Chapter One

1.0 Introduction to open channel flow

When the flow takes place in a channel or pipe such that the water has a free surface exposed to the atmosphere, we spoke of open channels, curvets, spillways, and similar human made structures are designed & analyzed by the method of open channel hydraulics.

The primary differences b/n the confined flow in pipes & open channel flow is that the pipe flow is closed channel, which is the top surface is covered by solid boundary, it is not exposed to atmospheric pressure but open channel flow is exposed to atmospheric pressure. In open channels the cross-sectional area of the flow is variable that depends on many parameters of the flow. For this reason hydraulic computations related to open channel flow are more complicated.

The prime motivating force (the force causing motion) for open channel flow is gravity or the slope provided at the bottom (bed).

Let’s compare the two flow types using figure.

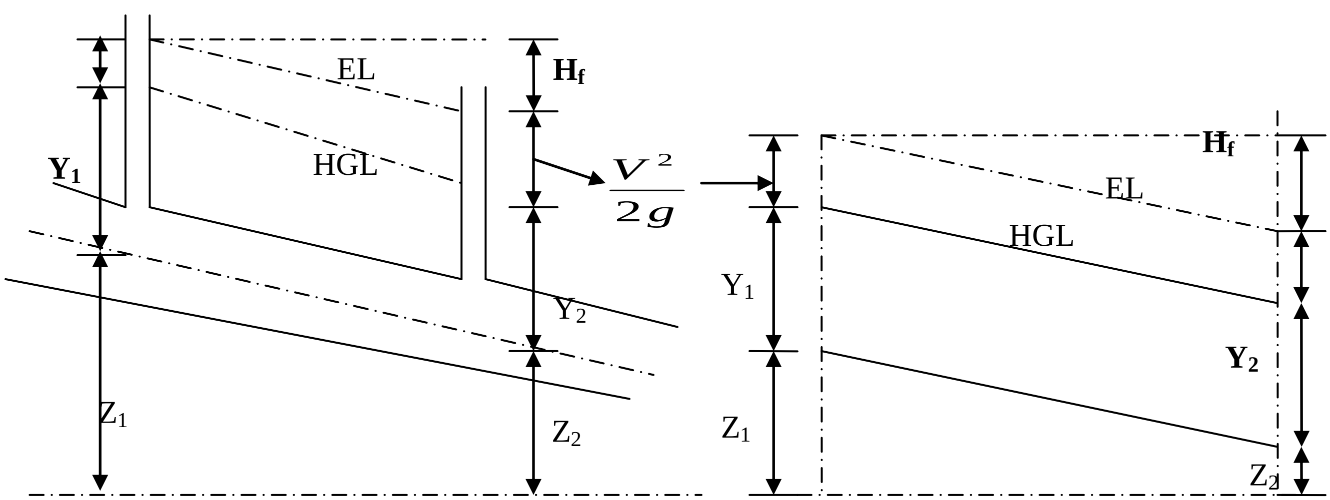

Fig 1(a) Pipe flow

Fig. 1(b) Open channel flow

Where HGL - Hydraulic grade line (coincide with water surface)

EGL - Energy grade line

H*f* - head loss due to friction

$V^2/2g$ - velocity head

Despite the similarity between these two flows it is much more difficult and complex to solve problems of the open channel case. This is due to the fact that the flow condition in open channel flow varies as per time and place. When we say the flow condition it includes depth of flow, cross-sectional area and slope of the channel. In turn the depth of flow , discharge and slope of the channel and water surface are related to each other.

In addition the bed roughness varies greatly leading the selection of friction coefficient to uncertainty. The cause of flow in open channel the gravitational forces and viscous shear forces along the channel wetted perimeter resists flow.

