CHAPTER 10

Rings, Disks, and cylinders subjected to rotation and thermal gradients

9.0. INTRODUCTION

This chapter deals with the study of stresses developed due to rotation in circular discs and cylinders. The machine members of the rotating type and bodies like circular discs, cylinders, flywheels etc. invariably rotate at high speeds. Due to rotation, these members are subjected to centrifugal forces. The stresses are setup in the material of these members due to centrifugal forces.

Also in case of thin disc, the stress in axial direction is zero. But in case of thick disc (or long cylinder) the stress in axial direction (i.e., longitudinal direction) will not be zero.

9.1. EXPRESSION FOR STRESSES IN A ROTATING THIN DISC

In case of thin disc, only two stresses namely circumferential and radial stresses are existing.

Expression for circumferential stresses

$$\sigma_c = \frac{C_1}{2} - \frac{C_2}{r^2} - \frac{\rho^2 \times \omega^2 \times r^2}{8}(1+3\mu)\ldots(1)$$

Expression for radial stresses

$$\sigma_r = \frac{C_1}{2} + \frac{C_2}{r^2} - \frac{\rho^2 \times \omega^2 \times r^2}{8}(3+\mu)\ldots(2)$$

The constants C_1 and C_2 are obtained from boundary conditions. In equations (1) and (2), L.H.S. is in N/m^2, hence every term on R.H.S. should be in N/m^2. Hence C_1 and C_2 will be in N/m^2.

9.2. <u>Expression for Stresses in rotating Solid Disc</u>

From equations (1) and (2), it is clear that the stresses set up in a rotating disc will become infinite at the centre of the disc where r = 0. But the stresses at the centre, can not be infinite. For a rotating solid disc, the stresses at any radius r are :

$$\sigma_c = \frac{\rho \times \omega^2}{8}\left[(3+\mu)r_2^2 - (1+3\mu)r^2\right] \qquad \ldots(3)$$

and

$$\sigma_r = \frac{\rho \times \omega^2}{8}(3+\mu)\left[r_2^2 - r^2\right] \qquad \ldots(4)$$

Where σ_c = Circumferential stress,

σ_r = Radial stress,

ω = Angular velocity = $\frac{2\pi N}{60}$

μ = poisson's ratio

At the centre of rotating solid disc, the radial and circumferential stress are maximum and are equal. The expression is given as:

$$(\sigma_r)_{max} = (\sigma_c)_{max} = \frac{\rho \times \omega^2}{8}(3+\mu)r_2^2 \qquad \ldots(5)$$

At the outer radius, radial stress is zero, but circumferential stress is not zero. By substituting $r = r_2$ in equation (3), we get circumferential stress at outer radius.

This is equal to :

$$\sigma_c = \frac{\rho \times \omega^2 \times r_2^2}{4}(1-\mu) \ldots(6)$$

9.3. Expression of stresses for Disc with Central Hole

(I) The stresses at any radius are:

$$\sigma_r = \frac{\rho \times \omega^2}{8}(3+\mu)\left[\left(r_1^2 + r_2^2\right) - \frac{r_1^2 r_2^2}{r^2} - r^2\right] \ldots(8)$$

and

$$\sigma_c = \frac{\rho \times \omega^2}{8}(3+\mu)\left[\left(r_1^2 + r_2^2\right) + \frac{r_1^2 r_2^2}{r^2} - \left(\frac{1+3\mu}{3+\mu}\right) r^2\right] \ldots(9)$$

(ii) The circumferential stress is maximum at inner radius and is given by

$$(\sigma_c)_{\max} = \frac{\rho \times \omega^2}{8}(3+\mu)\left[r_2^2 + \left(\frac{1-\mu}{3+\mu}\right) r_1^2\right] \ldots(10)$$

(iii) The radial stress is maximum at the radius given by

$$r = \sqrt{r_1 \times r_2} \ldots(11)$$

(iv) The maximum radial stress is given by

$$(\sigma_r)_{\max} = \frac{\rho \times \omega^2}{8}(3+\mu)\left[(r_2 - r_1)^2\right] \ldots(12)$$

(v) The value of σ_c at the outer radius is

$$(\sigma_c)_{at\, r=r_2} = \frac{\rho \times \omega^2}{4}(3+\mu)\left[r_1^2 + \left(\frac{1+\mu}{3+\mu}\right) r_2^2\right] \ldots(13)$$

9.4. Expression of stresses for Disc with a pin hole at the center

The maximum radial and circumferential stresses in a disc with a pin hole at the center are :

$$(\sigma_r)_{max} = \frac{\rho \times \omega^2 \times r_2^2}{8}(3+\mu) \ldots(14)$$

$$(\sigma_c)_{max} = \frac{\rho \times \omega^2 \times r_2^2}{4}(3+\mu) \ldots(15)$$

The maximum circumferential stress in a disc with a pin hole at the centre is two times the maximum circumferential stress in a solid disc.

