

CHAPTER ONE

Basic Principles of Electrical Machines Analysis

ELECTROMAGNETIC PRINCIPLES

Electric machines and electromechanical devices are made up of coupled electric and magnetic circuits. An Electrical machine converts energy from one form to another. Electrical machines based on power flow can be classified into DC and AC Machines and these also can be classified each into Motor and Generator.

- Motor- Converts Electrical energy into Mechanical energy
- Generator- Converts Mechanical into Electrical energy

Any motor and generator has three main parts

1. Rotor (rotating part)
2. Stator (stationary part)
3. Enclosure (protect the stator & the rotor)

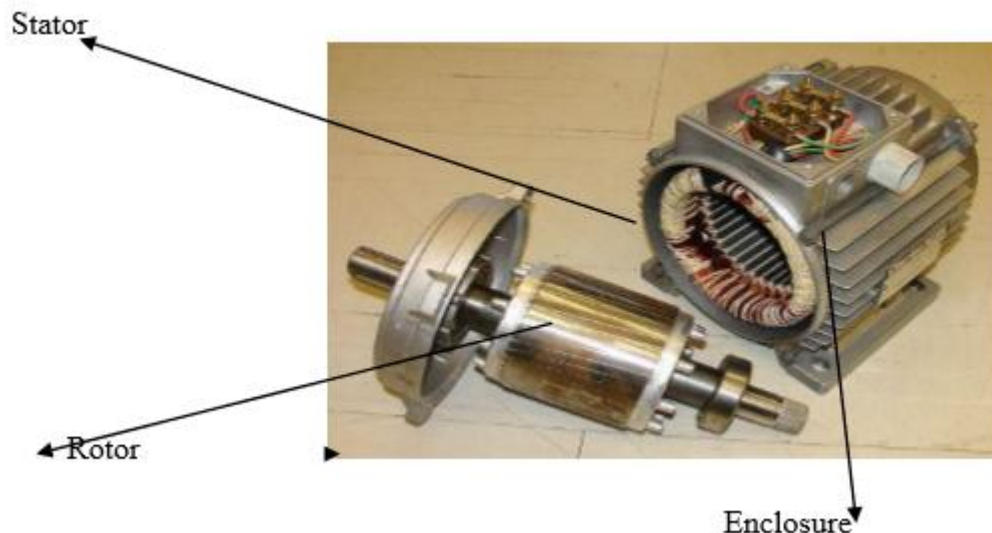


Fig 1 Main parts of Electrical Machine

1. THE MAGNETIC ASPECTS OF MACHINES

1.1. Magnetics

Magnet is a substance that attracts the iron pieces and pieces of some other metals. A magnet can be classified into Permanent and Temporary magnet.

- **Permanent magnet** is a magnet which retains its property of magnetism indefinite period of time. This is made up of cobalt, steel or tungsten steel. The significance of permanent magnet is that it can produce magnetic flux in a magnetic circuit in the absence of an

external source. It is used in moving coil instruments, energy meters, loud speakers, microphone etc.

Permanent magnetization or residual flux density (B_r) and coercivity are the two important qualities of permanent magnet.

- ◆ **Residual flux density (B_r)** it is the flux density trapped in closed magnetic structure if the applied emf (and therefore the magnetic field intensity H) were reduced to zero.
- ◆ **Coercivity** it is the measure of mmf or H which, applied to the magnetic curved, carried reduce its flux density to zero. Its value is negative and in units of kA/m.

➤ **Temporary magnet (Electromagnet)**

The material used here is soft iron or silicon steel. A soft iron piece with a coil acts as a magnet as long as current flows through the coil. It is used in electrical machine such as motor and generator.

- **Magnetic Lines of Force** are a magnet lines which travels from South Pole to North Pole inside the magnet.
- **Magnetic field** is a region around the magnetic force of lines.

1.2. Magnetic Circuit

Magnetic circuit is a closed path followed by magnetic lines of forces or simply a path for magnetic flux, just as an electric circuit provides a path for the flow of electric current. Source of magnetic fluxes are electric current and permanent magnets. In electric machines, currentcarrying conductors interact with magnetic fields, resulting in electromechanical energy conversion.

A. Magnetic flux (ϕ) is the total number of lines of force in a magnetic field. Source of ϕ are Electromagnet and Permanent. ϕ produced due to the flow of current in wire is Electromagnet. The direction of ϕ can be found from the Right hand thumb rule.



- *It states that when a current carrying conductor is held on the right hand, then the thumb indicates the direction of current and the fingers pointing in the direction of Magnetic flux.*

SI unit of ϕ is Weber (wb)

B. Magnetic Flux Density (B)

A more useful measure of the magnetic effect is the magnetic flux density, which is the magnetic flux per unit cross-sectional area. We will consider two equal areas through which the magnetic flux penetrates at right angles near one end of the permanent magnet along its centerline. From the illustration it becomes apparent that there is a greater amount of magnetic flux passing through an area that is nearer the magnet pole. In other words, the magnetic flux density increases as we approach closer to the end of the magnet. However, it must be noted that the magnetic flux density inside the magnet is uniformly constant. Magnetic flux density is measured in units of tesla (T) and is given the symbol B. One tesla is equal to 1 weber of magnetic flux per square meter of area. We can state that

$$B = \frac{\phi}{A} \left(\frac{\text{wb}}{\text{m}^2} \right)$$

Where B = magnetic flux density, T

ϕ = magnetic flux, Wb

A = area through which ϕ penetrates perpendicularly, m²

Example 1 The total magnetic flux out of a cylindrical permanent magnet is found to be 0.032 mWb. If the magnet has a circular cross section and a diameter of 1 cm, what is the magnetic flux density at the end of the magnet?

