Chapter Six: Deep beams and Corbels

6.1 Deep Beams

Deep beams are structural elements loaded as beams but having a large depth/thickness ratio and a shear span/depth ratio not exceeding 2 for simple span and 2.5 for continuous span, where the shear span is the clear span \( l_n \) of the beam for distributed load and the distance between the point of application of the load and the face of the support \( a \) for concentrated load. This definition is somewhat arbitrary. A better definition is: A deep beam is a beam in which a significant amount of the load is carried to the supports by a compression thrust joining the load and the reactions. This occurs if the above proportions are maintained.

Deep beams are usually found in transfer girders (girders support one or more columns transferring it laterally to other columns) used in multi storey buildings to provide column offsets, in foundation walls, pile caps, walls of rectangular tanks and bins, floor diaphragms, and shear walls.

Deep beams are usually loaded along the top edge with reactions provided at the bottom (See Fig. 6.1a). However in some cases, e.g., the side walls of storage bins, the loads may be applied along the bottom edge (See Fig. 6.1b). Deep beams may be simply supported or continuous.

![Deep beam diagrams](image)

**Fig. 6.1 Placement of loads on deep beams**

**Behavior of Deep Beams**

Because of the proportions of deep beams, they behave as two-dimensional rather than one dimensional member and are subjected to a two-dimensional state of stress. As a result plane sections before bending do not necessarily remain plane after bending. The resulting strain distribution is no longer considered linear, and shear deformations that are neglected in normal beams become significant compared to pure flexure. Consequently the stress block becomes nonlinear even at the elastic stage and the flexural stress at the bottom is constant over much of the span (See Fig 6.3b).

In the case of a single span beam supporting a concentrated load at mid span (See Fig. 6.2b), the principal compressive stresses act roughly parallel to the lines joining the load and the supports and the largest principal tensile stresses act parallel to the bottom of the beam. The stress trajectories in Fig. 6.2b can be simplified to the pattern given in Fig. 6.2c, which can further simplified to the model shown in Fig. 6.2d. If such a beam were tested, the crack pattern would be as shown in Fig. 6.2e.
The stress trajectories, distribution of horizontal flexural stresses and the truss models for an uncracked elastic, single span beam supporting a uniform load are shown in Fig. 6.3.

In both cases the cracks would almost be vertical or follow the direction of the compression trajectories, with the beam almost shearing off from the support in a total shear failure.

Fig. 6.2 Single span deep beam with concentrated load

Fig. 6.3 Uniformly loaded single span deep beam
The flexural strengths of deep beams can be predicted with sufficient accuracy using the same methods employed for beams of normal proportions. The equivalent rectangular stress block and the associated parameters can be employed without change.

Shear strengths of deep beams may be as much as 2 or 3 times that predicted using code equations for normal beams. For normal beams, it was explained that shear transferred in diagonally cracked beams is usually assumed to take place by four mechanisms: (a) direct transfer in the uncracked concrete compression zone, (b) aggregate interlock, (c) dowel action of the main flexural reinforcement, and (d) direct tension in the web steel. For deep beams, however, a significant part of the load is transferred directly from the point of application to the supports by diagonal compression struts (See Fig. 6.4a). Diagonal cracks that form roughly in a direction parallel to a line from the load to the support isolate a compression strut, which acts with the horizontal compression in the concrete and the tension in the main reinforcement to equilibrate the loads. The geometry of this mechanism and the relative importance of each contribution to the shear strength clearly depend on the proportion of the member as well as the placement of the loads and reactions.

Due to the above special features of deep beams:
- The main flexural steel is placed near the tension edge, as usual, although because of the greater depth of the tension zone, it may be advisable to distribute such steel over, say, the bottom third of the member.
- Because the ultimate strength of deep beams depends upon strut-and-tie action, in which the main steel is fully stressed over nearly its entire length rather than only at the maximum moment section, special attention must be paid to the anchorage of such steel. Hooks and bends are normally used, even though deformed bars are specified.
- Due to the steep gradient of principal stress trajectories in deep beams, while it is important to include vertical stirrups, they are apt to be less effective than horizontal web steel placed as shown in Fig. The horizontal bars are effective not only because they act in the direction perpendicular to the diagonal crack, thus improving shear transfer by aggregate interlock, but also because they contribute shear transfer by dowel action. Thus, horizontal reinforcement is needed throughout the height of the beams, in addition to the vertical shear reinforcement along the span.

(a) Loads reactions, and internal forces  (b) cross sections  (c) reinforcement

Fig. 6.3 Deep beam carrying concentrated load
Figure 6.4a presents stress trajectories of the principal tensile and compressive stresses in a continuous deep beam. Comparing this diagram to Figure 6.2b for the simply supported case, one can observe the similarity of the steepness of the tensile stress trajectories at mid span. At the continuous supports, the total section is in tension.

