Chapter 5

Operational Amplifiers

5.1 Introduction

An operational amplifier is designed so that it performs some mathematical operations when external components, such as resistors and capacitors, are connected to its terminals. Thus,

> An op amp is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

The op amp is an electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes.

Op amps are commercially available in integrated circuit packages in several forms. A typical one is the eight-pin dual in-line package (or DIP), shown in Fig. 5.2(a). Pin or terminal 8 is unused, and terminals 1 and 5 are of little concern to us. The five important terminals are:

1. The inverting input, pin 2.
2. The noninverting input, pin 3.
3. The output, pin 6.
4. The positive power supply V+, pin 7.
5. The negative power supply V−, pin 4.

The circuit symbol for the op amp is the triangle in Fig. 5.2(b); as shown, the op amp has two inputs and one output. The inputs are marked with minus (−) and plus (+) to specify inverting and noninverting inputs, respectively. An input applied to the noninverting terminal will appear with the same polarity at the output, while an input applied to the inverting terminal will appear inverted at the output.

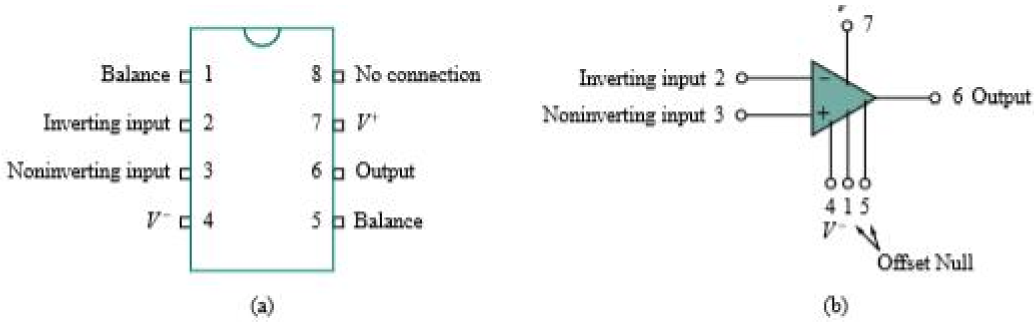


Figure 5.2 A typical op amp: (a) pin configuration, (b) circuit symbol.

5.2 Ideal OP AMP

To facilitate the understanding of op amp circuits, we will assume ideal op amps. An op amp is ideal if it has the following characteristics:

1. Infinite open-loop gain, A ≈ ∞.

2. Infinite input resistance, Ri ≈ ∞.

3. Zero output resistance, Ro ≈ 0.

5.3 Inverting Amplifier

In this and the following sections, we consider some useful op amp circuits that often serve as modules for designing more complex circuits. The first of such op amp circuits is the inverting amplifier shown in Fig.5.10. In this circuit, the noninverting input is grounded, vi is connected to the inverting input through R1, and the feedback resistor Rf is connected between the inverting input and output. Our goal is to obtain the relationship between the input voltage vi and the output voltage vo. Applying KCL at node 1,

$$i_1 = i_2 \qquad \Longrightarrow \qquad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

But $v_1 = v_2 = 0$ for an ideal op amp, since the noninverting terminal is grounded. Hence,

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f}$$

or

$$v_o = -\frac{R_f}{R_1} v_i$$

The voltage gain is $A_v = v_o/v_i = -R_f/R_1$. The designation of the circuit in Fig. 5.10 as an *inverter* arises from the negative sign. Thus,

An inverting amplifier reverses the polarity of the input signal while amplifying it.

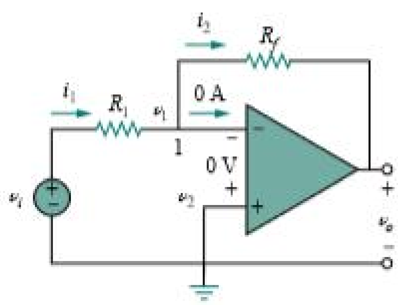


Figure 5.10 The inverting amplifier.

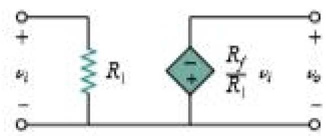


Figure 5.11 An equivalent circuit for the inverter in Fig. 5.10.

Example 1:

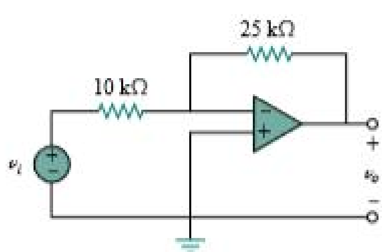


Figure 5.12 For Example 5.3.

Refer to the op amp in Fig. 5.12. If $v_i = 0.5$ V, calculate: (a) the output voltage v_o, and (b) the current in the 10 kΩ resistor.

Solution:

(a) Using Eq.

