CHAPTER 5 - DIGITAL MODULATION

Forms of Digital Modulation

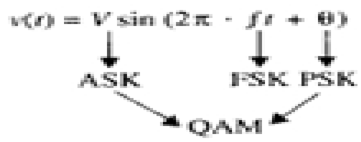
amplitude shift keying (ASK) is produced if the information signal is digital and the amplitude of the carrier is varied proportional to the information signal.

frequency shift keying(FSK) is produced if the frequency (f) is varied proportional to the information signal.

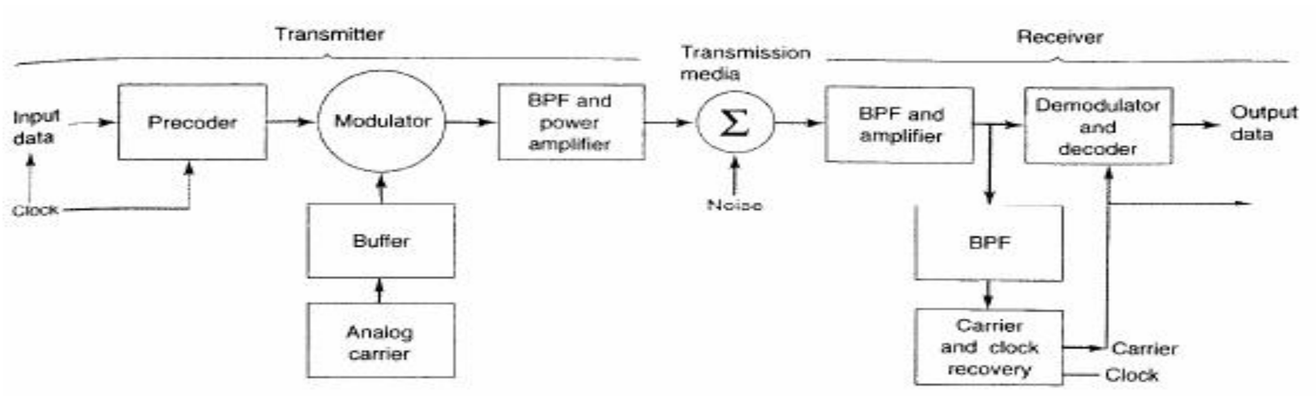
phase shift keying (PSK) is produced if the phase of the carrier is varied proportional to the information signal.

quadrature amplitude modulation(QAM) results if both the amplitude and the phase are varied proportional to the information signal.

Mathematical Expression



A simplified block diagram for a digitalmodulation system



In the transmitter, the **precoder**performs level conversion andthen encodes the incoming data into groups of bits thatmodulate an analog carrier.

The **modulated carrier** is shaped (filtered), amplified, andthen transmitted through the transmission medium to thereceiver.

The **transmission medium** can be a metallic cable, optical fibercable, Earth's atmosphere, or a combination of two or moretypes of transmission systems.

In **the receiver**, the incoming signals are filtered, amplified,and then applied to the demodulator and decoder circuits,which extracts the original source information from themodulated carrier.

The **clock and carrier recovery circuits** recover the analogcarrier and digital timing (clock) signals from the incomingmodulated wave since they are necessary to perform the de-modulation process.

Information Capacity, Bits, and Bit Rate

$$I \alpha B \times t$$

where I= information capacity (bits per second)

B = bandwidth (hertz)

t **=** transmission time (seconds)

Shannon limit for information capacity $I = B \log_2\left(1 + \frac{S}{N}\right) \qquad I = 3.32B \log_{10}\left(1 + \frac{S}{N}\right)$

where I = information capacity (bps)

B = bandwidth (hertz)

S/N = signal-to-noise power ratio

Ex. For a standard telephone circuit with a signal-to-noise power ratio of 1000 (30 dB) and a bandwidth of 2.7 kHz, the Shannon limit for information capacity is?

M-ary Encoding

M-ary is a term derived from the word *binary*

$$N = \log_2 M \qquad 2^N = M$$

Where N = number of bits necessary

M = number of conditions, levels, or combinations

Baud and Minimum Bandwidth

Baud refers to the rate of change of a signal on the transmission medium after encoding and modulation have occurred.

Baud = 1/Ts where baud = symbol rate (baud per second)

Ts = time of one signaling element (seconds)

Minimum NyquistBandwidth - the minimum theoretical bandwidth necessary to propagate a signal

$f_b = B \log_2 M$ where fb = channel capacity (bps)

B = minimum Nyquist bandwidth (hertz)

M = number of discrete signal or voltage levels

$$B = \frac{f_b}{\log 2\ M} = \frac{fb}{N}$$

The **Baud** andthe **ideal minimum Nyquist bandwidth** have the same valueand are equal to the bit rate divided by the number of bitsencoded**.**

AMPLITUDE-SHIFT KEYING

The simplest digital modulation technique where a binary information signal directly modulates the amplitude of an analog carrier.Sometimes called *digital amplitude modulation* (DAM).