Types of channels

- **Natural channels**: These channels naturally exist without the influence of human beings. E.g. Rivers, streams, tidal estuaries, aqueducts.
  Aqueducts are under ground conduits which carry water with free surface.
- **Artificial channels**: Such channels are formed by man's activity for various purposes. E.g. irrigation channel, navigation channel, sewerage channel, culverts, power canal…… etc.

The above two channels can have either of the following features:

Prismatic channel: - channels with constant shape and slope.

Non-prismatic channels: - channels with varying shape and slope.

Generally the natural channels fall into the non prismatic group. That is why intensive study of the behavior of flow in natural channels requires other fields of studies like, sediment transport, geomorphology, hydrology, river engineering.

Types of flow in open channel

According to the characteristics of the flow with respect to time and place, different categories can be set.

A. Steady flow:- Here the criterion is time. A flow can be said steady if the fluid characteristics like velocity, pressure density, depth of flow doesn't change or if it can be assumed constant between the time of consideration.

$$\frac{\delta V}{\delta t} = 0 \quad , \quad \frac{\delta p}{\delta t} = 0 \quad \text{and} \quad \frac{\delta y}{\delta t} = 0$$

B. Unsteady flow:- Here the fluid characteristics vary with time such that

$$\frac{\delta V}{\delta t} \neq 0 \quad , \quad \frac{\delta p}{\delta t} \neq 0 \quad \text{and} \quad \frac{\delta y}{\delta t} \neq 0$$

C. Uniform flow:- A space as a criterion is used . Open channel flow is said to be uniform if the depth of flow, velocity remains constant or the same at every section of the channel. Uniform flow may be steady or unsteady, depending on whether or not the depth changes with time.

$$\frac{\delta V}{\delta s} = 0 \quad , \text{and} \quad \frac{\delta y}{\delta s} = 0$$

D. Non uniform flow: - In case when the velocity, depth of flow in a channel changes with space:

$$\frac{\delta V}{\delta s} \neq 0 \quad , \text{and} \quad \frac{\delta y}{\delta s} \neq 0$$

E. Steady uniform flow: - The depth of flow does not change during time interval and space under consideration.

F. Unsteady uniform flow: - This is a rare phenomenon when the depth of flow fluctuates while remaining parallel to the channel bottom.

G. Unsteady uniform flow: - This is a flow in which the depth is varying time but not with space.

H. Unsteady non uniform flow: - Is the flow in which the depth is varying with space and time.

Geometric elements of open channel section

Geometric elements are properties of a channel section that can be defined entirely by the geometry of the section and the depth of flow. The most used geometric properties include:

1. **Depth of flow(y**): it the vertical distance from the lowest point of the channel to the free surface.
2. **Top width (T):** it is the width of channel section at free surface.
3. **Stage (h):** is the elevation or vertical distance of the free surface above a datum.
4. **Wetted perimeter (p)**: it is the length of the channel boundary which is in contact with water.
5. **Wetted area (A)**: is the cross-sectional area of the flow normal to the direction of flow.
6. **Hydraulic radius(hydraulic mean depth)(R) :** it is the ratio of wetted area to its wetted perimeter

$$R = \frac{A}{P}$$

7. **Hydraulic depth(D):** the ratio of wetted area to the top width,

$$D=\frac{A}{T}$$

8. **Section factor (Z):** is the product of the wetted area and the two-third power of the hydraulic radius

$$Z=A\sqrt{D}=A\sqrt{\frac{A}{T}}=\left(\frac{A^3}{T}\right)^{\frac{1}{2}}=AR^{\frac{2}{3}}$$