Solution

The total flux = 0.032×10^{-3} Wb, cross-sectional area of magnet:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.01)^2}{4} = 78.53 \times 10^{-6} \text{ m}^2$$

$$B = \frac{\Phi}{A} = \frac{0.032 \times 10^{-3}}{78.53 \times 10^{-6}} = 0.407 \text{ T}$$

Note that this magnetic flux density exists only at the immediate end of the magnet. As we move away from the end of the magnet, the magnetic flux spreads out, and therefore the magnet flux density decreases.

C. Magneto-motive Force (mmf)

We have seen that an increase in the magnitude of current in a coil or a single conductor results in an increase in the magnetic flux. If the number of turns in a coil are increased (with the current remaining constant), there is an increase in magnetic flux. Therefore, the magnetic flux is proportional to the products of amperes and turns. This ability of a coil to produce magnetic flux is called the **magnetomotive force**. Magnetomotive force is abbreviated MMF and has the units of ampere-turns (At). The magnetomotive force is given the symbol F_m . Strictly speaking, the units of MMF are amperes because turns are dimensionless quantities. However, from a pedagogical standpoint, we prefer and shall use

throughout this book the units of ampere-turns (At) for MMF. We may write

$$\mathcal{F}_m = NI$$

where $\mathcal{F}_m$ = magnetomotive force (MMF), At

N = number of turns in the winding

I = electric current passing through the coil

Magnetomotive force in the magnetic circuit is analogous to electromotive force in an electric circuit.

D. **Magnetic Flux Intensity (H)** is the mmf per unit length and given by :

$$H = \frac{mmf}{l} \text{ (AT/M)} \text{ and also given by: } H = \frac{F}{\phi} \text{ (N/wb)}$$

E. **Reluctance (R)** is an opposition offered by magnetic material to the flow of magnetic flux. It is defined as the ratio of mmf to the flux.

$$R = \frac{l}{\mu A} \quad \text{or} \quad R = \frac{mmf}{\phi}$$

Where l = mean or average length of the core, (m)

μ = permeability of the magnetic material

A = Area of cross section, (m²)

F. **Permeability (μ)** is the magnetic conductivity of iron as compared to that of air.

The permeability of magnetic material is given by:

$$\mu = \frac{B}{H}$$

μ also given by $\mu = \mu_r \mu_0$ where μ_r = relative permeability

μ_0 = permeability of free space ($4\pi \times 10^{-7}$

H/m) Typical value of μ_r is from 2000-80000 and the value of μ_r for air, free space and vacuum is 1

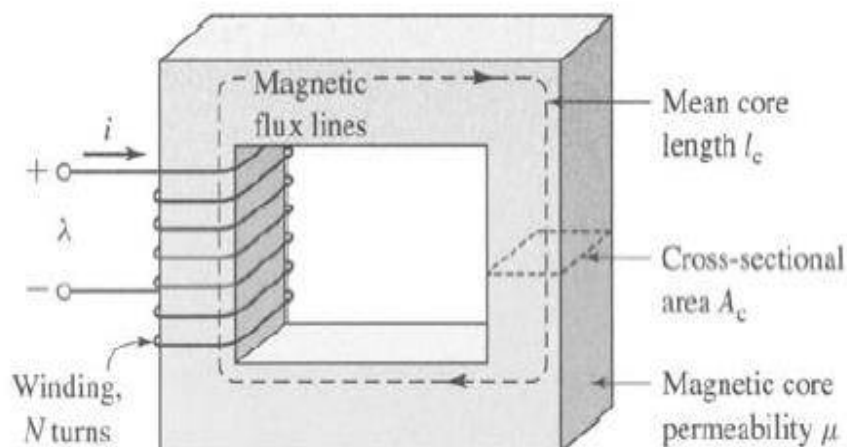


Figure 2 Simple magnetic circuits

A simple example of a magnetic circuit is shown in Fig. 2. The core is assumed to be composed of magnetic material whose permeability is much greater than that of the surrounding air ($\mu \gg \mu_0$). The core is of uniform cross section and is excited by a winding of N turns carrying a current of i amperes. This winding produces a magnetic field in the core, as shown in the figure.

Because of the high permeability of the magnetic core, an exact solution would show that the magnetic flux is confined almost entirely to the core, the field lines follow the path defined by the core, and the flux density is essentially uniform over a cross section because the cross-sectional area is uniform. The magnetic field can be visualized in terms of flux lines which form closed loops interlinked with the winding.

As applied to the magnetic circuit of Fig.2, the source of the magnetic field in the core is the ampere-turn product Ni . In magnetic circuit terminology Ni is the *magneto motive force* (mmf) F acting on the magnetic circuit. Although Fig. 2 shows only a single coil, transformers and most rotating machines have at least two windings, and Ni must be replaced by the algebraic sum of the ampere-turns of all the windings.

The magnetic circuit
 $l_c = 30$ cm, and $N =$
 reluctances $\mathcal{R}_c$ and $\mathcal{R}_g$
 find (b) the flux ϕ and

■ **Solution**

a. The reluctances $\mathcal{R}_c$

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.3}{70,000 (4\pi \times 10^{-7}) (9 \times 10^{-4})} = 3.79 \times 10^3 \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{5 \times 10^{-4}}{(4\pi \times 10^{-7}) (9 \times 10^{-4})} = 4.42 \times 10^5 \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

b. From Eq. 1.4,

$$\phi = B_c A_c = 1.0 (9 \times 10^{-4}) = 9 \times 10^{-4} \text{ Wb}$$

c. From Eqs. 1.6 and 1.15,

$$i = \frac{\mathcal{F}}{N} = \frac{\phi (\mathcal{R}_c + \mathcal{R}_g)}{N} = \frac{9 \times 10^{-4} (4.46 \times 10^5)}{500} = 0.80 \text{ A}$$

Fig 3 Magnetic circuit with air gap

Finally for any magnetic circuit with total reluctance the magnetic flux is given by:

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_c + \mathcal{R}_g}$$

Example 1

1.3 Electro Mechanical Energy Conversion Process

Converting of energy from one form of energy to another form of energy is called energy conversion. Therefore, electromechanical energy conversion is the conversion of energy either from electrical to mechanical or from mechanical to electrical.