9.5. DISC OF UNIFORM STRENGTH

A disc which has equal values of circumferential and radial stresses at all radii, is known as a disc of uniform strength. Hence for a disc of uniform strength, $\sigma_r = \sigma_c$ = constant = σ for all radii. The thickness of the disc of uniform strength will not be constant. It will be varying.

The thickness of a disc of uniform strength is given by

$$t = t_o e^{\frac{-\rho \times \omega^2 \times r^2}{2\sigma}} \quad \ldots(16)$$

where t_o = Thickness at r = 0.

Examples

1. A steel disc of uniform thickness and of diameter 900 mm is rotating about its axis at 3000 r.p.m. Determine the radial and circumferential stresses at the centre and outer radius. The density of material is 7800 kg/m^2 and Poisson's ratio = 0.3.

SolutionGiven: Diameter = 900mm :. Radius of disc, r = $\frac{900}{2}$ = 450mm = 0.45 m

Speed, N = 3000 r.p.m.

$\therefore$ Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60} = 100\pi\ rad/s$

Density, $\rho = 7800\ kg/m^3$

Poisson's ratio, $\mu = 0.3$

The radial and circumferential stresses are given by

$$\sigma_r = \frac{C_1}{2} + \frac{C_2}{r^2} - \frac{\rho \times \omega^2 \times r^2}{8}(3 + \mu) \quad \text{...(i)}$$

and

$$\sigma_c = \frac{C_1}{2} - \frac{C_2}{r^2} - \frac{\rho \times \omega^2 \times r^2}{8}(1 + 3\mu) \quad \text{...(ii)}$$

As the stresses cannot have infinite value at r = 0, hence C_2 should be zero. Hence equations (i) and (ii) becomes as

$$\sigma_r = \frac{C_1}{2} - \frac{\rho \times \omega^2 \times r^2}{8}(3 + \mu) \quad \text{...(iii)}$$

and

$$\sigma_c = \frac{C_1}{2} - \frac{\rho \times \omega^2 \times r^2}{8}(1 + 3\mu) \quad \text{...(iv)}$$

Also at the outer radius, $\sigma_r = 0$. This means at r = 0.45 m, $\sigma_r = 0$. Substituting these values in equation (iii), we get

$$0 = \frac{C_1}{2} - \frac{\rho \times \omega^2 \times (0.45)^2}{8}(3 + \mu)$$

or

$$\frac{C_1}{2} = \frac{\rho \times \omega^2 \times (0.45)^2}{8}(3 + \mu) \quad \text{or} \quad C_1 = \frac{\rho \times \omega^2 \times (0.45)^2}{4}(3 + \mu)$$

$$= \frac{7800 \times (100\,\pi)^2 \times (0.45)^2 (3 + 0.3)}{4} = 128.6 \times 10^6\ N/m^2$$

Substituting the value of C_1 in equations (iii) and (iv), we get

$$\sigma_r = \frac{128.6 \times 10^6}{2} - \frac{\rho \times \omega^2 \times r^2}{8}(3 + \mu)$$

and

$$\sigma_c = \frac{128.6 \times 10^6}{2} - \frac{\rho \times \omega^2 \times r^2}{8}(1 + 3\mu)$$

Atr=0,

$$\sigma_r = \sigma_c = \frac{128.6 \times 10^6}{2} = 64.3 \times 10^6 \text{ N/m}^2 = \mathbf{64.3\ MN/m^2}. \quad \textbf{Ans.}$$

Atr=0.45, σ_r=0

$$\sigma_c = \frac{128.6 \times 10^6}{2} - \frac{7800 \times (100\,\pi)^2 \times (0.45)^2}{8}(1 + 3 \times 0.3)$$

$$= 64.3 \times 10^6 - 37.024 \times 10^6 \text{ N/m}^2$$

$$= 27.276 \times 10^6 \text{ N/m}^2 = \mathbf{27.276\ MN/m^2}. \quad \textbf{Ans.}$$

2. If for the example 1, the disc is having a central hole of 150 mm diameter, then determine

(i) circumferential stress at inner radius and outer radius,

(ii) radius at which radial stress is maximum, and

(iii) maximum radial stress.