9. **Conveyance (K) :**

$$Q=VA \ldots\ldots\ldots\ldots\ldots\ldots V=\frac{1}{n}R^{\frac{2}{3}}S^{\frac{1}{2}}$$

$$Q=A\frac{1}{n}R^{\frac{2}{3}}S^{\frac{1}{2}}$$

$$=AR^{\frac{2}{3}}\frac{1}{n}S^{\frac{1}{2}}$$

$=KS^{\frac{1}{2}}$ S= bed slope

$K=\frac{1}{n}AR^{\frac{2}{3}}$ n= Mannings constant

$=CA\sqrt{R}$ c= Chezy's constant

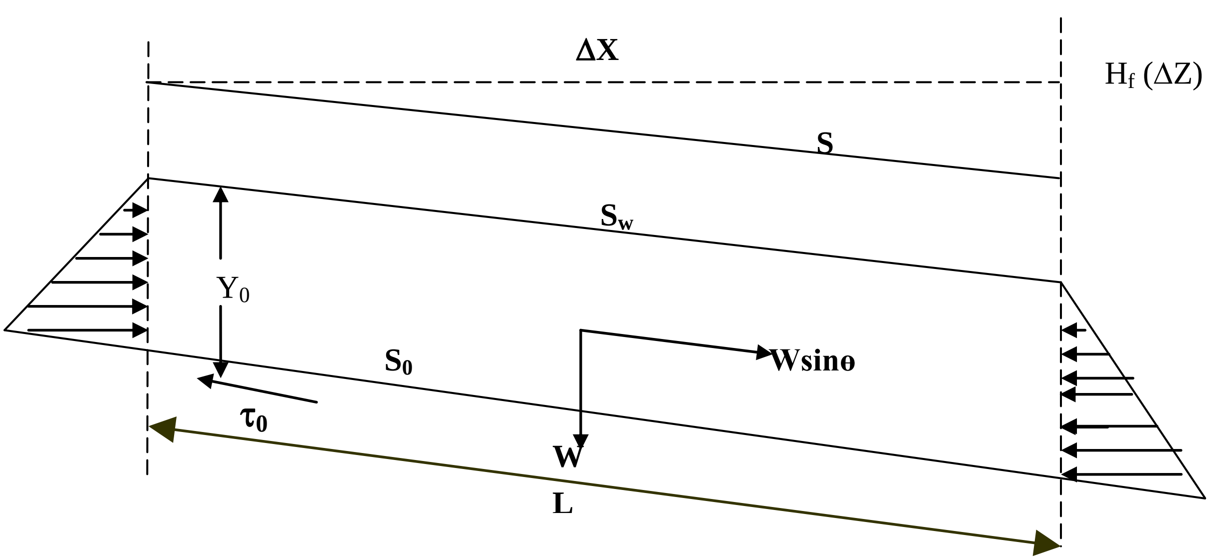

Fig. 1.2

Where S_0- bed slope of channel

S_w- Water surface slope

S- Slope of EGL

W – Weight of water

τ_0 – Shear force

L- Length of channel

Uniform flow is the result of exact balance between the gravity and friction force

$$W\sin\theta = \bar{\tau}_o .P.L \text{.................................(1)}$$

$$\gamma A\, L \sin\theta = \bar{\tau}_o .P.L$$

But $\sin\theta = hf/L = S$, solving for $\bar{\tau}_o$,

$$\bar{\tau}_o = \gamma \frac{A}{P}.S = \gamma R.S \text{ .... (2)}$$

Where γ- unit weight of the water

The shear stress is assumed proportional to the square of the mean velocity,

or $\tau_o = kV^2$..(3)

Therefore, $kv^2 = \gamma RS$

$$V^2 = \frac{\gamma}{k} RS \quad ,$$

Let $\frac{\gamma}{k} = C^2$ -constant (b/c γ&k- are constant)

$$V = C\sqrt{RS} \text{.. (4)}$$

This is the Chezy –formula

C= chezy coefficient (chezy's resistance factor)

V= Average velocity of flow

Manning Formula

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \text{...(5)}$$

- The best as well as most widely used formula for uniformly for uniform flow.

n- is the roughness coefficient.

A relation between the Chezy's C and Manning's n may be obtained by comparing eqn (4) & (5)

$$C = \frac{R^{\frac{1}{6}}}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

- The value of n ranges from 0.009 (for smooth straight surfaces) to 0.22 (for very dense flood plain forests).