Mathematically:

$$v_{(ask)}(t) = [1 + v_m(t)]\left[\frac{A}{2}\cos(\omega_c t)\right]$$

Where $v_{ask}(t)$ =amplitude-shift keying wave

$v_m(t)$ = digital information (modulating) signal (volts)

A/2 = unmodulated carrier amplitude (volts)

ω_c = analog carrier radian frequency (radians per second, $2\pi f_c t$

The modulating signal [vm(t)] *is* a normalized binary waveform;

where + 1 V = logic 1

-1 V = logic 0.

Therefore, for a logic 1 input, vm(t) = + 1 V, substituting

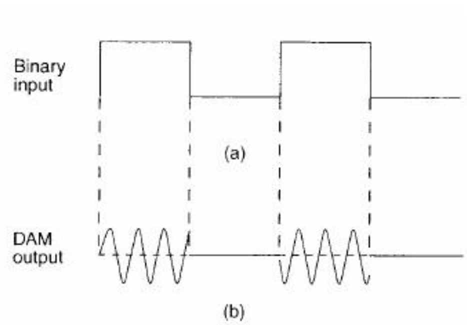
$$v_{(ask)}(t) = [1 + 1]\left[\frac{A}{2}\cos(\omega_c t)\right] = A\cos(\omega_c t)$$

and for a logic 0 input, vm(t) = - 1 V, substituting

$$v_{(ask)}(t) = [1 - 1]\left[\frac{A}{2}\cos(\omega_c t)\right]$$

Thus, the modulated wave$v_{ask}(t)$, is either A $\cos(\omega_c t)$ = "on" or 0 = "off"

The waveform Digital Amplitude Modulation



-for every change in the input binary data stream, there is one change in the ASK waveform

- The rate of change of the ASK waveform (baud) is the same as the rate of change of the binary input (bps)

B = fb /1 = fb baud = fb /1 = fb

Ex. Determine the baud and minimum nyquist bandwidth necessary to pass a10kbps binary signal using ASK, for ASK, N = 1?

FREQUENCY-SHIFT KEYING

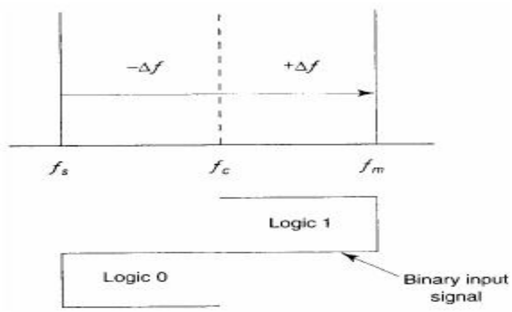
- is a form of constant-amplitude angle modulation similar to standard frequency modulation (FM) except themodulating signal is a binary signal that varies between two discrete voltage levels. Sometimes called *binary FSK* (BFSK)

Mathematically: $v_{fsk}(t) = V_c \cos\{2\pi[f_c + v_m(t)\,\Delta f]t\}$

Where $v_{fsk}(t)$ = binary FSK waveform
V_c = peak analog carrier amplitude (volts)
f_c = analog carrier center frequency (hertz)
Δf = peak change (shift) in the analog carrier frequency (hertz)
$v_m(t)$ = binary input (modulating) signal (volts)

The modulating signal is a normalized binary waveform
where logic 1 = + 1 V
logic 0 = -1 V.
Thus, for a logic l input, $v_m(t) = +1$
for a logic 0 input, vm(t) = -1

$$v_{fsk}(t) = V_c \cos[2\pi(f_c + \Delta f)t]$$
$$v_{fsk}(t) = V_c \cos[2\pi(f_c - \Delta f)t]$$

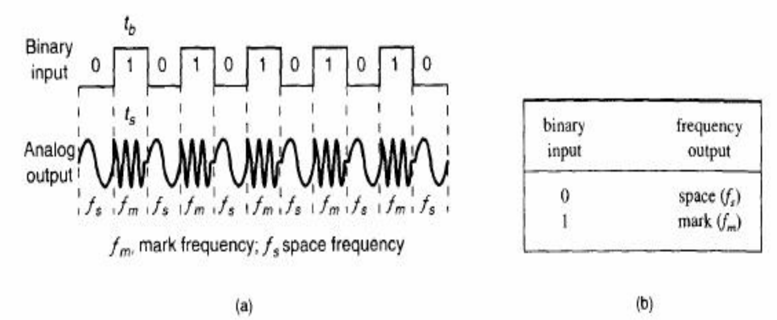


- the carrier center frequency (fc) is shifted (deviated) up and down in the frequency domain

Frequency deviation is expressed mathematically:

$$\Delta f = |f_m - f_s| / 2$$

Where: Δf = frequency deviation (hertz)
$|f_m - f_s|$ = absolute difference between the mark and space frequencies (hertz)
$f_m = f_{USB};\ f_s = f_{LSB}$



FSK in the time domain: (a) waveform: (b) truth table

FSK Bit Rate, Baud, and Bandwidth

The baud for binary FSK can also be determined bysubstituting N = 1

baud = fb / 1 = fb

The minimum bandwidth for FSK is given as

$B = |(f_s - f_b) - (f_m - f_b)|$
$= |(f_s - f_m)| + 2f_b$
$\mathbf{B = 2(\Delta f + f_b)}$

Example:
Determine (a) the peak frequency deviation, (b) minimumbandwidth, and (c) baud for a binary FSK signal with a markfrequency of 49 kHz, a space frequency of 51 kHz, and an input bitrate of 2 kbps.