Solution. Given:

From example 1, Outer radius, r_2 = 0.45 m,Angular speed, $\omega = (100\ \pi)$ rad/s

ρ = 7800kg/m^3, μ = 0.3

Inner diameter = 150 mm, $\therefore$ Inner radius, $r_1 = \frac{150}{2}$ = 75 mm = 0.075 mm

(i) Circumferential stress (σ_c) at inner and outer radii

The circumferential stress in a disc with central hole is maximum at the inner radius. This stress is given by

$$(\sigma_c)_{max} = \frac{\rho \times \omega^2}{4}(3+\mu)\left[r_2^2 + \left(\frac{1-\mu}{3+\mu}\right)r_1^2\right]$$

$$= \frac{7800 \times (100\,\pi)^2}{4}(3+0.3)\left[0.45^2 + \left(\frac{(1-0.3)}{3+0.3}\right)(0.075)^2\right]$$

$$= 635.109 \times 10^6\,[0.2025 + 1.193 \times 10^{-3}]$$

$$= 129.367 \times 10^6\ \text{N/m}^2 = \mathbf{129.367\ MN/m^2}.\quad \textbf{Ans.}$$

The circumferential stress at the outer radius is given by

$$\sigma_c = \frac{\rho \times \omega^2}{4}(3+\mu)\left[r_1^2 + \left(\frac{1-\mu}{3+\mu}\right)r_2^2\right]$$

$$= \frac{7800 \times (100\,\pi)^2 \times (3+0.3)}{4}\left[0.075^2 + \left(\frac{1-0.3}{3+0.3}\right) \times 0.45^2\right]$$

$$= 635.109 \times 10^6\,[5.625 \times 10^{-3} + 0.04295]$$

$$= 30.85 \times 10^6\ \text{N/m}^2 = \mathbf{30.85\ MN/m^2}.\quad \textbf{Ans.}$$

(ii) Radius at which radial stress is maximum

The radius at which radial stress is maximum is given by

$$r = \sqrt{r_1 \times r_2} = \sqrt{0.075 \times 0.45} = \mathbf{0.1837\ m.}\quad \textbf{Ans.}$$

(iii) Maximum radial stress

The maximum radial stress is given by

$$(\sigma_r)_{max} = \frac{\rho \times \omega^2}{8}(3+\mu)(r_2 - r_1)^2$$

$$= \frac{7800 \times (100\,\pi)^2}{8}(3+0.3)(0.45 - 0.075)^2$$

$$= 317.5545 \times 10^6\,(0.375)^2$$

$$= 44.656 \times 10^6\ \text{N/m}^2 = \mathbf{44.656\ MN/m^2}.\quad \textbf{Ans.}$$

3. Problem 22.5. For the examples 1 and 2, find the maximum shear stress in the solid disc and in the disc with a central hole.

Solution

The maximum shear stress at any radius is given by, $\tau_{\max} = \frac{1}{2}(\sigma_c - \sigma_r)$

(i) Solid disc

In case of solid disc the stresses calculated in example 1 at the centre are $\sigma_r = \sigma_c$ = 64.3 MN/m^2, whereas the stresses at outer radius are $\sigma_r = 0$ and $\sigma_c = 27.276$ MN/m^2. Hence principal stresses at the centre are: 64.3 MN/m^2, 64.3 MN/m^2, 0

Shear stress at centre = $\frac{1}{2}(64.3 - 64.3) = 0$

The principal stresses at the outer radius are 27.276 MN/m^2, 0, 0

Shear stress at outer radius = $\frac{1}{2}$ (27.276- 0) = 13.638 MN/m^2

Maximum shear stress is at the outer radius and equal to 13.638 MN/m^2

$\therefore \tau_{\max}$ = 13.638 MN/m2. **Ans.**

(ii) <u>Disc with a central hole</u>

From example 2, the calculated values of stresses are:

At the inner radius, $\sigma_c = 129.367$ MN/m^2, $\sigma_r = 0$

At the outer radius, $\sigma_c = 30.85$ MN/m^2, $\sigma_r = 0$

Principal stresses at inner radius are: 129.367 MN/m^2, 0, 0

Principal stresses at outer radius are : 30.85 MN/m^2, 0, 0.

