected to reversed axial loading. We know that for reversed axial loading, the load correction factor (Ka) is 0.8

If σ_e is the endurance limit for reversed bending load, then endurance limit for reversed axial load,

$$\sigma_{ea} = \sigma_e \times K_a \times K_{sur} = \sigma_e \times 0.8 \times 0.9 = 0.72\ \sigma_e$$

We know that design stress,

$$\sigma_d = \frac{\sigma_{ea}}{F.S.} = \frac{0.72\ \sigma_e}{2} = 0.36\ \sigma_e \textbf{ Ans.}$$

Fatigue Stress Concentration Factor

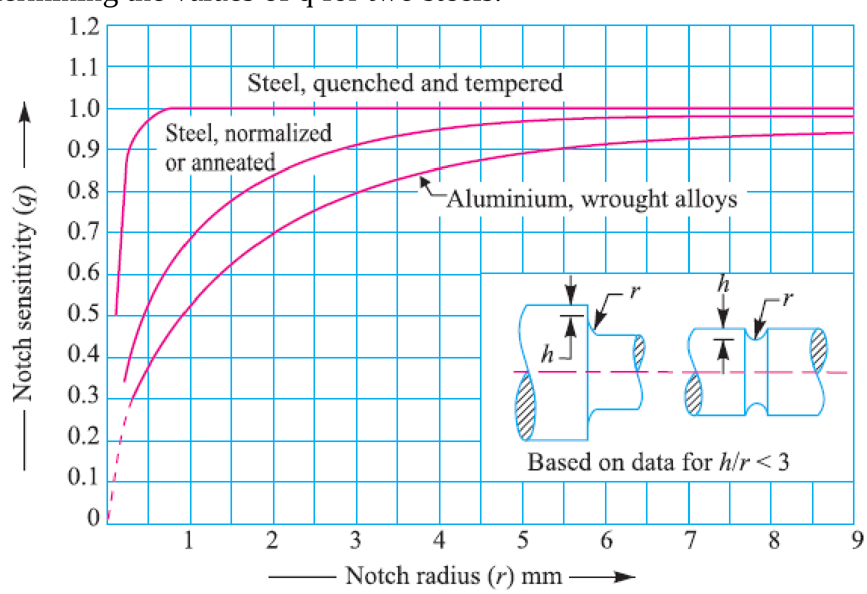
When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term **notch sensitivity** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig. 6.14, may be used for determining the values of q for two steels.



When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

or $$K_f = 1 + q\,(K_t - 1)$$ ...[For tensile or bending stress]

and $$K_{fs} = 1 + q\,(K_{ts} - 1)$$ ...[For shear stress]

Where Kt = Theoretical stress concentration factor for axial or bending loading, and
Kts = Theoretical stress concentration factor for torsional or shear loading.

<u>Combined Steady and Variable Stress</u>

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. 6.15 as functions of variable stress (σv) and mean stress (σm). The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative).Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

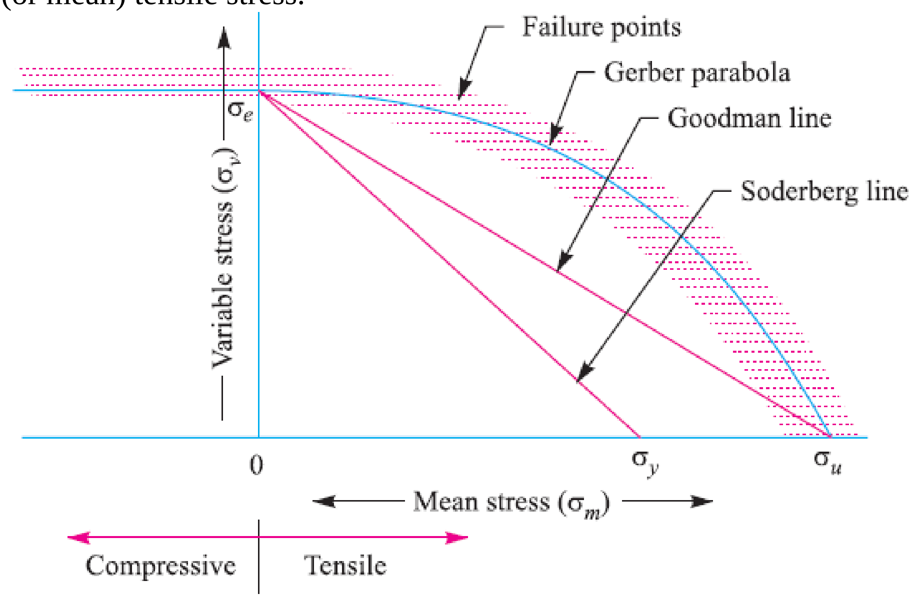


Fig. 6.15. Combined mean and variable stress.