$\mathbf{f_a = f_b / 2}$ where f_a = highest fundamental frequency of the binary input signal(hertz)
f_b = input bit rate (bps)

The formula used for modulation index in FM is also valid forFSK, so

$\mathbf{m_f = \Delta f / f_a}$

substituting:

$m_f = |(f_m - f_s)/2| / f_b/2$

$\mathbf{m_f = |f_m - f_s| / f_b}$

Using a Bessel table, determine the minimum bandwidth for thesame FSK signal with a mark frequency of49 kHz, a space frequency of 51 kHz, and an input bit rate of 2 kbps.

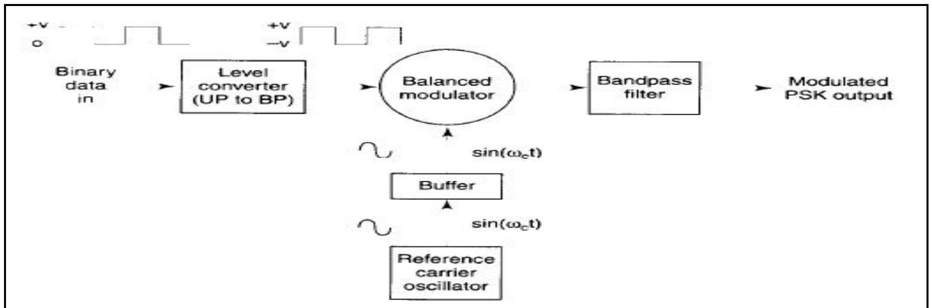
PHASE-SHIFT KEYING - another form of *angle-modulated, constant-amplitude* digital modulation.
Types: a) BPSK b) QPSK c) 8PSK d) 16PSK

A) The simplest form of PSK is *binary phase-shift keying* (**BPSK**), where N = 1 and *M = 2.* Therefore, with BPSK, two phases ($2^1 = 2$) are possible for the carrier.

There are two possible phases, Logic 1 and logic 0, the phase of the output carrier shifts between two angles that are separated by 180°.

BPSK TRANSMITTER:

The balanced modulator acts as a phase reversing switch. Depending on the logic condition of the digital input, the carrier is transferred to the output either in phase or 180° out of phase with the reference carrier oscillator.



(a)Balanced ring modulator;

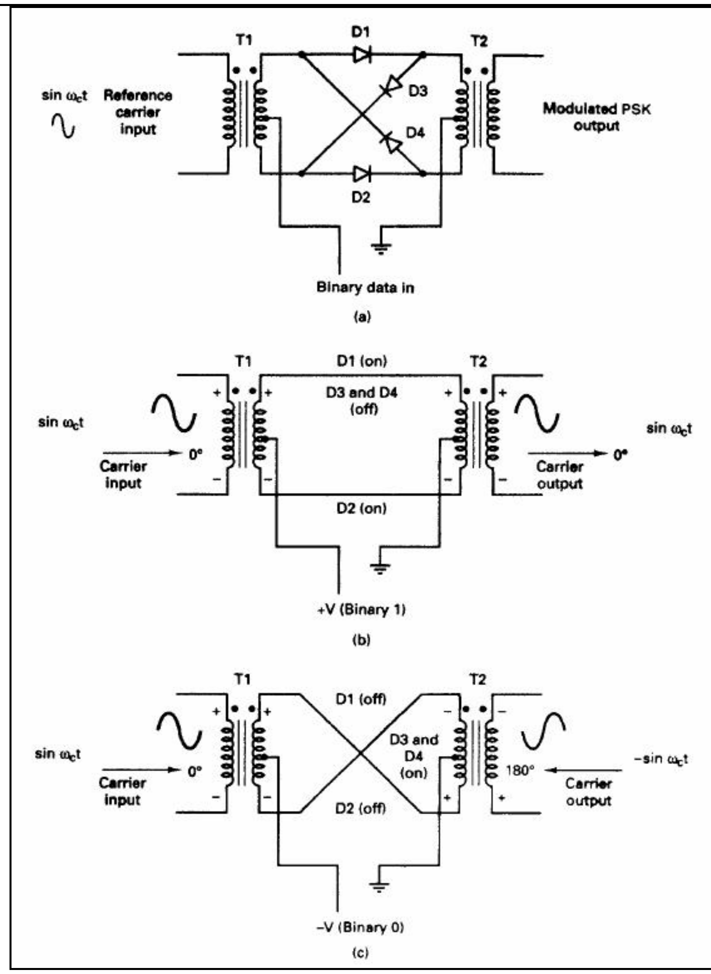
For the balanced modulator to operate properly, the digital input voltage must be much greater than the peak carrier voltage (V_c).