Hence maximum shear stress will be at the inner radius.

$$\therefore \quad \sigma_{\max} = \frac{1}{2}(\sigma_c - \sigma_r)$$

$$= \frac{1}{2}(129.367 - 0) = \mathbf{64.683\ MN/m^2}. \quad \textbf{Ans.}$$

4.The minimum thickness of a turbine rotor is 9mm at a radius of300 mm. If the rotor is to be designed for a uniform stress of 200 MN/m^2, find the thickness of the rotor at a radius of 25 mm when it is running at 9000 r.p.m. Take p = 8000 kg/m^3.

Solution. Given:

Thickness, t_1 = 9 mm when r_1 = 300 mm = 0.3 m

Uniform stress, $\sigma = 200\ MN/m^2 = 200 \times 10^6\ N/m^2$

Density, $\rho = 8000\ kg/m^3$

Speed, N = 9000 r.p.m.

$\therefore$ Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 9000}{60} = 300\pi\ rad/s$

Find thickness when radius = 25 mm = 0.025 m

Using equation (16), we get

$$t = C \times e^{-\frac{\rho\times\omega^2\times r^2}{2\sigma}} \qquad \text{...(i)}$$

(i) When t = 9 mm ; radius, r = 300mm = 0.3 m

Hence above equation becomes,

$$9 = C \times e^{\frac{-\rho\times\omega^2\times(0.3)^2}{2\sigma}}$$

$$= C \times e^{\frac{-\rho\times\omega^2\times 0.09}{2\sigma}}$$

or
$$C = \frac{9}{e^{\frac{-\rho\times\omega^2\times 0.09}{2\sigma}}}$$

$$= 9 \times e^{\frac{\rho\times\omega^2\times 0.09}{2\sigma}} \qquad \text{...(ii)}$$

(ii) When thickness is t, radius is 0.025 m

Substituting the above values in equation (i), we get

$$t = C \times e^{\frac{-\rho \times \omega^2 \times (0.025)^2}{2\sigma}}$$

$$= \left(9 \times e^{\frac{\rho \times \omega^2 \times 0.09}{2\sigma}}\right) \times e^{-\frac{\rho \times \omega^2 \times 0.000625}{2\sigma}}$$

$$\left[\because \text{ From equation } (ii), C = 9 \times e^{\frac{\rho \times \omega^2 \times 0.09}{2\sigma}}\right]$$

$$= 9 \times e^{\left(\frac{\rho \times \omega^2 \times 0.09}{2\sigma} - \frac{\rho \times \omega^2 \times 0.000625}{2\sigma}\right)}$$

$$= 9 \times e^{\frac{\rho \times \omega^2 (0.09 - 0.000625)}{2\sigma}}$$

$$= 9 \times e^{\frac{\rho \times \omega^2}{2\sigma} \times 0.089375} = 9 \times e^{\frac{8000 \times (300\,\pi)^2 \times 0.089375}{2 \times 200 \times 10^6}}$$

$$= 9 \times e^{1.587} = 9 \times 4.892 = \mathbf{44.03\ mm.} \quad \textbf{Ans.}$$

9.6. <u>Combined rotational and thermal stresses in uniform discs and thick cylinders</u>

If the temperature of any component is raised uniformly then, provided that the material is free to expand, expansion takes place without the introduction of any so-called thermal or temperature stresses. In cases where components, e.g. discs, are subjected to thermal gradients, however, one part of the material attempts to expand at a faster rate than another owing to the difference in temperature experienced by each part, and as a result stresses are developed. These are analogous to the differential expansion stresses experienced in compound bars of different materials.

