CHAPTER -Three

5.0- Flow through pipes

Introduction

Pipes were introduced in the earliest days of the practice of hydraulics. Their common place use to day makes it of great importance that the lows governing the flow in them should be fully understood.

Water is conveyed from its source, normally in pressure pipelines, to water treatment plants where it enters the distribution system & finally arrives at the consumer. In addition oil, gas, irrigation water, sewerage can be conveyed by pipeline system.

Some loss of energy is inevitable in the flow of any real fluid. In the case of flow in a horizontal uniform pipeline this is evidenced by the fall of pressure in the direction of flow. Predicting the energy loss per unit length is essential to efficient pipeline design.

The prime concern in the analysis of real flows is to account for the effect of friction. The effect of friction is to decrease the pressure, causing a pressure ' loss' compared to the ideal, frictionless flow case. The loss will be divided into *major losses* (due to friction in fully developed flow in constant area portions of the system) & *minor losses* (due to flow through valves, elbow fittings & frictional effects in other non-constant –area portions of the system).

Fig.1 Flow in the pipes (circular pipe)

h_f = Head loss (major +minor)

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+hf \quad \text{--}1$$

I) **Major Losses** (Head loss in conduits of constant cross-section)

Fig.2

For equilibrium in steady flow, the summation of forces acting an any fluid element must be equal to zero, i.e. $\sum F=0$,

$$p_1A - p_2A + W\sin\alpha - \overline{\tau_o}(pL) = 0$$

$$\sin\alpha = \frac{(z_1 - z_2)}{L}$$

$$p_1A - p_2A + \gamma AL\frac{(z_1 - z_2)}{L} - \overline{\tau_o}(p.L) = 0$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} + (z_1 - z_2) - \overline{\tau_o}.\frac{p.L}{\gamma A} = 0$$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + \overline{\tau_o}.\frac{p.L}{\gamma A}$$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + \overline{\tau_o}.\frac{p.L}{\gamma A} \text{.......................................}(**)(2)$$

Form the above eqn. (1)

$$h_{lf} = \overline{\tau_o}.\frac{p}{A}.\frac{L}{\gamma} = \left(\frac{p_1}{\gamma} + z_1\right) - \left(p_2/\gamma + z_2\right)$$

$$h_\ell = \overline{\tau_o}.\frac{L}{R.\gamma} \text{..}(3)$$

This eqn. is applicable to any shape of uniform cross-sections, regardless of whether the flow is laminar or turbulent.

The average shear stress $\overline{\tau_o}$ is a function of ρ,μ,ν& some characteristic liner dimension, hydraulic radius R.Thus:

$\overline{\tau_o}$ =ϕ(ρ,μ,ν, R)

By dimensional analysis:

$$\overline{\tau_o} = \rho v^2\phi\left(\frac{\mu}{\rho Rv}\right) = \rho v^2\phi(\text{Re})$$

let ϕ (Re) = ½ C_f (dimensionless term)

$$\overline{\tau_o} = C_f.\rho\frac{V^2}{2} \text{..}(4)$$

From eqn. (3): $h_\ell = C_f.\frac{L}{R}.\frac{V^2}{2g}$ ....(5)

(Applied for any shape of smooth walled conduits).

For circular conduits (pipe) flowing full;

R= ¼ D,

Therefore, $$h_L = C_f.\frac{4L}{D}.\frac{V^2}{2g} = f.\frac{L}{D}.\frac{v^2}{2g} \qquad (6)$$

Where, f = 4Cf =8ϕ (Re)

This eqn. is applicable for both smooth-walled and rough walled conduits. It is known as pipe –friction equation, and commonly referred to as the Darcy-Weisbach equation.

Friction factor, f, is dimensionless & must be determined by experiments.