(b) logic 1 input;

If the binary input is a logic 1(positive voltage), diodes D 1 and D2 are forward biased and on, while diodes D3 and D4 are reverse biased and off. With the polarities shown, the carrier voltage is developed across transformer T2 in phase with the carrier voltage across T1. Consequently, the output signal is in phase with the reference oscillator.

(c) logic 0 input

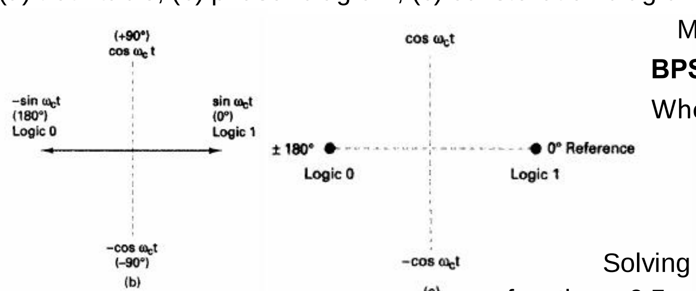
If the binary input is a logic 0 (negative voltage), diodes Dl and D2 are reverse biased and off, while diodes D3 and D4 are forward biased and on. As a result, the carrier voltage is developed across transformer T2 180° out of phase with the carrier voltage across T 1.



BPSK modulator: (a) truth table; (b) phasor diagram; (c) constellation diagram

Binary input	Output phase
Logic 0	180°
Logic 1	0°

(a)



Mathematically, BPSK modulator output:

BPSK output = [sin (2πfat)] x [sin (2πfct)]

Where: fa = maximum fundamental frequency of binary input (hertz)
fc = reference carrier frequency (hertz)

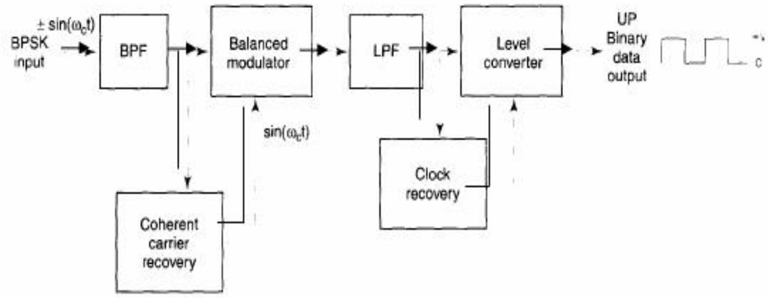
Solving for the trig identity for the product of two sine functions, **0.5 cos[2π(fc – fa)t] – 0.5cos[2π(fc + fa)t]**

Bandwidth of BPSK:

Example: For a BPSK modulator with a carrier frequency of 70 MHz and an input bit rate of 10 Mbps, determine the maximum and minimum upper and lower side frequencies, draw the output spectrum, determine the minimum Nyquist bandwidth, and calculate the baud.

BPSK RECEIVER

- The input signal maybe + sin ωct or - sin ωct.
- The balanced modulator is a product detector; the output is the product of the two inputs (the BPSK signal and the recovered carrier)



Mathematically, the demodulation process is as follows.

For a BPSK input signal of **+sin $\omega_c t$ (logic 1)**, the output of the balanced modulator is

$$\text{output} = (\sin \omega_c t)(\sin \omega_c t) = \sin^2 \omega_c t$$

Or

$$\sin^2 \omega_c t = 0.5(1 - \cos 2\omega_c t) = 0.5 - 0.5\cos 2\omega_c t$$

filtered out

leaving output = + 0.5 V = logic 1

For a BPSK input signal of **-sin $\omega_c t$ (logic 0)**, the output of the balanced modulator is

$$\text{output} = (-\sin \omega_c t)(\sin \omega_c t) = \sin_2 \omega_c t$$

or

$$\sin_2 \omega_c t = -0.5(1 - \cos 2\omega_c t) = 0.5 + 0.5\cos 2\omega_c t$$

filtered out

leaving

output = - 0.5 V = logic 0

Quaternary Phase-Shift Keying (QPSK) is an M-ary encoding scheme where N = 2 and *M*= 4**.** Produce four different input combinations,: 00, 01, 10, and 11. The binary input data are combined into groups of two bits, called *dibits*. Each dibit code generates one of the four possible output phases (+45°,+135°, -45°, and -135°).