1.2 Hydraulic Efficiency channel cross section

A channel section is said to be efficient if it gives the maximum discharge for the given shape, area and roughness.

The velocity in an open channel is:

V= f(R,S)……………………………………..(a)

Q=A*V=Af(R,S)……………………………..(b)

Equation (b) indicates that for the given area of cross section and slope the discharge Q will be maximum when R is maximum.

Since, R = A/P, R will be maximum when P is minimum for a given area.

We can conclude that for most efficient and economical channel section the wetted perimeter should be minimum& also frictional resistance,τo is minimum.

For example, a rectangular channel of depth Y and width, B

A=BY………………………….(i)

P=B=2Y……………………….(ii)

From eqn. (i), B=A/Y

Substituting in (ii) P=A/Y+2Y…………(iii)

For maximum Q, P- is minimum.

$$\frac{dp}{dY} = 0 \Rightarrow \frac{d}{dY}(A/Y = 2Y) = 0$$

$$\Rightarrow -A/Y = 2 = 0$$

$$\Rightarrow A = 2Y^2 = B*Y$$

So, B=2Y (or Y=B/2)

Thus the rectangular channel is most efficient and economical when the depth of water is one half of the width of the channel and the discharge flow will be maximum.
Accordingly, the most efficient channel shape is the semi circle. The usual shape for new channel & canal is the rectangular or trapezoidal. Such that the inscribed semi circle is tangential to the bed & side.

1.3 Specific Energy

For any cross section, shape, the specific energy (E) at a particular section is defined as the energy head to the channel bed as datum. Thus,

$$E = Y + \alpha \frac{V^2}{2g} \quad \text{..(1)}$$

(α- is kinetic energy correction factor ≅1)

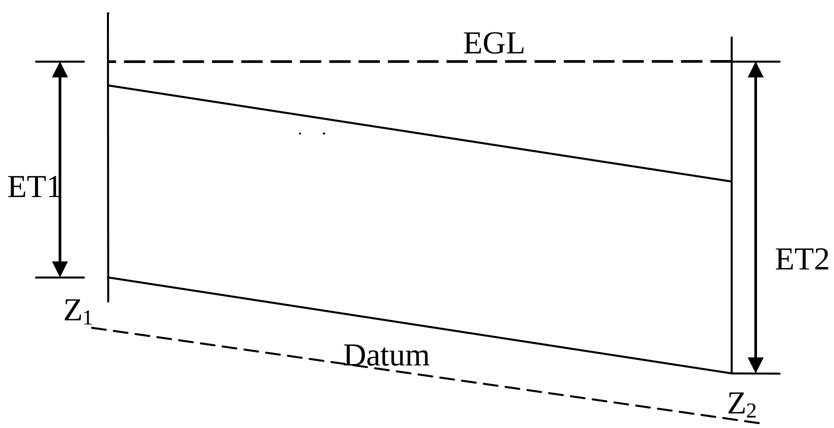

Fig 1.3 Specific Energy at a particular section

For a rectangular channel, the value of flow per unit width is Q/B=q, and average velocity

$$V = Q/A = \frac{qB}{BY} = \frac{q}{Y}$$

Therefore eqn (1) becomes:

$$E = y + \frac{\left(q/y\right)^2}{2g} = y = \frac{q^2}{2gy^2} \quad \text{.. (2)}$$

$(E - y)Y^2 = \frac{q^2}{2g}$ (For the case of constant q)………………………… (3)

A plot of E Vs Y is a hyperbola like with asymptotes (E-Y) =0 i.e. E=Y and y=0. Such a curve is known as specific energy diagram.