From eqn. (3) $$h_l = \overline{\tau_o}\frac{L}{R\gamma} = \tau_o * L * \frac{1}{(r_o/2)} * \frac{1}{\gamma} = 2\frac{\tau_o\ L}{r_o\gamma} \qquad (7)$$

From eqn. (4) $$\overline{\tau_o} = \tau_o = C_f.\rho\frac{V^2}{2} = \frac{f}{4}.\rho\frac{V^2}{2}$$

$$\tau_o = \frac{f}{4}.\gamma.\frac{V^2}{2g} \qquad (8)$$

Fig. 5.2 velocity profile & distribution of shear stress

To determine the velocity profile for laminar flow in a circular pipe, the expression

$$\tau = \mu\frac{du}{dy} = -\mu\frac{du}{dr},\ substitude in\ to\ eq\underline{n}\ (7)$$

$$h_L = \frac{2\tau_o.L}{r_o.\gamma} = -\mu.\frac{du}{dr}.\frac{2L}{r\gamma}$$

$$du = -\frac{h_L.\gamma}{2\mu.L}.\gamma.dr \Rightarrow Intergrating\ this\ eq\underline{n}.$$

$$u = -\frac{h_l.\gamma}{4\mu.L}.r^2 + C$$

$$u = U_{\max}\ at\ r = 0\ ,\ therfore,$$

$$C = U_{\max}$$

$$\Rightarrow u = U_{\max} - \frac{h_L.\gamma}{4\mu.L}.r^2 = U_{\max} - kr^2$$

At the boundary velocity is zero (i.e., u=0 at r = r_o)

$$0 = u_{max} - kr_o^2 \Rightarrow K = \frac{u_{max}}{r_o^2} = \frac{V_c}{r_o{}^2}$$

$$\therefore \quad u = V_c - \frac{V_c}{r_o^2} * r^2 = V_c\left(1 - \frac{r^2}{r_o^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

From the above expression $V_c = u_{max} = \frac{h_L.\gamma}{4\mu.L}.r_0^2 = \frac{h_L.\gamma}{16\mu.L}.D^2$ -------------10

Where $V_c = U_{max}$ = center line velocity of pipe.

The mean velocity (V) is half of the centerline velocity (V_c)

$$\therefore \quad V = \frac{h_L.\gamma}{32\mu.L}.D^2 \text{ --11}$$

$$\Rightarrow h_l = 32.\frac{\mu}{\gamma}.\frac{L}{D^2}.V = 32.\upsilon.\frac{L}{gD^2}.V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

This is the loss of head in friction known as Hagen-poi Seville low.
From eqn. 6&12

$$f = 64.\frac{v}{DV} = \frac{64}{\text{Re}} \qquad (for\ laminar\ flow) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Head loss: - $h_{lf} = \left(\frac{64}{\text{Re}}\right).\frac{L}{D}.\frac{\overline{V}^2}{2g}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$

Experimental Investigation on friction losses in Turbulent flow: -

In fully developed turbulent flow, the pressure drop, Δp, due to friction in a horizontal constant area pipe depends upon the diameter, D, the pipe length, L, the pipe roughness, ∈, the average velocity, $\overline{V}$, the fluid density, ρ, and the fluid viscosity, μ.

$$\Delta\rho = \phi(v, D, \rho, \mu, \varepsilon)$$

$$\frac{\Delta\rho}{\rho v^2} = \phi\left(\frac{\mu}{\rho v D}, \frac{L}{D}, \frac{\varepsilon}{D}\right)$$

By dimensional analysis.

$$\frac{h\ell}{\frac{v^2}{g}} = \frac{L}{D}\phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

$$\frac{h\ell}{\frac{1}{2}\frac{v^2}{g}} = \frac{L}{D}\phi_1\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

$$\therefore f = \phi_1\left(\text{Re}, \frac{\varepsilon}{D}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

- Blasius had concluded that there were two types of pipe friction in turbulent flow. The first is the smooth pipes where the viscosity effects predominate so that the friction factor is dependent solely on the Reynolds number (f=ϕ (Re). He deduced the following expression for the friction in smooth pipes:

$$f = \frac{0.316}{\mathrm{Re}^{1/4}} \quad \text{..(16)}$$

The second type was relevant to rough pipes where the viscosity & roughness effects influence the flow & the friction factor (f) is dependent both on the Reynolds no. & a parameter of relative roughness. ($^{\varepsilon}/_{D}$).