QPSK transmitter

where logic 1 = + 1 V
logic 0 = -1 V

I balanced modulator (+sin $\omega_c t$, - sin $\omega_c t$)

Q balanced modulator (+cos $\omega_c t$, -cos $\omega_c t$)

The output of the linear summer:

QI@00

I = -1 sin $\omega_c t$

Q = -1 cos $\omega_c t$

QI@01

I = +1 sin $\omega_c t$

Q = -1 cos $\omega_c t$

QI@10

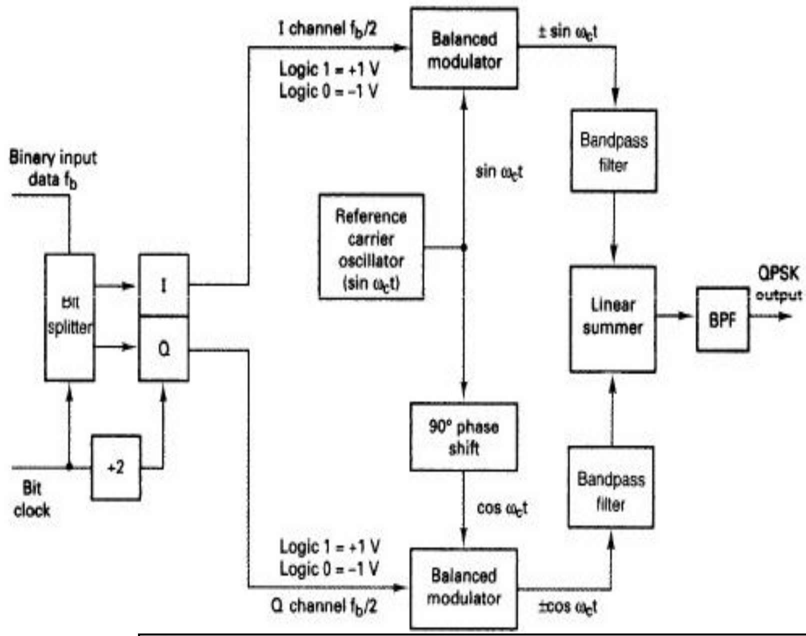
I = -1 sin $\omega_c t$

Q = +1 cos $\omega_c t$

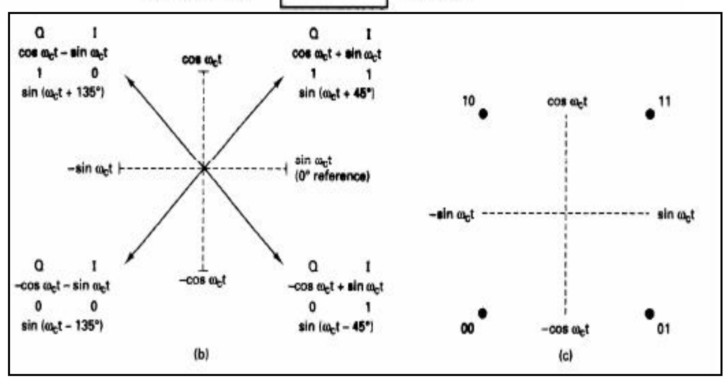
QI@11

I = +1 sin $\omega_c t$

Q = +1 cos $\omega_c t$



Binary Input Q	I	Amplitude	Output Phase
0	0	1.414	
0	1	1.414	
1	0	1.414	
1	1	1.414	



QPSK receiver

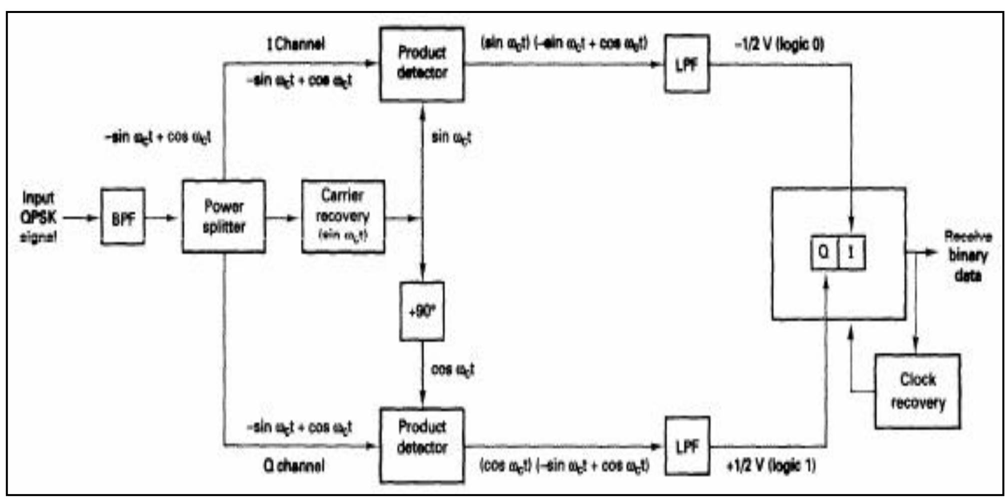
Four possible input signal

-1 sin $\omega_c t$ -1 cos $\omega_c t$

+1 sin $\omega_c t$ -1 cos $\omega_c t$

-1 sin $\omega_c t$ +1 cos $\omega_c t$

+1 sin $\omega_c t$ +1 cos $\omega_c t$



@I channel

$$I = \underbrace{(-\sin \omega_c t + \cos \omega_c t)}_{\text{QPSK input signal}} \underbrace{(\sin \omega_c t)}_{\text{carrier}}$$