Design of deep beams for shear, EBCS 2, 1995

Definitions and Limitations

- For a given shear span, a principal load is a concentrated load which causes 50 percent or more of the shear at the support of that shear span.
- The shear span $a_v$ shall be taken equal to the distance from the center of the principal load to the center of the support. This span shall not be more than 1.15 times the clear distance from the face of the load to the face of the support.
- The shear spans $l_s$ for beams supporting uniform load shall be taken equal to the distance from the point of zero shear to the center of the support but not more than 1.15 times the clear distance from the point of zero shear to the face of the support.

Shear strength of Deep shear Spans

- The shear resistance of deep shear spans $S_{rd}$ shall be obtained as the sum of the resistances of the concrete $V_{cd}$ and the vertical and horizontal stirrups $V_s$ and $V_h$, respectively.
- The applied shear $V_{sd}$ shall not exceed $V_{rd}$, the limiting value of ultimate shear, i.e.:

$$V_{rd} = 0.25 f_{cd} b_w d$$

Shear Carried by Deep shear Spans

1) For deep shear span supporting a principal load:
   a) The shear resistance $V_{rd}$ shall be computed at $A_v/2$, the shear reinforcement required at this section shall be used throughout the entire shear span.
   b) The shear force $V_c$ carried by the concrete shall be given by:

$$V_c = \beta \times 0.25 f_{cd} K_1 K_2 b_w d$$

Where: $\beta = \frac{2d}{a_v} \geq 1$
$K_1 = (1 + 50 \rho) \leq 2$
$K_2 = 1.6 - d \geq 1$, $d$ in m,
\[ \rho = \frac{A_s}{b_w d} \]

c) The shear force \( V_c \) transferred by vertical stirrups shall be given by

\[ V_c = \frac{A_v f_yd (a_v - d/2)}{S_v} \leq \frac{A_{vd} f_yd}{S_v} \]

d) The shear force \( V_h \) transferred by horizontal stirrups shall be given by

\[ V_h = \frac{A_{vh} f_yd (3d/2 - a_v)}{S_h} \leq \frac{A_{vd} f_yd}{S_h} \]

Where:
- \( A_v \) is the area of vertical stirrups
- \( A_{vh} \) is the area of horizontal stirrups
- \( S_v \) is the spacing of the vertical stirrups (\( S_v \leq d/4 \))
- \( S_h \) is the spacing of the horizontal stirrups (\( S_h \leq d/3 \))

2) For deep shear spans not supporting a principal load, beams supporting uniform loads, the above provision apply with \( a_v/2 \) replaced by \( l_v/3 \).

**Design of deep beams for Flexure**

**Simply Supported Beams**

EBCS 2 does not specify a design procedure but requires a rigorous nonlinear analysis for the flexural analysis and design of deep beams. The simplified provisions presented in this section are based on the recommendation of the Euro-International Concrete Committee (CEB). Though the lever arm in a cracked section may slightly be increased, for design of simple span beam, one may use:

\[ Z = 0.2(L+2h) \text{ for } 1 \leq L/h \leq 2 \]

And

\[ Z = 0.6L \text{ for } L/h > 1 \]

Where \( L \) is the effective span measured center to center of supports and shall be less than 1.15 times the clear span.

The bending moment may be obtained using elastic theory in the same way as normal beams and

\[ M_{rd} = A_s f_yd Z \]

The reinforcements are detailed in such a way that:
- Flexural reinforcement bars are uniformly placed using relatively small size bars over a vertical distance of \( (0.25h-0.05L) \leq 0.2h \) for positive moment, where the vertical distance is measured from the bottom extreme face of the cross section.
- As the centrally applied load is disposed primarily by arch action, it necessitates very good anchorages and the extension of the entire flexural reinforcement to the supports.

**Continuous Beams:**

Continuous deep beams can be treated in the same manner as simply supported deep beams, except that additional reinforcement has to be provided for the negative moment at the support.
The concentration of the tensile stress trajectories at the support regions of the continuous deep beam necessitates a concentration of well-anchored horizontal shear reinforcement. The required total flexural reinforcement area:

For multiple spans, the lever arm for both negative and positive moment may be computed using:

\[ Z = 0.2(L+1.5h) \text{ for } 1 \leq L/h \leq 2.5 \]
\[ Z = 0.5L \text{ for } L/h < 1 \]

The equation for simply supported beam holds true for multi span beams.