L.F Moody prepared a chart for determining friction factor for rough pipes experimentally, by plotting f versus Re curve for each value of $\frac{\varepsilon}{D}$.

(See Moody Chart)

The Colebrook has developed the formula:

$$\frac{1}{\sqrt{f}} = -0.809\ln\left(\frac{\in/D}{3.7} + \frac{2.523}{\mathrm{Re}\sqrt{f}}\right) \quad \text{..(17)}$$

the simplified eqn. of this eqn. is provided with the restriction placed on it:

$$f = \frac{1.325}{\left[\ell n(\frac{\in}{3.7D} + \frac{5.74}{R^{0.9}}\right]^2} \Rightarrow \left\{\begin{matrix} 10^{-6} \le \frac{\in}{D} \le 10^{-12} \\ 5000 \le \mathrm{Re} \le 10^{8} \end{matrix}\right\} ----------18$$

(for Rough pipes)

∴ Head loss in pipes is given by:

$$h_{\ell f} = f.\frac{L}{D}.\frac{V^2}{2g} \quad \text{(for all pipes rough smooth, laminar, \& turbulent)}$$

II**. Minor losses in the pipes**

Loss due to the local disturbances of the flow conduits such as changes in cross-section, bend, elbows, valves; joints, etc are called minor losses.

i) ***Loss of head at entrance***: -

A poorly designed inlet to a pipe can cause an appreciable head loss.

Fig.

$$h_{l\,i} = k\frac{V^2}{2g}\ , \text{ ---19}$$

The value of k depends on the edge of the in let of pipe.

Entrance type	inlet loss coefficient (k)
Rounded (bell mouthed)	0.04
Squared edged	0.5
Reentrant	0.8

ii) ***Loss of head at submerged discharges: (leave of pipe),*** ($\mathbf{h_{d'}}$)

When the fluid with a velocity V is discharged from the end of a pipe in to a large reservoir, (v=0), the entire kinetic energy of the coming flow is dissipated.

From the energy equilibrium:
Ha=Hc+ h_{loss}

Fig.

$$\frac{p_a}{\gamma} + y_a + \frac{V_a^2}{2g} = \frac{p_c}{\gamma} + \frac{V_c^2}{2g} + h_{\ell oss} \quad \text{(Taking datum through a)}$$

$$y + 0 + \frac{V^2}{2g} = 0 + y + 0 + h_{\ell oss}$$

$$\therefore h_{loss} = h_{d'} = \frac{V^2}{2g} \text{ ……………………………………………………….(20)}$$

iii***) Loss due to contraction*** (h_c)

Sudden contraction

There is a marked drop in pressure due to increase in velocity & to the loss of energy in turbulence.

$$h_C = k_C\frac{V_2^2}{2g}$$

Losses coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K_C	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

Gradual contraction

In order to reduces high losses, abrupt changes of cross section should be avoided. This is accomplished by changing from one diameter to the other by means of a smoothly curved transition or by employing the frustum of a cone.

$\therefore K_C=0.05-0.10$

For nozzle at the end of pipeline, kc =0.04-0.20

(iv) *Loss due to Expansion* (h_e)

Sudden Expansion

After the flow enters expanded pipe, there is excessive turblence & formation of eddies which causes loss of energy.

Let the pressure at section (2) in ideal case without friction is p_o(atmospheric pressure) Then:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_o}{\gamma}+\frac{V_2^2}{2g}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(*)$$

In actual case pressure at point (2) is p_2; then equating the resultant force on the body of fluid b/n section (1) & (2) to the time

$$p_1A_1-p_2A_2=\frac{\gamma}{g}(A_2V_2^2-A_1V_1^2)\quad \text{Rate of momentum b/n section (1) \& (2)}$$

$$\frac{p_2}{\gamma}=\frac{A_1}{A_2}\frac{p_1}{\gamma}+\frac{A_1}{A_2}\frac{V_1^2}{g}-\frac{V_2^2}{g}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(*)$$