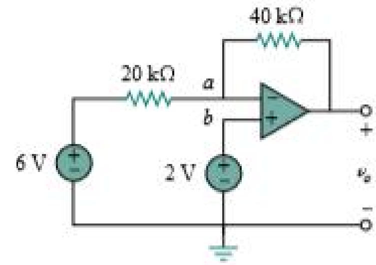
$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

(b) The current through the 10-kΩ resistor is

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50\ \mu\text{A}$$

Example 2:



Determine v_o in the op amp circuit

Solution:

Applying KCL at node a,

$$\frac{v_a - v_o}{40} = \frac{6 - v_a}{20}$$

$$v_a - v_o = 12 - 2v_a \quad \Longrightarrow \quad v_o = 3v_a - 12$$

But $v_a = v_b = 2$ V for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence,

$$v_o = 6 - 12 = -6 \text{ V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$

5.4 Noninverting Amplifier

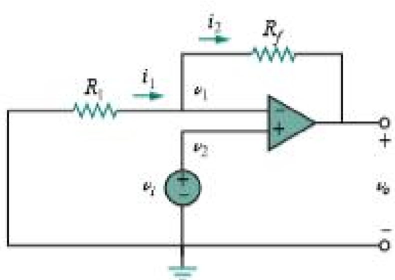


Figure 5.16 The noninverting amplifier.

Another important application of the op amp is the noninverting amplifier shown in Fig. 5.16. In this case, the input voltage v_i is applied directly at the noninverting input terminal, and resistor R_1 is connected between the ground and the inverting terminal. We are interested in the output voltage and the voltage gain. Application of KCL at the inverting terminal gives

$$i_1 = i_2 \quad \Longrightarrow \quad \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

But $v_1 = v_2 = v_i$.

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f}$$

or

$$\boxed{v_o = \left(1 + \frac{R_f}{R_1}\right) v_i}$$

The voltage gain is $A_v = v_o/v_i = 1 + R_f/R_1$, which does not have a negative sign. Thus, the output has the same polarity as the input.

A **noninverting amplifier** is an op amp circuit designed to provide a positive voltage gain.

Again we notice that the gain depends only on the external resistors.

Notice that if feedback resistor $R_f = 0$ (short circuit) or $R_1 = \infty$ (open circuit) or both, the gain becomes 1. Under these conditions ($R_f = 0$ and $R_1 = \infty$), the circuit in Fig. 5.16 becomes that shown in Fig. 5.17, which is called a *voltage follower* (or *unity gain amplifier*) because the output follows the input. Thus, for a voltage follower

$$v_o = v_i$$

Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another, as portrayed in Fig. 5.18. The voltage follower minimizes interaction between the two stages and eliminates interstage loading.

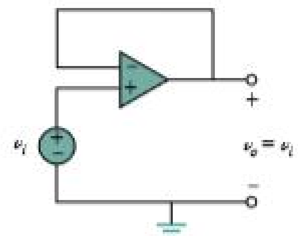


Figure 5.17 The voltage follower.

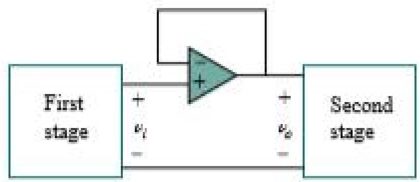


Figure 5.18 A voltage follower used to isolate two cascaded stages of a circuit.

Example 3:

For the op amp circuit in Fig. 5.19, calculate the output voltage v_o.

Solution:

We may solve this in two ways: using superposition and using nodal analysis.

METHOD 1 Using superposition, we let

$$v_o = v_{o1} + v_{o2}$$

where v_{o1} is due to the 6-V voltage source, and v_{o2} is due to the 4-V input. To get v_{o1}, we set the 4-V source equal to zero. Under this condition, the circuit becomes an inverter.

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

To get v_{o2}, we set the 6-V source equal to zero. The circuit becomes a noninverting amplifier

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$

Thus,

$$v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$$

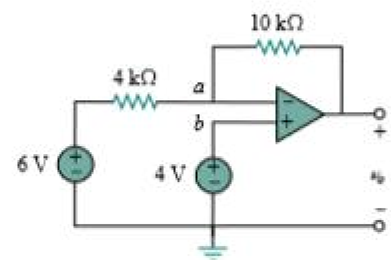


Figure 5.19

METHOD 2 Applying KCL at node a,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

But $v_a = v_b = 4$, and so

$$\frac{6-4}{4} = \frac{4 - v_o}{10} \qquad \Longrightarrow \qquad 5 = 4 - v_o$$

or $v_o = -1$ V, as before.

5.5 Summing Amplifier

Besides amplification, the op amp can perform addition and subtraction. The addition is performed by the summing amplifier covered in this section; the subtraction is performed by the difference amplifier covered in the next section.

> A **summing amplifier** is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.

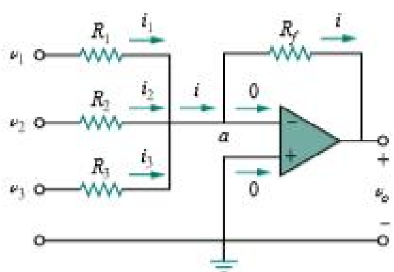


Figure 5.21 The summing amplifier.