The reinforcements are detailed in such a way that:

- Half the negative moment bars over supports should extend overall full length of adjacent spans.
- Negative bars are uniformly distributed in two bands in such a way that:
  \[ As1 = 0.5(L/h-1)As \text{ in the upper bands for } 0.2h \]
  \[ As2 = As - As1 \text{ in the lower band with in a depth of } 0.6h \text{ below the upper band (See the figure).} \]

Sequence of Deep Beams Design Steps for shear

The following is a recommended procedure for the design of shear reinforcement in deep beams based on ACI requirements. The sequence of steps should essentially be similar to that the web reinforcement design in normal beams. Additionally, flexural reinforcement has to be provided to resist the stresses due to bending.

1. Check whether the beam can be classified as a deep beam, that is, \( a/d < 2.5 \) or \( Ls/d < 5.0 \) for a concentrated or a uniform load, respectively.
2. Determine the critical section distances for shear from the face of support for concentrated load and for distributed load. Calculate the factored design shear force \( V_{sd} \) at the critical section, and check with the limiting value for diagonal compression failure.
3. Calculate the shear resisting capacity \( V_c \), of the plain concrete.
4. Calculate \( V_s \), if \( V_{sd} > V_c \) and choose \( S_v \) and \( S_h \) by assuming the size of shear reinforcement in both the horizontal and vertical directions.
5. Verify if the size and maximum spacing from step 4 satisfy the criteria of EBCS 2.
6. Select reasonable size and spacing of the shear reinforcement in both horizontal and vertical directions. Where possible, use welded wire fabric mats since they provide superior anchorage of the reinforcement to tied bar mats and are easier to handle and keep in position at both faces of the deep beam.
6.2 Brackets and Corbels

A bracket or corbel is a short member that cantilevers out from a column or wall to support a load. The corbel is generally built monolithically with the column or wall. It is widely used in pre-cast construction for supporting pre-cast beams at the columns. The term corbel is generally restricted to cantilevers having shear span to depth ratios, $a_v/d$, less than or equal to 1.

Brackets or corbels are designed mainly to provide for the vertical reaction $V_{sd}$ at the end of the supported beam, but unless special precautions are taken to avoid horizontal forces caused by restrained shrinkage, creep or temperature change, they must also resist a horizontal force $N_{sd}$.

Steel bearing plates or angles are generally used at the top surface of the brackets, as shown, to provide a uniform contact surface and to distribute the reaction. A corresponding steel bearing plate or angle is usually provided at the lower corner of the supported member.

**Structural Action**

The structural performance of a bracket can be visualized easily by means of the strut and tie model shown in Fig. the downward thrust of the load $V_{sd}$ is equilibrated by the vertical component of the reaction from the diagonal compression strut that carried the load down in to the column the out ward thrust at the top of the strut is balanced by the tension in the horizontal tie bars across the top of the bracket; these also take the tension, if any, imparted by the horizontal force $N_{sd}$. At the left end of the horizontal tie, the tension is equilibrated by the horizontal component of trust from the second compression strut shown. The vertical component of this strut requires the tensile forces shown acting downward at the left side of the supporting column.

The most common failure modes of corbels are: yielding of the tension tie (flexural tension), either under the load point or in the column; failure of the compression strut by crushing or shearing (diagonal splitting); and local failures under the bearing plate. If the tie reinforcement is hooked downward, as shown in Fig. , the concrete outside of the hook may split off, causing failure. If the corbel is to shallow at the outside end, there is a danger that cracking may extend through the corbel as shown in Fig.

**Design of Corbels, EBCS 2**

**Definitions and limitations**

1) Theses provisions apply to corbels having a shear span to depth ratio $a_v/d$ of unity or less.

2) The distance $d$ shall be measured at a section adjacent to the face of the support, but shall not be taken greater than twice the depth of the corbel at the outside edge of the bearing area.

**Design**

1) corbels with $0.4d \leq a_v \leq d$ may be designed using a simple strut and tie model
2) Corbels for which $a_v > d$ may be designed as cantilever

3) Unless special provision is made to limit horizontal forces on the support, or other justification is given, the corbel shall be designed for the vertical forces $F_v$, and the horizontal force $H_c \geq 0.2F_v$ acting at the bearing area.

4) The effective depth $d$ of the corbel shall be determined from consideration of shear

Consider the corbel element shown under $N_{vd}$ and internal forces developed due to such actions.

**Detailing**

As the bracket dimensions are small, special attention must be paid to provide proper anchorage for all bars. The main tensile bars of are $A_{sv}$ must develop their full yield strength directly under the load $N_{vd}$, and for this reason are usually anchored by welding to the bearing plate or angle.

The modes of failure due to direct shear along a plane more or less flush with the vertical face of the main part of the column can be controlled by shear friction reinforcement crossing such a crack, which includes the area $A_{sv}$, and $A_h$ (see Fig.)

The bars providing $A_h$ are placed in the form of closed hoops, and usually of about the same diameter as the stirrups, and serve mainly to improve stirrup anchorage at the outer end of the bracket.