The loss of head is given by the difference b/n the ideal &actual pressure heads at section 2. Thus,

$$he=\frac{(p_o-p_2)}{\gamma}$$

and from continuity eqn. $A_1V_1=A_2V_2\Rightarrow\frac{A_1}{A_2}=\frac{V_2}{V_1}$

$A_1V^2_1$=(A_1V_1) V1=(A_2V_2). V_1

$$he = \frac{(V - V_2)^2}{2g} + \left(1 - \frac{A_1}{A_2}\right)\frac{p_1}{\gamma} \Rightarrow \qquad take\ p_1\ is\ small$$

$$\therefore he = \frac{(V_1 - V_2)^2}{2g} = \left(\frac{D_2^2}{D_1^2} - 1\right)^2 \frac{V_2^2}{2g}$$

Gradual Expansion

To minimize the loss accompanying a reduction in velocity a diffuser may be used. (Diffuser is a curved outline, or it may be a frustum of cone.

Fig.

i) *Pipe friction Loss*:

$$h_{\ell e} = \int \frac{f}{D} \cdot \frac{V^2}{2g} dL$$

ii) Turbulence loss increase with the degree of divergence: the total loss for gradual expansion pipe is the sum of these two losses, marked K' (coeff.)

Therefore, $h_e' = K' \frac{(V_1 - V_2)^2}{2g}$

K'-is a function of cone angle α.

K'	0.4	0.6	0.95	1.1	1.18	1.09	1.0	1.0
α	20^0	30^0	40^0	50^0	60^0	90^0	120^0	180

(v***) Loss in pipe fittings***

The loss of head in pipe fittings is expressed as:

$$h_f = k_f \frac{V^2}{2g}$$

The values of "K_f" depends on the type of fittings.

Fitting	K
Globe valve, wide open	10
Angle valve, wide open	5
Close –return bend	2.2
T-through side outlet	1.8
Short-radius elbow	0.9

Medium radius elbow	0.75
Long radius elbow	0.60
Gate valve, wide open	0.19
Half open	2.06
Pump foot value	5.60
Standard branch flow	1.80

(vi) ***Losses in bend & Elbow***

In flow around a bend or elbow, because of centrifugal effects, there is an increase in pressure along the outer wall & a decrease in pressure along the inner wall.

The head loss produced by a bend or elbow is:

$$h_b = k_b \cdot \frac{V^2}{2g}$$

k_b-depends on the ratio of curvature , r to pipe diameter, D.

Solution of single –pipe flow problems

We have observed the frictional loss of energy in single-pipe flow, caused by both wall roughness of the pipes (major loss) and by pipe cross-section that disturbs the flow (minor losses).
Pipe flow problems may be solved by Hazen-Williams eqn, the Manning eqn. or the Darcy-weisbach equation.

The total head losses b/n two points is the sum of the pipe friction loss plus the minor losses, or

$$h_L = h_{Lf} + \sum h'$$

Where h_L= total head loss
h_{Lf} = major head loss
$\sum h'$ =total minor losses

The above eqn. (h_L) relates four variables. Any one of these may be unknown quantity in practical flow situation. These are:

i) L, Q, D known h_L unknown
ii) h_L,Q, D " L "

iii) h_L, Q, L, " D "
iv) h_L, L, D, " Q "

Example: 1
A 100m length of smooth horizontal pipe is attached to a large reservoir. What depth, d, must be maintained in the reservoir to produce a volume flow rate of 0.03m^3/sec of water? The inside dia. of the smooth pipe is 75mm. The in let of the pipe is square edged. The water discharges to the atmosphere.