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

1. Gerber method,
2. Goodman method, and
3. Soderberg method.

We shall now discuss these methods, in detail, in the following pages.

1. Gerber Method for Combination of Stresses

The relationship between variable stress (σv) and mean stress (σm) for axial and bending loading for ductile materials are shown in Fig. 6.15. The point σe represents the fatigue strength corresponding to the case of complete reversal (σm = 0) and the point σu represents the static ultimate strength corresponding to σv = 0. A parabolic curve drawn between the endurance limit (σe) and ultimate tensile strength (σu) was proposed by Gerber in 1874. Generally, the test data for ductile material fall closer to Gerber parabola as shown in Fig. 6.15, but because of scatter in the test points, a straight line relationship (i.e.Goodman line and Soderberg line) is usually preferred in designing machine parts.

According to Gerber, variable stress,

$$\sigma_v = \sigma_e \left[\frac{1}{F.S.} - \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. \right]$$

or
$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_u = Ultimate stress (tensile or compressive), and

σ_e = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor (K_f), the equation (*i*) may be written as

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}$$

6.20 Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line *AB* in Fig. 6.16, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

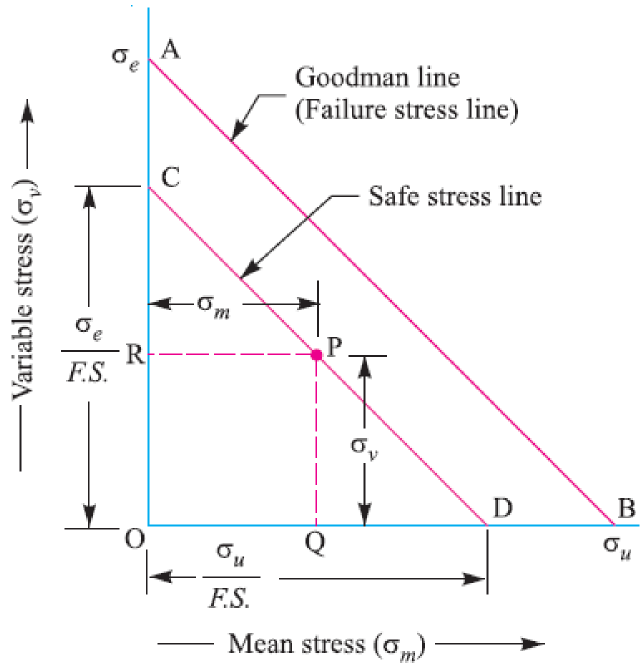


Fig. 6.16. Goodman method.

In Fig. 6.16, line *AB* connecting σ_e and σ_u is called **Goodman's failure stress line.** If a suitable factor of safety (F.S.) is applied to endurance limit and ultimate strength, a safe stress line CD may be drawn parallel to the line AB. Let us consider a design point P on the line CD. Now from similar triangles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad ...(\because QD = OD - OQ)$$

$$\therefore \qquad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.} \left[1 - \frac{\sigma_m}{\sigma_u / F.S.} \right] = \sigma_e \left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u} \right]$$

or
$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads. Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (i) must be altered to include this effect. In such cases, the fatigue stress concentration factor (Kf) is used to multiply the variable stress (σ_v). The equation **(i)** may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \quad \text{...}(ii)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation *(ii)* may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \quad \text{...}(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \quad \text{...}(\because\ \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

where K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

* Here we have assumed the same factor of safety ($F.S.$) for the ultimate tensile strength (σ_u) and endurance limit (σ_e). In case the factor of safety relating to both these stresses is different, then the following relation may be used :

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m}{\sigma_u / (F.S.)_u}$$

where $(F.S.)_e$ = Factor of safety relating to endurance limit, and

$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Notes : 1. The equation **(iii)** is applicable to ductile materials subjected to reversed bending loads (tensile or compressive). For brittle materials, the theoretical stress concentration factor (Kt) should be applied to the mean stress and fatigue stress concentration factor (Kf) to the variable stress. Thus for brittle materials, the equation **(iii)** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...}(iv)$$

2. When a machine component is subjected to a load other than reversed bending, then the endurance limit for that type of loading should be taken into consideration. Thus for reversed axial loading (tensile or compressive), the equations **(iii)** and **(iv)** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$ ...(For ductile materials)

and $$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$ ...(For brittle materials)

Similarly, for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$ ...(For ductile materials)

and $$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$ ...(For brittle materials)

where suffix '*s*' denotes for shear.