When we speak of electromechanical energy conversion, however, we mean either the conversion of electric energy into mechanical energy or vice versa. For example, an electric motor converts electric energy into mechanical energy. On the other hand, an electric generator transforms mechanical energy to electric energy.

Electromechanical energy conversion takes place via the medium of a magnetic field or an electric field, but most practical converters use magnetic field as the coupling medium between electrical and mechanical systems, this is because the

electric storing capacity of the magnetic field is much higher than that of the electric field.

Electromechanical energy conversion is a *reversible* process except for the losses in the system. The term "*reversible*" implies that the energy can be transferred back and forth between the electrical and the mechanical systems. However, each time we go through an energy conversion process, some of the energy is converted into heat and is lost from the system forever.

When the conversion takes place from electrical to mechanical form, the device is called the motor, and when the mechanical energy is converted to electrical energy, the device is called a generator. In these machines, conversion of energy from electrical to mechanical form or from mechanical to electrical form results from the following two electromagnetic phenomena:

1. When a conductor is allowed to move in a magnetic field, a voltage is induced in the conductor.
2. When a current-carrying conductor is placed in a magnetic field, then a mechanical force is experienced by the conductor.

In monitoring action, current flows through the conductors placed in a magnetic field. A force is produced on each conductor. The conductors are placed on the rotor which is free to move. An electromagnetic torque produced on the rotor is transferred to the shaft of the rotor and can be utilized to drive a mechanical load. Since the conductors rotate in the magnetic field, a voltage is induced in each conductor. In generating action, the rotor is driven by a prime mover. A voltage is induced in the rotor conductors. If an electrical load is connected to the winding formed by these conductors, a current will flow, delivering electric power to the load. Moreover, the current flowing through the conductors will interact with the magnetic field to produce a reaction torque, which will tend to oppose the torque developed by the prime mover.

When a current-carrying conductor is placed in a magnetic field, it experiences a force that tends to move it. If the conductor is free to move in the direction of the magnetic force, the magnetic field aids in the conversion of electric energy into mechanical energy. This is essentially the principle of operation of all electric motors. On the other hand, if an externally applied force makes the conductor move in a direction opposite to the magnetic force, the mechanical energy is converted into electric energy. Generator action is based upon this principle. In both cases, the magnetic field acts as a medium for the energy conversion.

The conversion of energy from one form into another satisfies the principle of conservation of energy. Therefore, the input energy W_i , is equal to the sum of the

useful output energy W_o , the loss in energy as heat W_l , and the change in the stored energy in the field W_f . That is,

$$W_i = W_o + W_l + W_f \quad \text{Eq. (1.1)}$$

Electric Machines converts Mechanical energy to electrical energy or vice versa.

Mechanical energy generator → Electric energy

Electrical energy motor → Mechanical energy:

Almost all practical motors and generators convert energy from one form to another through the action of a magnetic field.

The energy flow diagram is shown in Figure 1.1. The output energy and the loss in energy are considered positive quantities. The change in the stored energy, on the other hand, may be positive or negative, depending upon whether it is increasing or decreasing. In the above equation, if W_i represents the electrical energy input, then W_o , may be the electric equivalent of the mechanical energy output.

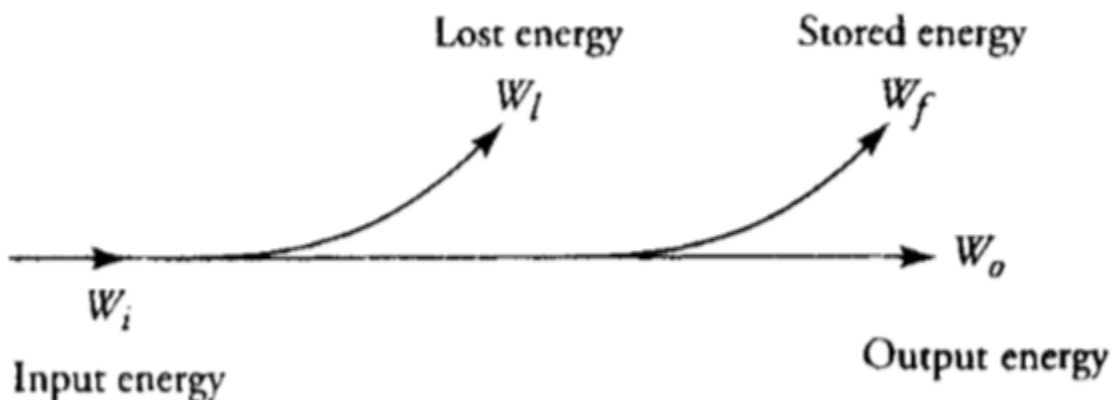


Figure y: Energy flow diagram

The system is said to be conservative or lossless if the loss in energy in the system is zero. In that case, Eq. (1.1) becomes

$$W_i = W_o + W_f$$

If the output energy of a system is zero, then the input energy must either

- Increase the stored energy of the system, or
- be dissipated as heat by the system, or
- Both of them

1.2 ENERGY BALANCE

The principle of conservation of energy states that energy is neither created nor destroyed; it is merely changed in form.