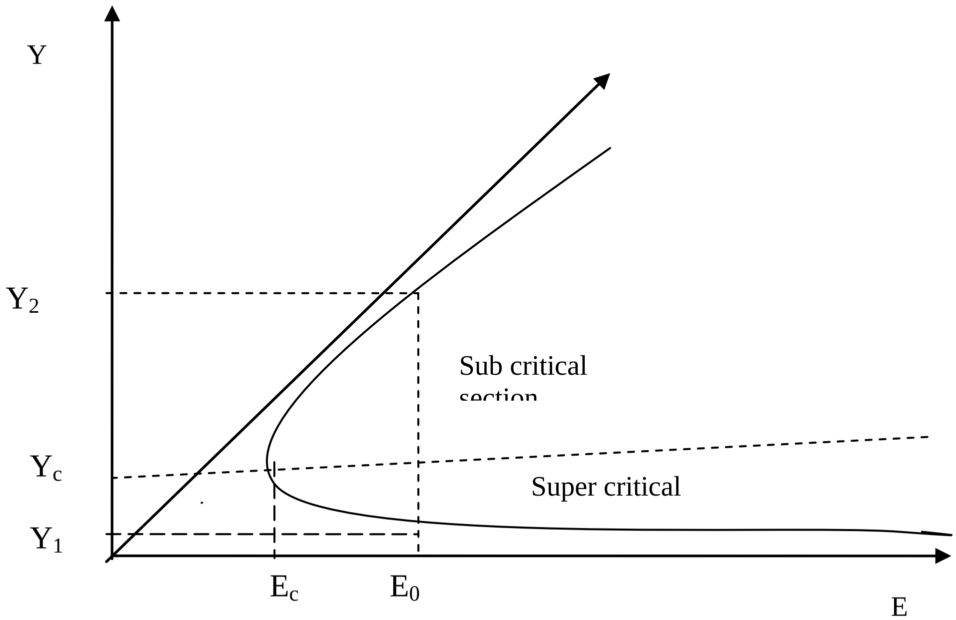

Fig 1.4 Specific Energy diagram

For a particular q, we see there are two possible values of Y for a given value of E. These are known as Alternative depths (for e.g. Y_1 & Y_2 on fig. above)

The two alternative depths represent two totally different flow regimes slow & deep on the upper limp of the curve (sub critical flow) & fast and shallow on the lower limb of the curve.(super critical flow)

1.4 Critical depth

From fig 1.4 above, at point C for a given q the value of E is a minimum and the flow at this point referred to as critical flow. The depth of flow at that point is the critical depth Yc & the velocity is the critical velocity Vc.

For example, a relation for critical depth in a wide rectangular channel can be found by differentiation E of eqn.2 with respect to Y to find the value of Y for which E is a minimum.

$$\frac{dE}{dY} = 1 - \frac{q^2}{gy^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (4)$$

And when E is a minimum Y=Yc and $dE/dy = 0$, so that

$$0 = 1 - \frac{q^2}{gY_c^{\ 3}} \Rightarrow q^2 = gy_c^{\ 3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Substituting q= vy = V_C*Yc, gives

$$V_c^{\ 2} = gy_c$$

$$\Rightarrow V_c = \sqrt{gy_c} = \frac{q}{y_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (6)$$

It may be expressed as:

$$y_c = \frac{V_{c^2}}{g} = \left(\frac{q^2}{g}\right)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (7)$$

From eqn (7) $\frac{V_c^{\ 2}}{2g} = \frac{y_c}{2}$, hence,

$$E_c = E_{\min} = y_c + \frac{V_c^{\ 2}}{2g} = y_c + \tfrac{1}{2} y_c = \tfrac{3}{2} y_c \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

And $y_c = \tfrac{2}{3} E_{\min}$ ……………………………………………………………..(9)

From eqn. (7): $q_{\max} = \sqrt{gy_c^{\ 3}}$ ……………………………………… ………….(10)

For non rectangular cross section the specific energy eqn.

$$E = y + \frac{Q^2}{2gA^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (11)$$