$$= (-\sin \omega_c t)(\sin \omega_c t) + (\cos \omega_c t)(\sin \omega_c t)$$

$$= -\sin^2 \omega_c t + (\cos \omega_c t)(\sin \omega_c t)$$

$$= -\frac{1}{2}(1 - \cos 2\omega_c t) + \frac{1}{2}\sin(\omega_c + \omega_c)t + \frac{1}{2}\sin(\omega_c - \omega_c)t$$

(filtered out) (equals 0)

$$I = -\frac{1}{2} + \frac{1}{2}\cos 2\omega_c t + \frac{1}{2}\sin 2\omega_c t + \frac{1}{2}\sin 0$$

$$= -\frac{1}{2}\text{V (logic 0)}$$

@Q channel

$$Q = \underbrace{(-\sin \omega_c t + \cos \omega_c t)}_{\text{QPSK input signal}} \underbrace{(\cos \omega_c t)}_{\text{carrier}}$$

$$= \cos^2 \omega_c t - (\sin \omega_c t)(\cos \omega_c t)$$

$$= \frac{1}{2}(1 + \cos 2\omega_c t) - \frac{1}{2}\sin(\omega_c + \omega_c)t - \frac{1}{2}\sin(\omega_c - \omega_c)t$$

(filtered out) (equals 0)

$$Q = \frac{1}{2} + \frac{1}{2}\cos 2\omega_c t - \frac{1}{2}\sin 2\omega_c t - \frac{1}{2}\sin 0$$

$$= \frac{1}{2}\text{V(logic 1)}$$

8-PSK

With *8-PSK,* the incoming bits are encoded in groups of three, called tribits ($2^3 = 8$), producing eight different input combinations: 000, 001, 010, 011, 100, 101, 110, 111 and producing eight different output phase (±22.5°, ±67.5, ±112.5, ±157.5).

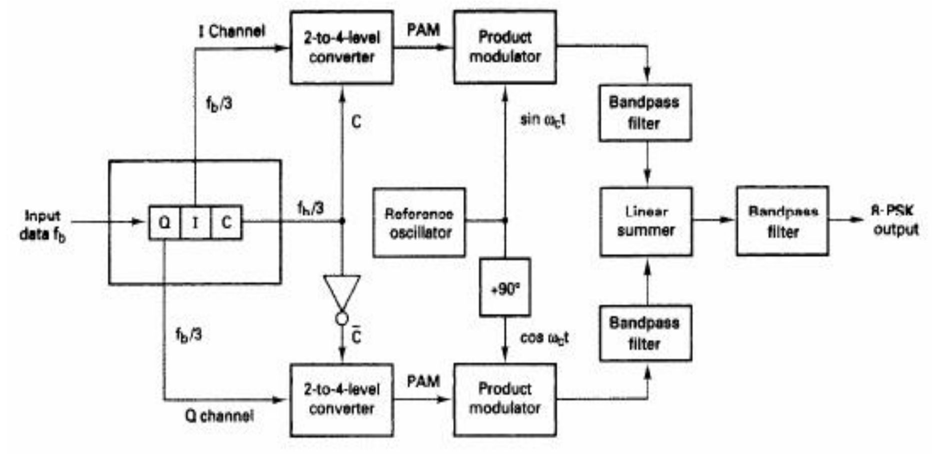
8-PSK transmitter

I	C	Output
0	0	−0.541 V
0	1	−1.307 V
1	0	+0.541 V
1	1	+1.307 V

(a)

Q	$\bar{C}$	Output
0	1	−1.307 V
0	0	−0.541 V
1	1	+1.307 V
1	0	+0.541 V

(b)