Energy balance represents the difference between energy intake and energy expenditure. When the energy intake exceeds the energy expenditure, there is a positive energy balance, which results in weight gain. When the energy intake is below the energy expenditure, there is a negative energy balance and weight loss results. Over the long term, energy balance is maintained in weight-stable individuals, even though on a day-to-day basis this balance may sometimes be positive and sometimes negative. Although life is not always that simple, basic manipulation of the energy balance equation will yield weight loss or gain. For example, if someone wishes to lose weight, it is important to increase the energy expenditure relative to the energy intake or decrease energy intake below daily energy expenditure. The opposite would be true if an individual is trying to achieve weight gain.

For an electro-mechanical system, following terms are important:

- (i) Electrical port (armature terminals): receiving / delivering electrical energy.
- (ii) Mechanical port (shaft): delivering / receiving mechanical energy.
- (iii) Coupling field: Magnetic field *or* Electric field.

Even though, theoretically, both the types of fields mentioned above are able to convert the energy, the magnetic medium is most popular since the voltage levels required are not very high, and the devices of given power rating are smaller in size and are economical. Hence, only those will be dealt with.

It is obvious that an electrical motor receives energy at the electrical port and delivers it at the mechanical port. While an electric generator receives the energy at the mechanical port and delivers it at the electrical port. It is also known Conversion of electrical energy into mechanical energy that the following losses take place in such systems and are dissipated away as heat:

- (i) i^2r Losses in the windings of the machines,
- (ii) Friction and wind-age losses,
- (iii) core-losses.

Energy balance equation is written for motor action below:

$$\left[\begin{array}{c} \text{Energy input} \\ \text{from electric} \\ \text{sources} \end{array} \right] = \left[\begin{array}{c} \text{Mechanical} \\ \text{energy} \\ \text{output} \end{array} \right] + \left[\begin{array}{c} \text{Increase in} \\ \text{energy stored} \\ \text{in magnetic field} \end{array} \right] + \left[\begin{array}{c} \text{Energy} \\ \text{converted} \\ \text{to heat} \end{array} \right]$$

For generator action, the energy balance equation is written as:

$$\left[\begin{array}{c} \text{Total mechanical} \\ \text{energy input} \end{array} \right] = \left[\begin{array}{c} \text{Electrical energy} \\ \text{output} \end{array} \right] + \left[\begin{array}{c} \text{Total energy} \\ \text{stored} \end{array} \right] + \left[\begin{array}{c} \text{Total energy} \\ \text{dissipated} \end{array} \right]$$

For lossless magnetic-energy-storage system;

Change in Electrical Energy = Change in Mechanical Energy + Change in Field-Energy

$$dW_{elec} = dW_{mech} + dW_{fld}$$

Where, dW_{elec} : Differential electrical energy input

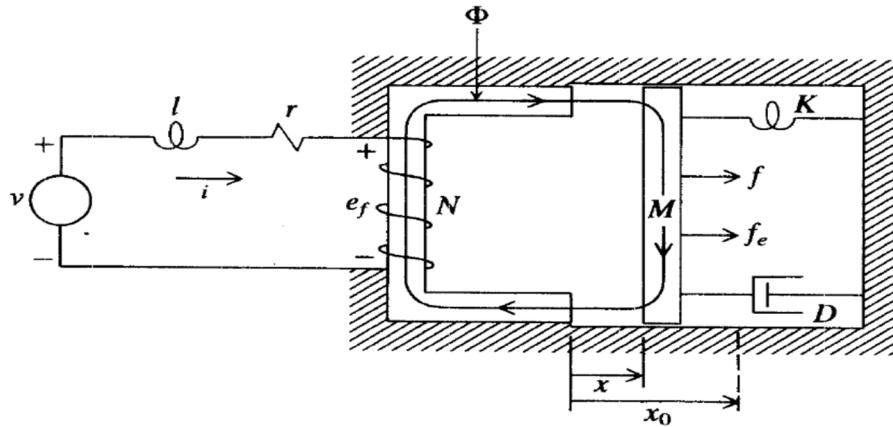
dW_{mech} : Differential mechanical energy output

dW_{fld} : Differential change in magnetic stored energy

Dynamic equations:

Consider the electromechanical system given below

Initially, the displacement x_0 is the zero and force f_e is zero.



The equation for the electric system is-

$$V = ri + L \frac{di}{dt} + e_f$$

e_f is voltage drop across coupling field

The total energy supplied by the electric source is-

$$W_E = \int Vi \, dt = \int \left[ri + L \frac{di}{dt} + e_f \right] i \, dt$$

The equation for the mechanical system is-

Spring

$$f_k = -K(x - x_0) \quad K: \text{Spring constant (N/m)}$$

Damper:

$$f_D = -D \frac{dx}{dt} \quad D: \text{Damping constant (N.s/m)}$$

Mass

$$f_M = -M \frac{d^2x}{dt^2} \quad M: \text{Mass (Kg)}$$

When there is displacement in the movable core, the dynamic behavior of translational mechanical system may be expressed by Newton's law of equation:

$$f_{fld} = M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + K(x - x_0) + f_e$$

The total energy equation for the mechanical system is-

$$W_M = \int f dx$$

Substituting f from the equation of motion-

$$W_M = \int f dx = \int \left[M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + K(x - x_0) - f_e \right] dx$$

Where;