[V=Q/A]

To find the critical depth,

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (12)$$

From fig 1.3 (b) dA = dy*T (at Yc, T= Tc)

Therefore the above equation becomes:

$$\frac{Q_{\max}{}^2 T_c}{g A_c{}^3} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(13)}$$

The critical depth must satisfy this equation

From eqn. (13) $Q^2 = \frac{g A_c{}^3}{T_c}$ and substitute in eqn. (11) then

$$E_c = y_c + \frac{A_c}{2T_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(14)}$$

eqn.(13) can be solved by trial & error for irregular section by plotting $f(y) = \frac{Q^2 T}{g A^3}$ and critical depth occurs for the value of y which makes f(y)=1

Sub critical, Critical and super critical flow

If specific energy curve for Q- constant is redraw along side a second curve of depth against discharge for constant E, will show the variation of discharge with depth.

a) For a given constant discharge fig (1.5a)

i) the specific energy curve has a minimum value Ec at point C with a corresponding depth Yc known as critical depth.

ii) For any other value of E there are two possible depth of flow known as alternative depth one of which is termed sub critical (y>Yc) and the other supercritical (Y<Yc).

y_2

y_c

y_1

b) For a given constant specific energy (fig.1.5(b))

i) the depth discharge curve shows that discharge is a maximum at the critical depth

ii) For all other discharges there are two possible depth of flow (sub- & super critical) for any particular value of E,

From eqn. (13) above if we substitute

Q= AV (continuity equation), we get

$$\frac{Q^2T}{gA^3}=1$$

$$\frac{A^2V^2T}{gA^3}=1 \Leftrightarrow \frac{V^2T}{gA}=1$$

but A/T = D (Hydraulic depth), then [D=Y for rectangular section)

$$\frac{V^2}{gy}=1 \Rightarrow V=\sqrt{gy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(*)$$

$$\frac{V}{\sqrt{gy}}=1 \Rightarrow \text{Froude number at critical state.}$$

$$F=\frac{V}{\sqrt{gy}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(**)$$

Thus, i) F= 1⇒critical flow

ii) F< 1⇒ sub critical flow

iii)F>1 ⇒Super critical

1.5 Hydraulic Jump

By far the most important of the local non-uniform flow phenomena is that which occurs when supercritical flow has its velocity reduced to sub critical. There is sudden rise in water level at the point where hydraulic jump occurs (Rapidly varied flow). This is an excellent example of the jump serving a useful purpose, for it dissipates much of the destructive energy of the high –velocity water, there by reducing downstream erosion. The turbulence with in hydraulic jumps has also been found to be very useful & effective for mixing fluids, & jumps have been used for this purpose in water treatment plant & sewage treatment plants.