For reversed torsional or shear loading, the values of ultimate shear strength (τ_u) and endurance shear strength (τ_e) may be taken as follows:

$$\tau_u = 0.8\ \sigma_u;\ \text{and}\ \tau_e = 0.8\ \sigma_e$$

Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σe) and the yield strength (σy), as shown by the line AB in Fig. 6.17, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.

Proceeding in the same way as discussed in Art 6.20, the line *AB* connecting σ_e and σ_y, as shown in Fig. 6.17, is called ***Soderberg's failure stress line***. If a suitable factor of safety (*F.S.*) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*. Now from similar triangles *COD* and *PQD*,

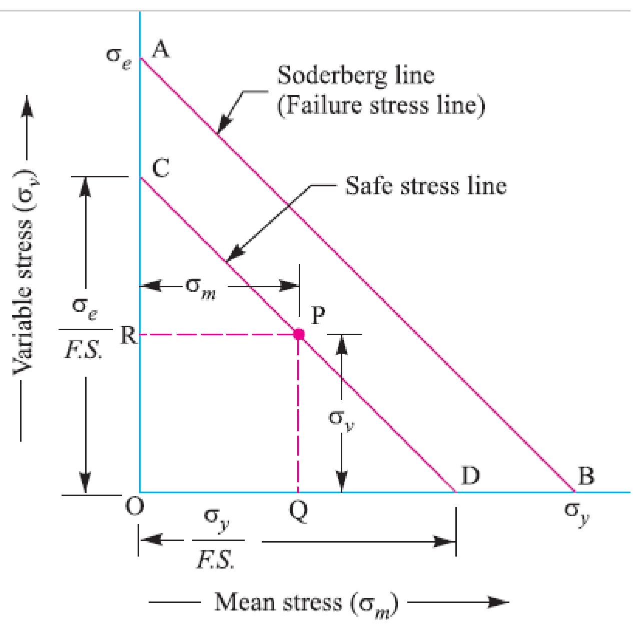


Fig. 6.17. Soderberg method.

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD}$$

...($\because$ $QD = OD - OQ$)

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or $$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$ ...(*i*)

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (Kf) should be applied to only variable stress (σ_v). Thus the equations **(i)** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

Considering the load factor, surface finish factor and size factor, the equation *(ii)* may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

Notes: 1. The Soderberg method is particularly used for ductile materials. The equation *(iii)* is applicable to ductile materials subjected to reversed bending load (tensile or compressive).

2. When a machine component is subjected to reversed axial loading, then the equation *(iii)* may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

3. When a machine component is subjected to reversed shear loading, then equation *(iii)* may be written as

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

where K_{fs} is the fatigue stress concentration factor for reversed shear loading. The yield strength in shear (τ_y) may be taken as one-half the yield strength in reversed bending (σ_y).

Example . A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m2 and – 150 MN/m2. Determine the value of minimum ultimate strength according to 1. Gerber relation; 2. Modified Goodman relation; and 3. Soderberg relation. Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2.

Solution. Given : σ1 = 300 MN/m2 ;
σ2 = – 150 MN/m2 ; σy = 0.55 σu ; σe = 0.5 σu ; F.S. = 2

Let σ_u = Minimum ultimate strength in MN/m^2.

We know that the mean or average stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{300 + (-150)}{2} = 75 \text{ MN/m}^2$$

and variable stress,

$$\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

1. ***According to Gerber relation***

We know that according to Gerber relation,

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u} = \frac{11\,250}{(\sigma_u)^2} + \frac{450}{\sigma_u} = \frac{11\,250 + 450\,\sigma_u}{(\sigma_u)^2}$$

$$(\sigma_u)^2 = 22\,500 + 900\,\sigma_u$$

or $$(\sigma_u)^2 - 900\,\sigma_u - 22\,500 = 0$$

$$\therefore \quad \sigma_u = \frac{900 \pm \sqrt{(900)^2 + 4 \times 1 \times 22\,500}}{2 \times 1} = \frac{900 \pm 948.7}{2}$$

$= 924.35$ MN/m^2 **Ans.** ...(Taking +ve sign)

2. *According to modified Goodman relation*

We know that according to modified Goodman relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

or
$$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\ \sigma_u} = \frac{525}{\sigma_u}$$

$\therefore$
$$\sigma_u = 2 \times 525 = 1050 \text{ MN/m}^2 \quad \textbf{Ans.}$$

3. *According to Soderberg relation*

We know that according to Soderberg relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

or
$$\frac{1}{2} = \frac{75}{0.55\ \sigma_u} + \frac{255}{0.5\ \sigma_u} = \frac{586.36}{\sigma_u}$$

$\therefore$
$$\sigma_u = 2 \times 586.36 = 1172.72 \text{ MN/m}^2 \quad \textbf{Ans.}$$