$M \frac{d^2x}{dt^2}$ = Kinetic energy stored in the mass

$D \frac{dx}{dt}$ = Heat loss due to the friction

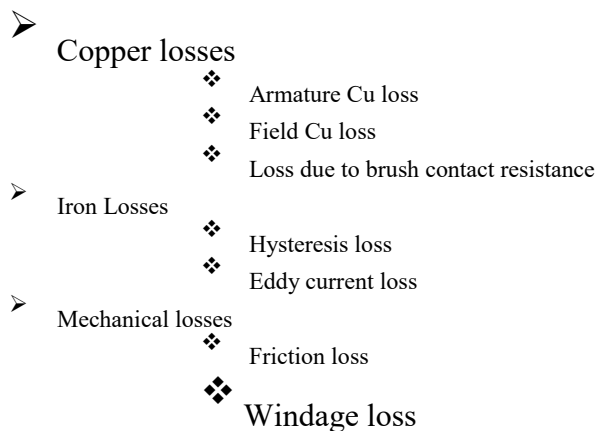
$K(x - x_0)$ = Potential energy stored in the spring

f_e = Total energy due to the coupling field from the mechanical system

Power flow diagram

The most convenient method to understand these losses in a generator or a motor is using the power flow diagram. The diagram visualizes the amount of power that has been lost in various types of losses and the amount of power, which has been actually converted into the output.

Losses in a rotating machine



The above tree categorizes various types of losses that occur in a dc generator or a dc motor. Each of these are explained in details below.

Copper losses

These losses occur in armature and field copper windings. **Copper losses** consist of Armature copper loss, Field copper loss and loss due to brush contact resistance.

Armature copper loss = $I_a^2 R_a$ (where, I_a = Armature current and R_a = Armature resistance) This loss contributes about 30 to 40% to full load losses. The armature copper loss is variable and depends upon the amount of loading of the machine.

Field copper loss = $I_f^2 R_f$ (where, I_f = field current and R_f = field resistance) In the case of a shunt wound field, field copper loss is practically constant. It contributes about 20 to 30% to full load losses.

Brush contact resistance also contributes to the copper losses. Generally, this loss is included into armature copper loss.

Iron losses (Core losses)

As the armature core is made of iron and it rotates in a magnetic field, a small current gets induced in the core itself too. Due to this current, eddy current loss and hysteresis loss occur in the armature iron core. Iron losses are also called as *Core losses or magnetic losses*.

Hysteresis loss is due to the reversal of magnetization of the armature core. When the core passes under one pair of poles, it undergoes one complete cycle of magnetic reversal. The frequency of magnetic reversal is given by, $f = P \cdot N / 120$ (where, P = no. of poles and N = Speed in rpm)

The loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. **Hysteresis loss** is given by, Steinmetz formula: $W_h = \eta B_{\max}^{1.6} f V$ (watts) where, η = Steinmetz hysteresis constant & V = volume of the core in m^3

Eddy current loss: When the armature core rotates in the magnetic field, an emf is also induced in the core (just like it induces in armature conductors), according to the Faraday's law of electromagnetic induction. Though this induced emf is small, it causes a large current to flow in the body due to the low resistance of the core. This current is known as eddy current. The power loss due to this current is known as eddy current loss. Eddy current loss depends on the various factors which are,

- Nature of material
- Maximum flux density
- Frequency
- Thickness of lamination
- Volume of magnetic material

✓ *Eddy current loss can be minimized by laminating the core.*

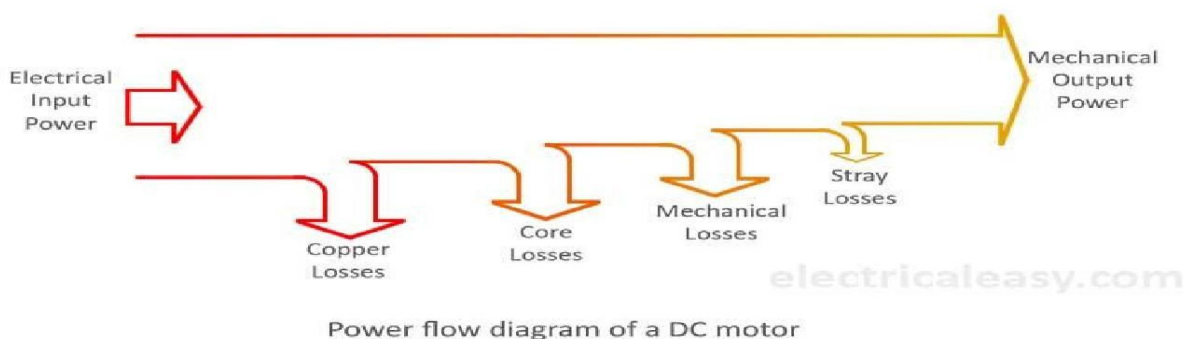
Mechanical Losses

Mechanical losses consist of the losses due to friction in bearings and commutator. Air friction loss of rotating armature also contributes to these. These losses are about 10 to 20% of full load losses.

Stray Losses

In addition to the losses stated above, there may be small losses present which are called as stray losses or miscellaneous losses. These losses are difficult to account. They are usually due to inaccuracies in the designing and modeling of the machine. Most of the times, stray losses are assumed to be 1% of the full load

- An example of power flow diagram for DC machine summarized as



The Equation can be summarized for the DC Motor:

$$\bullet P_{in} = P_{out} + P_{SCL} + P_{RCL} + (P_{rot} = P_{mech} + P_{core} + P_{stray})$$