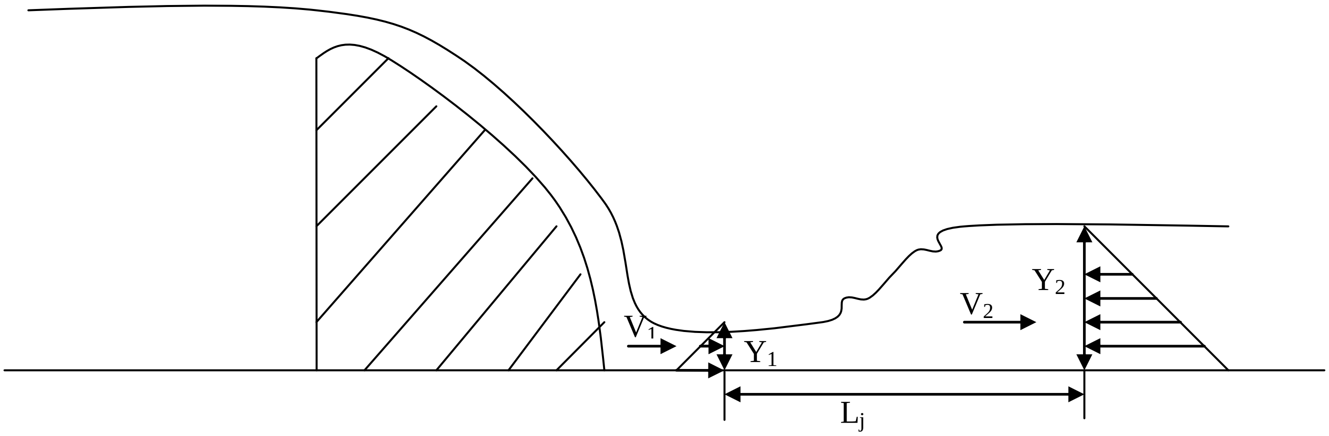

Fig 1.6 hydraulic jump on horizontal bed following over a spillway

- **Purposes of hydraulic jump:-**

i) To increase the water level on the d/s of the hydraulic structures

ii) To reduce the net up lift force by increasing the downward force due to the increased depth of water,

iii) To increase the discharge from a sluice gate by increasing the effective head causing flow,

iv) For aeration of drinking water

v) For removing air pockets in a pipe line

- **Analysis of hydraulic jump**

Assumptions

1) The length of the hydraulic jump is small, consequently, the loss of head due to friction is negligible,
2) The channel is horizontal as it has a very small longitudinal slope. The weight component in the direction of flow is negligible.
3) The portion of channel in which the hydraulic jump occurs is taken as a control volume & it is assumed the just before & after the control volume, the flow is uniform & pressure distribution is hydrostatic.

Let us consider a small reach of a channel in which the hydraulic jump occurs.

The momentum of water passing through section (1) per unit time is given as:

$$\frac{p_1}{t} = \frac{rQV_1}{g} = \rho QV_1 \text{ ....(i)}$$

Momentum at section (2) per unit time is:

$$\frac{p_2}{t} = \frac{rQV_2}{g} = \rho QV_2 \text{ ....(ii)}$$

Rate of change of momentum b/n section 1 & 2

$$\frac{\Delta P}{t} = \rho Q(V_2 - V_1) \text{ ....(iii)}$$

The net force in the direction of flow = F1-F2 ………………..(iv)

$$F_1 = \gamma A_1\overline{Y}_1, \qquad F_2 = \gamma A_2\overline{Y}_2$$

$\overline{Y}_1$ & $\overline{Y}_2$ are the center of pressure at section (1) & (2)

Therefore F_1-F_2 =ΔM =ρQ (V_2-V_1)

$$\gamma A_1\overline{Y}_1 - \gamma A_2\overline{Y}_2 = \frac{\gamma Q}{g}(V_2 - V_1) \text{ ....(v)}$$

From continuity eqn. Q= A*V, ⟹ V= Q/A, so

$$\gamma A_1\overline{Y}_1 - \gamma A_2\overline{Y}_2 = \frac{\gamma Q}{g}\left(\frac{Q}{A_2} - \frac{Q}{A_1}\right)$$

$$A_1\overline{Y}_1 - A_2\overline{Y}_2 = \frac{Q^2}{g}\left(1/A_2 - 1/A_1\right) \text{..}(iv)$$

Rearranging this eqn.:

$$\underbrace{\left[\frac{Q^2}{gA_1} + A_1\overline{Y}_1\right]}_{M_1} = \underbrace{\left[\frac{Q^2}{gA_2} + A_2\overline{Y}_2\right]}_{M_2} = \text{Constant. …………… (vii)}$$

$M_1$and M_2 are the specific forces at section (1) & (2) indicates that these forces are equal before & after the jump.