Expressions for stresses in discs

Radial stress

$$\sigma_r = A - \frac{B}{r^2} - \frac{\rho r^2 \omega^2}{8}(3 + \nu) - \frac{Ea}{r^2} \int Tr\, dr$$

Circumferential stress

$$\sigma_H = A + \frac{B}{r^2} - (1 + 3\nu)\frac{\rho r^2 \omega^2}{8} - EaT + \frac{Ea}{r^2} \int Tr\, dr$$

Expressions for stresses in cylinders

$$\sigma_r = A - \frac{B}{r^2} - \frac{EaT}{2(1 - \nu)}$$

$$\sigma_H = A + \frac{B}{r^2} - \frac{EaT}{2(1 - \nu)} - \frac{Eab}{2(1 - \nu)}$$

EXAMPLE

1.An initially unstressed short steel cylinder, internal radius 0.2 m and external radius 0.3 m, is subjected to a temperature distribution of the form T = a + b $\log_e r$ to ensure constant heat flow through the cylinder walls. With this form of distribution the radial and circumferential stresses at any radius r, where the temperature is T, are given by

$$\sigma_r = A - \frac{B}{r^2} - \frac{\alpha ET}{2(1 - \nu)}$$

$$\sigma_H = A + \frac{B}{r^2} - \frac{\alpha ET}{2(1 - \nu)} - \frac{E\alpha b}{2(1 - \nu)}$$

If the temperatures at the inside and outside surfaces are maintained at 200°C and 100°C respectively, determine the maximum circumferential stress set up in the cylinder walls. For steel, E = 207 GN/m^2, ν = 0.3 and α = 11 x 10^{-6} per °C.

Solution

$$T = a + b\log_e r$$

$\therefore$ $$200 = a + b\log_e 0.2 = a + b(0.6931 - 2.3026)$$

$$200 = a - 1.6095\ b \quad (1)$$

also $$100 = a + b\log_e 0.3 = a + b(1.0986 - 2.3026)$$

$$100 = a - 1.204\ b \quad (2)$$

(2) − (1), $$100 = -0.4055\ b$$

$$b = -246.5 = -247$$

Also $$\frac{E\alpha}{2(1-\nu)} = \frac{207 \times 10^9 \times 11 \times 10^{-6}}{2(1-0.29)}$$

$$= 1.6 \times 10^6$$

Therefore substituting in the given expression for radial stress,

$$\sigma_r = A - \frac{B}{r^2} - 1.6 \times 10^6 T$$

At $r = 0.3$, $\sigma_r = 0$ and $T = 100$

$$0 = A - \frac{B}{0.09} - 1.6 \times 10^6 \times 100 \quad (3)$$

At $r = 0.2$, $\sigma_r = 0$ and $T = 200$

$$0 = A - \frac{B}{0.04} - 1.6 \times 10^6 \times 200 \quad (4)$$

(4) − (3), $$0 = B(11.1 - 25) - 1.6 \times 10^8$$

$$B = -11.5 \times 10^6$$

and from (4),

$$A = 25B + 3.2 \times 10^8$$

$$= (-2.88 + 3.2)10^8 = 0.32 \times 10^8$$

substituting in the given expression for hoop stress,

$$\sigma_H = 0.32 \times 10^8 - \frac{11.5 \times 10^6}{r^2} - 1.6 \times 10^6 T + 1.6 \times 10^6 \times 247$$

At $r = 0.2$, $\sigma_H = (0.32 - 2.88 - 3.2 + 3.96)10^8 = \mathbf{-180\ MN/m^2}$

At $r = 0.3$, $\sigma_H = (0.32 - 1.28 - 1.6 + 3.96)10^8 = \mathbf{+140\ MN/m^2}$

The maximum tensile circumferential stress therefore occurs at the outside radiusand has a value of 140 MN/m^2. The maximum compressive stress is 180 MN/m^2 at the inside radius.

2.(a) Derive expressions for the hoop and radial stresses developed in a solid disc of radius R when subjected to a thermal gradient of the form T = Kr. Hence determine the positionand magnitude of the maximum stresses set up in a steel disc of 150 mm diameter when the temperature rise is 150°C. For steel, α = 12 x 10—6 per °C and
E = 206.8 GN/m^2.

(b) How would the values be changed if the temperature at the centre of the disc was increased to 30°C, the temperature rise across the disc maintained at 150°C and the thermal gradient now taking the form T = a + br?