Where P_{in} –Electrical power in pu

P_{SCL} –Power stator copper loss

P_{rot} –rotational power losses

P_{core} –Power core loss

P_{out} –Mechanical out put power

P_{RCL} –Power rotor copper loss

P_{mech} – Mechanical power loss

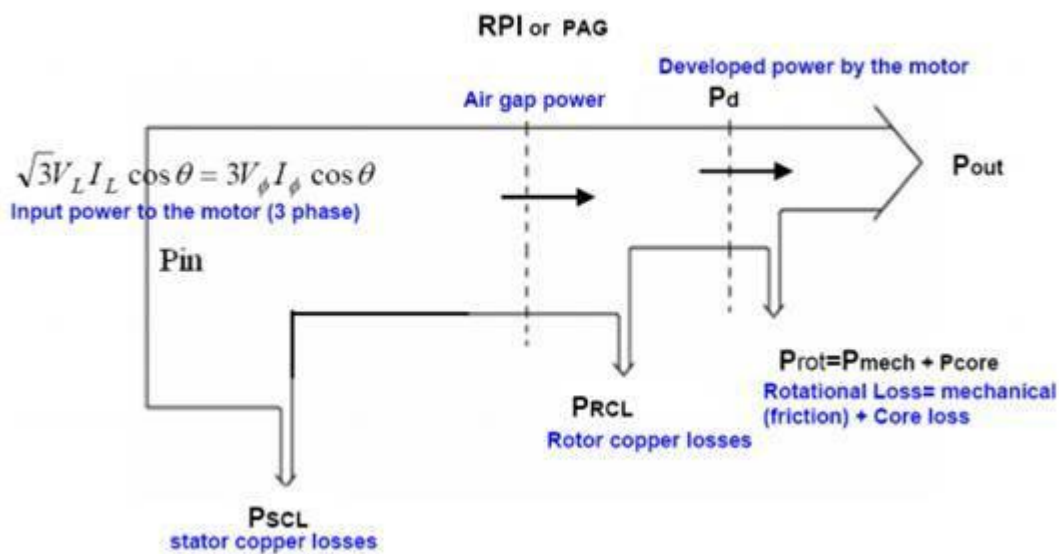
P_{stray} – Stray power loss

Example 2

A 480 V, 50 hp, three phase induction motor is drawing 60 A at 0.85 pf lagging. The stator copper losses are 2 kW and the rotor copper losses are 700 W. The friction loss is 600 W and the core losses are 1800 W, find:

1. The air gap power.
2. The converted power.
3. The output power.
4. The efficiency of the motor.

Solution: using power flow diagram of an Induction motor



$$a) P_{in} = \sqrt{3} V_T I_L \cos(\theta)$$

$$P_{in} = \sqrt{3} (480)(60)(0.85) = 42.4 \text{ kW}$$

$$P_{AG} = P_{in} - P_{SCL} = 42.4 - 2 = 40.4 \text{ kW}$$

$$b) P_d = P_{AG} - P_{RCL} = 40.4 - 0.7 = 39.7 \text{ kW}$$

$$c) P_{out} = P_d - P_{rot} = 39.7 - 2.4 = 37.3 \text{ kW}$$

$$d) \eta = \frac{P_{out}}{P_{in}} = \frac{37.3}{42.4} = 88\%$$

1.4.1. Motional EMF (Dynamically or Speed EMF)

Figure 7 shows three conductors *a*, *b*, *c* moving in a magnetic field of flux density *B* in the directions indicated by arrow. Conductor “*a*” is moving in a direction perpendicular to its length and perpendicular to the flux lines. Therefore, it cuts the lines of force and a motional emf is induced in it. Let the conductor move by a distance *dx* in a time *dt*. If the length of conductor is *l*, the area swept by the conductor is *l dx*. Then change in flux linking the coil

$$= d\phi = B \cdot l \cdot dx$$

Since there is only one conductor

$$e = \frac{d\phi}{dt} = \frac{B l dx}{dt}$$

Since dx/dt is *v*, i.e. velocity of conductor

$$e = B l v \text{ volts} \dots\dots\dots (9)$$

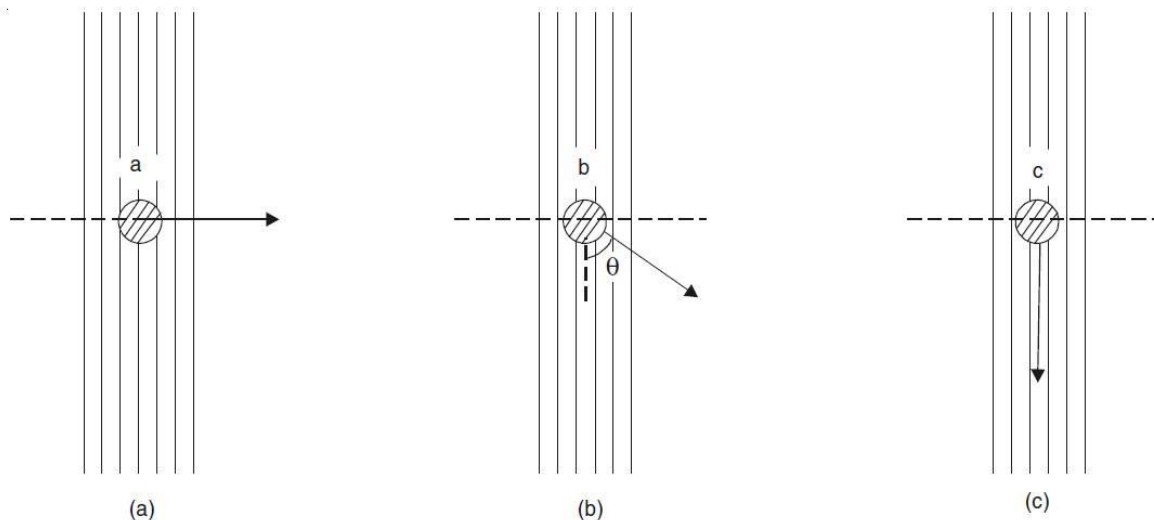


Fig. 7 Motion of a conductor in a magnetic field.