Y_1= initial depth

Y_2 = sequent depth

Hydraulic jump in a rectangular channel

A_1=By_1 ⎤ the section has uniform width (B)
A_2= By_2 ⎦

$\bar{Y}_1 = \frac{Y_1}{2}$, $\bar{Y}_2 = \frac{Y_2}{2}$

Now from eqn. (Vii) above:

$$\frac{Q^2}{gBy_1} + By_1\left(\frac{y_1}{2}\right) = \frac{Q}{gBy_2} + By_2 * \left(y_2/2\right)$$

$$\frac{Q^2}{gBy_1} + \frac{By_1^2}{2} = \frac{Q^2}{Bgy_2} + \frac{By_2^2}{2} \text{...}(viii)$$

Flow per unit width of q= Q/B ⇨ Q=qB, then eqn. (viii) becomes

$$\frac{q^2B^2}{Bgy_1} + \frac{By_1^2}{2} = \frac{q^2B^2}{Bgy_2} + \frac{By_2^2}{2}$$

$$\frac{q^2}{g}\left[\frac{1}{y_1} - \frac{1}{y_2}\right] = \frac{y_2^2 - y_1^2}{2} \text{ ………………………………… (.ix)}$$

$$\frac{2q^2}{g} = y_1y_2 \quad \frac{\left(y_2^2 - y_1^2\right)}{\left(y_2 - y_1\right)}$$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (x)$$

$$y_2 y_1^2 + y_1 y_2^2 - \frac{2q^2}{g} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xi)$$

This is quadratic eqn. & the solution is given as

$$y_1 = \frac{-y_2}{2} + \sqrt{\left(\frac{y_2}{2}\right)^2 + \frac{2q^2}{gy_2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xii)(a)$$

$$y_2 = \frac{-y_1}{2} + \sqrt{\left(y_1/2\right)^2 + \frac{2q^2}{gy_2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

$$y_1 = y_2/2\,(-1 + \sqrt{1 + \frac{8q^2}{gy_2^3}}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

$$y_2 = y_1/2\,(-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xii)(d)$$

The ratio of conjugate depths;

$$y_1/y_2 = 1/2(-1 + \sqrt{1 + \frac{8q^2}{gy_2^3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xii)(e)$$

$$y_2/y_1 = 1/2(-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (f)$$

$$F_1 = \frac{V_1}{\sqrt{gy_1}} =, F_2 = \frac{V_2}{\sqrt{gy_2}} \; \frac{q/y_2}{gy_2} = \frac{q}{\sqrt{gy_2^3}}$$

$$Therefore \;\; \frac{y_1}{y_2} = 1/2\,(-1 + \sqrt{1 + 8F_2^2}) \ldots\ldots\ldots\ldots\ldots\ldots (g)$$

$$\frac{y_2}{y_1} = 1/2(-1 + \sqrt{1 + 8F_1^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (h)$$

➢ Energy dissipation in a Hydraulic Jump

The head loss $h_{l.f}$ caused by the jump is the drop in energy from section (1) to (2) or:

$h_{lf} = \Delta E = E_1 - E_2$

$$= \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)a$$

$$= \left(y_1 + \frac{q^2}{2gy_1^2} \right) - \left(y_2 + \frac{q^2}{2gy_2^2} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

$$= \frac{q^2}{2g} \left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2} \right) - (y_2 - y_1) \ldots\ldots\ldots\ldots\ldots (c)$$

From eqn. (x) substituting: $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$ in to this eqn. & by rearranging:

$$h_{lf} = \Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Therefore power lost = γ Q h_{lf} (kw)…………………(3)

➢ Types of Hydraulic jump

Hydraulic jumps are classified according to the upstream Froude number and depth ratio.

F_1	Y_2/y_1	Classification
<1	1	Jump impossible
1-1.7	1-2	Undular jump (standing wave)
1.7-2.5	2-3.1	Weak jump
2.5-4.5	3.1-5.9	Oscillating jump
4.5-9.0	5.9-12	Steady jump (45-70% energy loss)
>9.0	>12	Strong or chopping jump (=85% energy loss)