The summing amplifier, shown in Fig. 5.21, is a variation of the inverting amplifier. It takes advantage of the fact that the inverting configuration can handle many inputs at the same time. We keep in mind that the current entering each op amp input is zero. Applying KCL at node a gives

$$i = i_1 + i_2 + i_3$$

But

$$i_1 = \frac{v_1 - v_a}{R_1}, \qquad i_2 = \frac{v_2 - v_a}{R_2}$$

$$i_3 = \frac{v_3 - v_a}{R_3}, \qquad i = \frac{v_a - v_o}{R_f}$$

We note that $v_a = 0$

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

indicating that the output voltage is a weighted sum of the inputs. For this reason, the circuit in Fig. 5.21 is called a *summer.*
the summer can have more than three inputs.

Example 4:

Calculate v_o and i_o in the op amp circuit in Fig. 5.22.

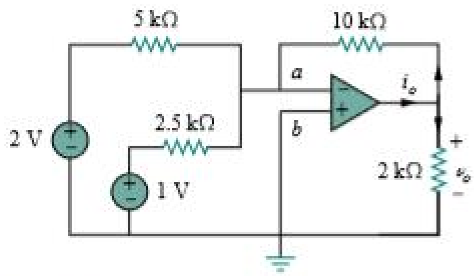


Figure 5.22

Solution:

This is a summer with two inputs.

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8 \text{ V}$$

The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8$ V across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{ mA} = -0.8 - 0.4 = -1.2 \text{ mA}$$

5.6 Differential Amplifier

Difference (or differential) amplifiers are used in various applications where there is need to amplify the difference between two input signals. They are first cousins of the *instrumentation amplifier*, the most useful and popular amplifier

A **difference amplifier** is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

Consider the op amp circuit shown in Fig. 5.24. Keep in mind that zero currents enter the op amp terminals. Applying KCL to node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

or

$$v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$$

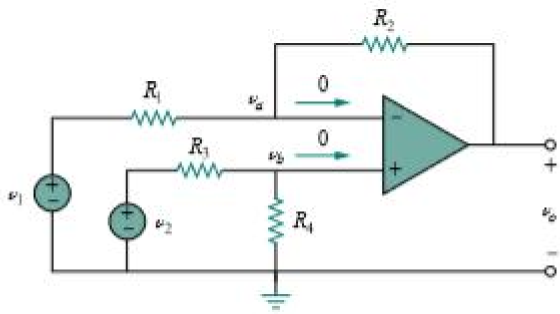


Figure 5.24 Difference amplifier.

Applying KCL to node b,

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

or

$$v_b = \frac{R_4}{R_3 + R_4} v_2$$

But $v_a = v_b$.

$$v_o = \left(\frac{R_2}{R_1} + 1\right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

or

$$v_o = \frac{R_2 \,(1 + R_1/R_2)}{R_1 \,(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_o = 0$ when $v_1 = v_2$. This property exists when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Thus, when the op amp circuit is a difference amplifier, comes

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

If $R_2 = R_1$ and $R_3 = R_4$, the difference amplifier becomes a *subtractor*, with the output

$$v_o = v_2 - v_1$$

Example 5:

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Solution:

The circuit requires that

$$v_o = 3v_2 - 5v_1$$

This circuit can be realized in two ways.

DESIGN 1 If we desire to use only one op amp, we can use the op amp circuit of Fig. 5.24.

$$\frac{R_2}{R_1} = 5 \qquad \Longrightarrow \qquad R_2 = 5R_1$$

Also,

$$5\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \qquad \Longrightarrow \qquad \frac{\frac{6}{5}}{1 + R_3/R_4} = \frac{3}{5}$$

or

$$2 = 1 + \frac{R_3}{R_4} \qquad \Longrightarrow \qquad R_3 = R_4$$

If we choose $R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$, then $R_2 = 50\ \text{k}\Omega$ and $R_4 = 20\ \text{k}\Omega$.

DESIGN 2 If we desire to use more than one op amp, we may cascade an inverting amplifier and a two-input inverting summer, as shown in Fig. 5.25. For the summer,

$$v_o = -v_a - 5v_1 \tag{5.7.4}$$

and for the inverter,

$$v_a = -3v_2 \tag{5.7.5}$$

Combining Eqs. (5.7.4) and (5.7.5) gives

$$v_o = 3v_2 - 5v_1$$

which is the desired result. In Fig. 5.25, we may select $R_1 = 10\ \text{k}\Omega$ and $R_2 = 20\ \text{k}\Omega$ or $R_1 = R_2 = 10\ \text{k}\Omega$.

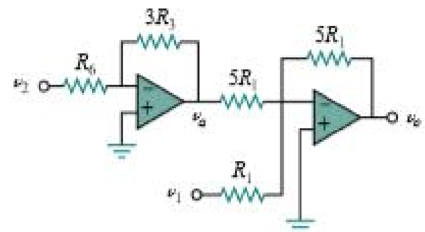


Figure 5.25