Where e = emf induced,
volts B = flux density, tesla
 v = velocity of conductor, metres/second
 l = length of conductor, metres.

The motion of conductor b (Fig. 7 b) is at an angle θ to the direction of the field. If the conductor moves by a distance dx , the component of distance travelled at right angles to the field is ($dx \sin \theta$) and, proceeding as above, the induced emf is

$$e = Bl v \sin \theta \text{ volts ... (10)}$$

Equation (10) includes Eq. (9) because when $\theta = 90^\circ$, the two equations become identical. In Fig 7 (c) the motion of conductor c is parallel to the field. Therefore, in this case, no flux is cut, θ is zero and induced emf is also zero. Dynamically induced emf is also known as speed emf or motional emf or rotational emf

Generally the dynamically induced EMF is found by Flemings Right Hand Rule.

Stats that:

Stretch the fore finger, middle finger and thumb of right hand mutually perpendicular to each other. If fore finger represents the direction of magnetic field, thumb represent the direction of motion of conductor then the middle finger will represent the direction of induced EMF.

1.4.2. Statically Induced EMF (Transformer EMF)

In this case the conductors and the electromagnet are stationary but EMF is induced by vary the flux. This is achieved by changing the flux associated with a coil by increasing or decreasing the current through it rapidly. Statically induced EMF is further classified as

- I. Self induced emf
- II. Mutually induced emf

1.5. Self Inductance

The property of a coil which opposes any change in the current passing through it is called Self inductance. It depends on

1. Shape and number of turns
2. Relative permeability of the material
3. The speed with the magnetic field changes

$$e = L \frac{di}{dt} = N \frac{d\phi}{dt}$$

$$L = N \frac{d\phi}{di}$$

If rate of change of flux is constant, then

$$\frac{d\phi}{di} = \frac{\phi}{i}$$

and

$$L = \frac{N\phi}{i}$$

$$\phi = \frac{\text{MMF}}{\text{Reluctance}} = \frac{Ni}{l/\mu a}$$

Therefore the Self inductance is given by:

$$L = N \frac{\left[\frac{Ni}{(l/\mu a)} \right]}{i} = \frac{N^2 \mu a}{l}$$

1.6. Mutual Inductance

When the flux of one coil is links another coil, a mutually induced emf appears across the second coil. The mutually induced emf can be written as

$$e = M \frac{di}{dt}$$

1.7. Forces and Torque in Magnetic Field

The Lorentz Force Law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \dots\dots\dots (11)$$

gives the force $\mathbf{F}$ on a particle of charge q in the presence of electric and magnetic fields. In SI units, $\mathbf{F}$ is in *newtons*, q in *coulombs*, E in *volts per meter*, $\mathbf{B}$ in *teslas*, and v , which is the velocity of the particle relative to the magnetic field, in *meters per second*.

Thus, in a pure electric-field system, the force is determined simply by the charge on the particle and the electric field

$$\mathbf{F} = q\mathbf{E} \quad \dots\dots\dots (12)$$

The force acts in the direction of the electric field and is independent of any particle motion.

In pure magnetic-field systems, the situation is somewhat more complex. Here the force

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad \dots\dots\dots (13)$$

Figure 8 (a) shows a conductor lying in a magnetic field of flux density B . The conductor is carrying a current (entering the page). This current sets up a flux in clockwise direction. The external field is in a downward direction. As seen in Fig. 8 (a) the field of the conductor assists the external field on the right hand side of the conductor and opposes it on the left hand side. This produces a force on the conductor towards left. If the direction of current is reversed (Fig. 8 (b)), the flux due to this current assumes counter-clockwise direction and the force on the conductor is towards right. In both cases, the force is in a direction perpendicular to both the conductor and the field and is maximum if the conductor is at right angles to the field. The magnitude of this force is

$$F = B I l \text{ Newton's..... (14)}$$

Where B is flux density in tesla, I is current in amperes and l is the length of conductor in metres. If the conductor is inclined at an angle θ to the magnetic field, the force is

$$F = B I l \sin \theta \text{ Newton's..... (15)}$$

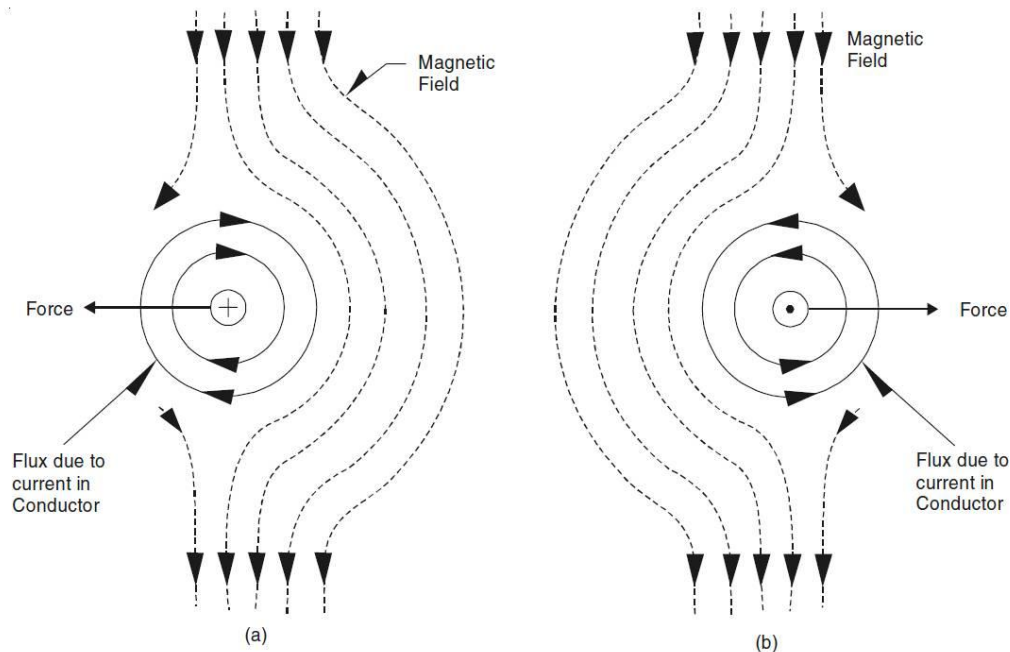


Fig. 8 Force on a conductor in a magnetic field (a) current into the page, (b) current out of the page