The I or Q bit determines the polarity of the output analog

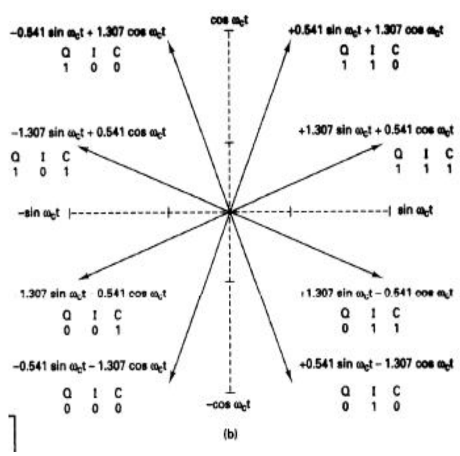
Signal: logic 1=+V and logic 0 = -V

The C determines the magnitude

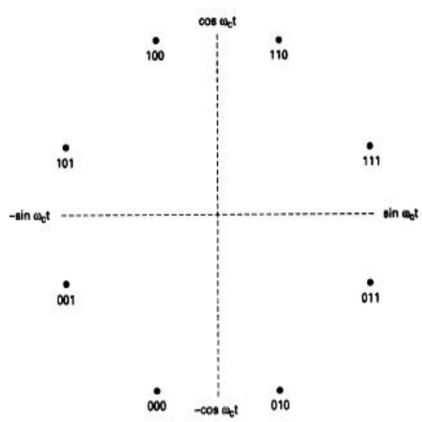
logic 1= 1.307 V and logic 0 =0.541 V

The output of the linear summer:

QIC@000	QIC@001	QIC@010	QIC@011
IC = -0.541 sin $\omega_c t$	IC = -1.307 sin $\omega_c t$	IC = +0.541 sin $\omega_c t$	IC = +1.307 sin $\omega_c t$
QC = -1.307 cos $\omega_c t$	QC = -0.541 cos $\omega_c t$	QC = -1.307 cos $\omega_c t$	QC = -0.541 cos $\omega_c t$



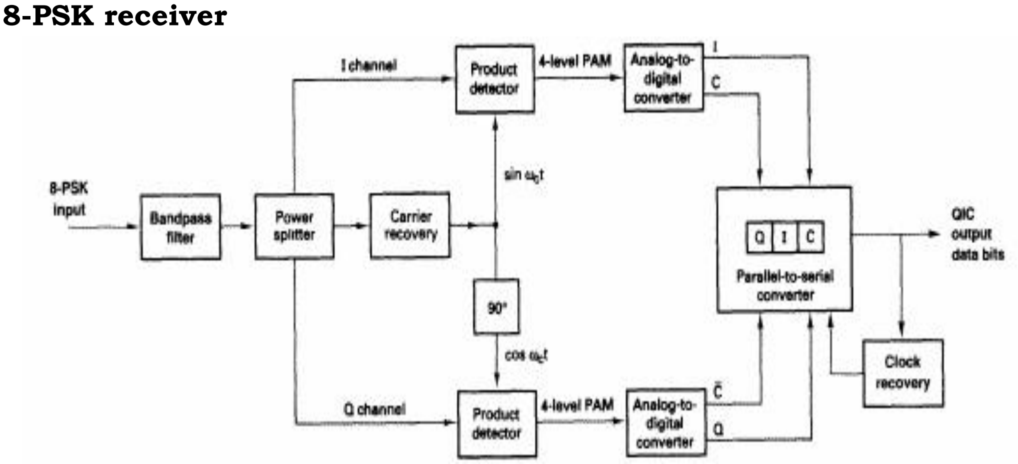
(b)



(c)

Q	I	C	Output phase
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

8-PSK receiver

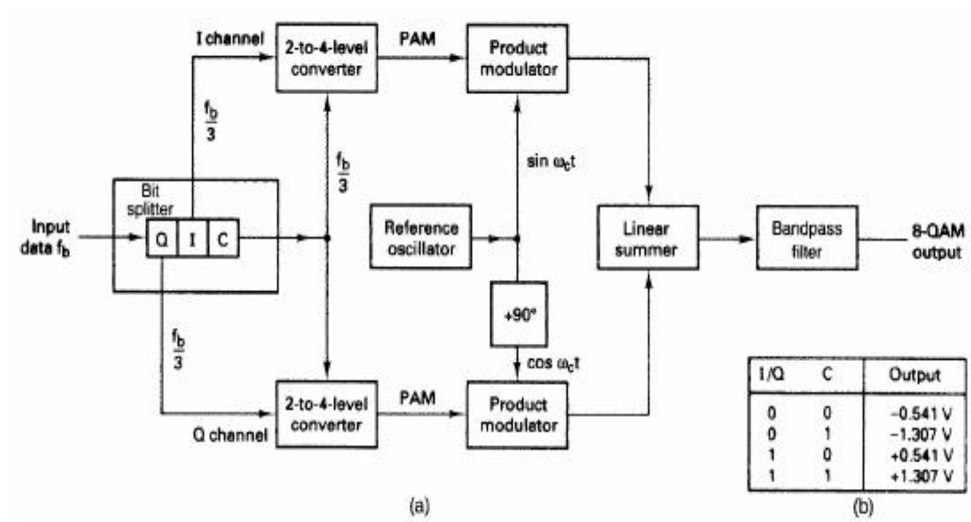


QUADRATURE – AMPLITUDE MODULATION

8-QAM is an M-ary encoding technique where M = 8. Unlike 8-PSK, the output signal from an 8-QAM modulator is not a constant-amplitude signal. There are two output amplitudes, and only four phases are possible (+45°,+135°, -45°, and -135°)