<u>Solution</u>

(a) The hoop and radial stresses are given by:

$$\sigma_r = A - \frac{B}{r^2} - \frac{\alpha E}{r^2}\int Tr\,dr \qquad (1)$$

$$\sigma_H = A + \frac{B}{r^2} + \frac{\alpha E}{r^2}\int Tr\,dr - \alpha ET \qquad (2)$$

In this case $$\int Tr\,dr = K\int r^2 dr = \frac{Kr^3}{3}$$

the constant of integration being incorporated into the general constant A.

Now in order that th)stresses at the centre of the disc, where r = 0, shall not be infinite,

$$\therefore \quad \sigma_r = A - \frac{B}{r^2} - \frac{\alpha EKr}{3} \qquad (3)$$

$$\sigma_H = A + \frac{B}{r^2} + \frac{\alpha EKr}{3} - \alpha EKr \qquad (4)$$

Now in order that the stresses at the centre of the disc, where r = 0, shall not be infinite, B must be zero and hence B/r^2 is zero. Also $\sigma_r = 0$ at r = R.

Therefore substituting in (3),

$$0 = A - \frac{\alpha EKR}{3} \text{ and } A = \frac{\alpha EKR}{3}$$

Substituting in (3) and (4) and rearranging,

$$\sigma_r = \frac{\alpha EK}{3}(R - r)$$
$$\sigma_H = \frac{\alpha EK}{3}(R - 2r)$$

The variation of both stresses with radius is linear and they will both have maximum values at the centre where r = 0.

$$\therefore \qquad \sigma_{r_{max}} = \sigma_{H_{max}} = \frac{\alpha EKR}{3}$$
$$= \frac{12 \times 10^{-6} \times 206.8 \times 10^9 \times K \times 0.075}{3}$$

Now T = Kr and T must therefore be zero at the center of the disc where r is zero. Thus, with a known temperature rise of 150°C, it follows that the temperature at the outside radius must be 150°C.

$$\therefore \qquad 150 = K \times 0.075$$
$$\therefore \qquad K = 2000^\circ/\text{m}$$

$$\text{i.e.} \qquad \sigma_{r_{max}} = \sigma_{H_{max}} = \frac{12 \times 10^{-6} \times 206.8 \times 10^9 \times 2000 \times 0.075}{3}$$
$$= \mathbf{124\ MN/m^2}$$

(b) With the modified form of temperature gradient,

$$\int Tr\,dr = \int (a + br)r\,dr = \int (ar + br^2)\,dr$$
$$= \frac{ar^2}{2} + \frac{br^3}{3}$$

Substituting in (1) and (2),

$$\sigma_r = A - \frac{B}{r^2} - \frac{\alpha E}{r^2}\left[\frac{ar^2}{2} + \frac{br^3}{3}\right] \quad (5)$$

$$\sigma_H = A + \frac{B}{r^2} + \frac{\alpha E}{r^2}\left[\frac{ar^2}{2} + \frac{br^3}{3}\right] - \alpha ET \quad (6)$$

Now $T = a + br$

Therefore at the inside of the disc where r = 0 and T = 30°C,

$$30 = a + b(0) \quad (7)$$

and $a = 30$

At the outside of the disc where T = 180°C,

$$180 = a + b(0.075) \quad (8)$$

(8) − (7) $150 = 0.075b \quad \therefore \ b = 2000$

Substituting in (5) and (6) and simplifying,

$$\sigma_r = A - \frac{B}{r^2} - \alpha E(15 + 667r) \quad (9)$$

$$\sigma_H = A + \frac{B}{r^2} + \alpha E(15 + 667r) - \alpha ET \quad (10)$$

Now for finite stresses at the centre,

B=0

Also, at r = 0.075, σ_r= 0 and T = 180°C

Therefore substituting in (9),

$$0 = A - 12 \times 10^{-6} \times 206.8 \times 10^9 (15 + 667 \times 0.075)$$

$$0 = A - 12 \times 206.8 \times 10^3 \times 65$$

$$\therefore \quad A = 161.5 \times 10^6$$

From (9) and (10) the maximum stresses will again be at the centre where r = 0,

i.e. $\sigma_{rmax} = \sigma_{Hmax} = A - \alpha ET = 124$ MN/m^2, as before.

N.B. The same answers would be obtained for any linear gradient with a temperature difference of 150°C. Thus a solution could be obtained with the procedure of part (a) using the form of distribution $T = Kr$ with the value of T at the outside taken to be 150°C (the value at $r = 0$ being automatically zero).