Figure 9 shows a coil carrying I and lying in a magnetic field of flux density B . From the discussion in fig 8, it is seen that an upward force is exerted on the right hand conductor and a downward force on the left hand conductor. Equation (14) gives the force on each conductor and the total force is

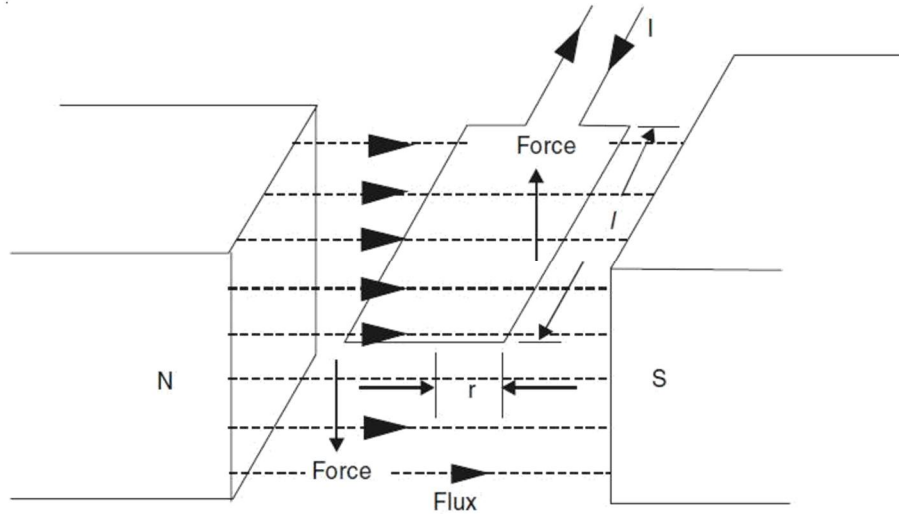


Fig. 9 Torque on a coil in a magnetic field.

$$F = 2B I l \text{ Newton's}$$

If the coil has N turns, the total force is

$$F = 2N B I l \text{ Newton's}$$

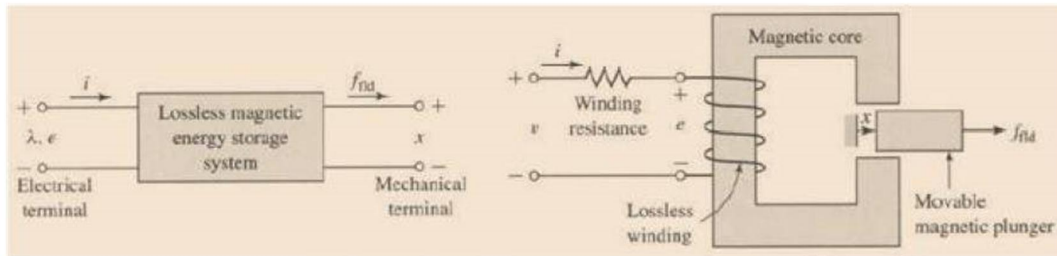
The torque is acting at a radius of r meters and is given by

$$\text{Torque} = 2N B I l r \text{ Newton's -meters}$$

1.8. Singly Excited System

Consider a singly excited linear actuator as shown below. The winding resistance is R . At a certain time instant t , we record that the terminal voltage applied to the excitation winding is v , the excitation winding current i , the position of the movable plunger x , and the force acting on the plunger F with the reference direction chosen in the positive direction of the x axis, as shown in the diagram. After a time interval dt , we notice that the plunger has moved for a distance dx under the action of the force F . The mechanical done by the force acting on the plunger during this time interval is thus

$$dw_m = Fdx$$



Singly Excited system energy conversion

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$$dw_e = dw_f + dw_m = vidt - Ri^2 dt$$

Since,

$$e = \frac{d\lambda}{dt} = v - Ri$$

So,

$$dw_f = dw_e - dw_m = eidt - Fdx = id\lambda - Fdx$$

we can also write,

$$e = \frac{d\lambda}{dt} = v - Ri$$

1.9. Doubly Excited System

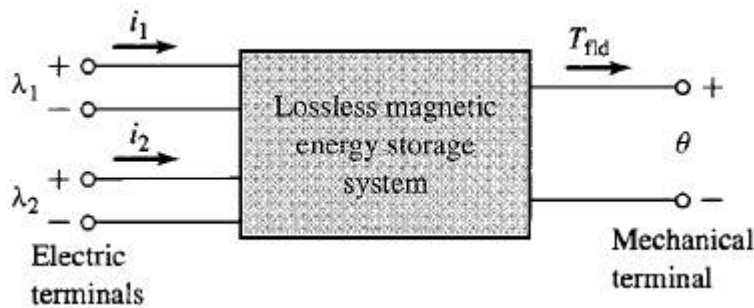


Fig 10 multiply excited energy system