8-QAM transmitter

Q	I	C	amplitude	Output phase
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



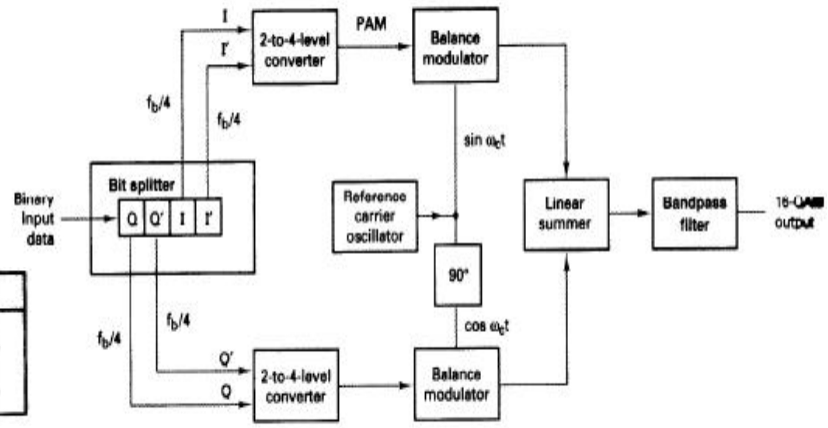
An 8-QAM receiver is almost identical to the 8-PSK receiver

16-QAM

16-QAM is an M-ary system where M =16. The input data are acted on in groups of four ($2^4 = 16$). Both the phase and the amplitude of the transmit carrier are varied. Listed are possible output phases (±15°,±45°,±75°, ±105, ±135°,±165°)

The I and Q bits determine the polarity logic 1 = positive
logic 0 = negative

The I' and Q' determine the magnitude logic 1 = 0.821V
logic 0 = 0.22 V



I	I'	Output
0	0	−0.22 V
0	1	−0.821 V
1	0	+0.22 V
1	1	+0.821 V

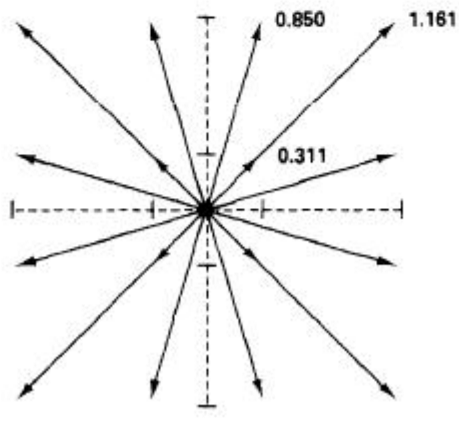
(a)

Q	Q'	Output
0	0	−0.22 V
0	1	−0.821 V
1	0	+0.22 V
1	1	+0.821 V

(b)

Binary input				16-QAM output	
Q	Q'	I	I'		
0	0	0	0	0.311 V	−135°
0	0	0	1	0.850 V	−165°
0	0	1	0	0.311 V	−45°
0	0	1	1	0.850 V	−15°
0	1	0	0	0.850 V	−105°
0	1	0	1	1.161 V	−135°
0	1	1	0	0.850 V	−75°
0	1	1	1	1.161 V	−45°
1	0	0	0	0.311 V	135°
1	0	0	1	0.850 V	165°
1	0	1	0	0.311 V	45°
1	0	1	1	0.850 V	15°
1	1	0	0	0.850 V	105°
1	1	0	1	1.161 V	135°
1	1	1	0	0.850 V	75°
1	1	1	1	1.161 V	45°

(a)



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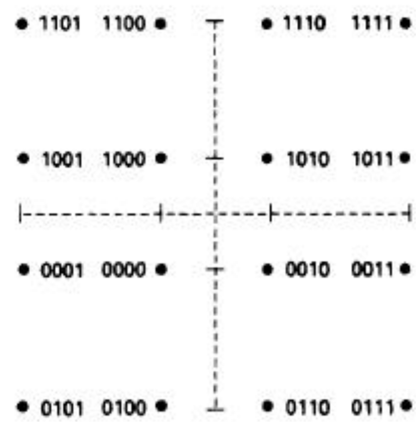

(c)

ASK, FSK, PSK and QAM summary

Modulation	Encoding Scheme	Outputs Possible	Minimum Bandwidth	Baud
ASK	Single bit	2	f_b	f_b
FSK	Single bit	2	f_b	f_b
BPSK	Single bit	2	f_b	f_b
QPSK	Dibits	4	$f_b/2$	$f_b/2$
8-PSK	Tribits	8	$f_b/3$	$f_b/3$
8-QAM	Tribits	8	$f_b/3$	$f_b/3$
16-PSK	Quadbits	16	$f_b/4$	$f_b/4$
16-QAM	Quadbits	16	$f_b/4$	$f_b/4$