Soln.

$$\left(\frac{p_1}{\gamma}+\frac{v_1{}^2}{2g}+Z_1\right)-\left(\frac{p_2}{\gamma}+\frac{v_2{}^2}{2g}+Z_2\right)=h_{LT}$$

h_{LT} = h_{Lf} +h_{Lm}

$$= f\frac{L}{D}\frac{V^2}{2g}+k\frac{V^2}{2g}$$

But $p_1=p_2= P_{atm}$, $V1\cong 0_1$, $V_2=V$, $Z_2=0$ (measured from the center of the pipe line, then $z_1=d$.

$$h_{LT} = d-\frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g}+k\frac{V^2}{2g}$$

$$d=\frac{v^2}{2g}\left[f\frac{L}{D}+K+1\right]$$

$$V_2 = V = \frac{Q}{A_2}=\frac{4Q}{\Pi D_2^2}, then$$

$$d=\frac{8Q^2}{\pi^2 D^4 g}\left[f\frac{L}{D}+k+1\right]$$

$$Let\ \rho = 1000kg/m^3, \mu = 1x10^{-3}kg/m.s,$$

$$\mathrm{Re}=\frac{\rho VD}{\mu}=\frac{4\rho Q}{\pi\mu D}=\frac{4}{\pi}*\frac{1000*0.03}{1x10^{-3}*0.075}=5.10x10^5$$

For smooth pipe from Moody diagram , f=0.0131, then k=0.5 for square-edged.

$$\therefore\ d=\frac{8}{\pi^2}*\frac{(0.03)^2}{(0.075)^4*9.81}*\left[0.0131*\frac{100}{0.075}+0.5+1\right]$$

$$d = 44.6m$$

Pipe line with Pump or Turbine

If a pump pumps a fluid from lower level reservoir to the higher level reservoir, it lifts the fluid the height ΔZ, and it overcome the friction loss in the suction & discharge piping.

The pump lifts the fluid a height ($\Delta Z + \sum h_l$) . Hence, the power delivered to the liquid by the pump is $\gamma Q(\Delta Z + \sum h_{L})$. The power required to run the pump is greater than this, depending on the efficiency of the pump. The total pumping head, hp, for this case is:

$$h_p = \Delta Z + \sum h_L .$$

If the pump discharges a stream through a nozzle, kinetic energy head of $V_2^2/2g$ is required.

Total pumping head is:-

$$h_p = \Delta Z + \frac{V_2^2}{2g} + \sum h_L$$

V_2-velocity of the nozzle.

Pipeline system

- ***Pipes In Series***

When two pipes of different sizes or roughness are so connected that the fluid flows through one pipe& then through the other, they are said to be connected in series.

As observed in the following fig there is head H, between two reservoirs for a given discharge flow:

Fig. pipes connected in series.

Applying the energy eqn. From A to B, including all losses, gives:

$$\frac{P_A}{\gamma}+Z_A+\frac{V_A^{\,2}}{2g}=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+hi+h_{f1}+he'+h_{f2}+h_d,$$

$$H+0+0=0+0+0+ki\frac{V_1^2}{2g}+f_1\frac{L_1}{D_1}\frac{V_1^{\,2}}{2g}+\frac{(V_1-V_2)^2}{2g}+f_2\frac{L_2}{D_2}\frac{V_2^{\,2}}{2g}+\frac{V_2^{\,2}}{2g}$$

$$From\ quantity\ eqn.:\qquad V_1D_1^{\,2}=V_2D_2^{\,2}$$

$$H=\frac{V_1^2}{2g}\left\{ki+f_1\frac{L_1}{D_1}+\left[1-\left(D_1/D_2\right)^2\right]^2+f_2\frac{L_2}{D_2}\left(\frac{D_1}{D_2}\right)^4+\left(\frac{D_1}{D_2}\right)^4\right\}$$

- **Equivalent pipes**

Series pipes can be solved by the method of equivalent lengths. Two pipe systems are said to be equivalent when the same head loss produces the same discharge in both systems.

$$\therefore hf_1=f_1\frac{L_1}{D_1^{\,5}}\frac{8Q_1^{\,2}}{\Pi^2 g}$$

for a second pipe $hf_2 = \dfrac{f_2L_2}{D_2^{\,5}}\dfrac{8Q_2^{\,2}}{\Pi^2 g}$

For two pipes to be equivalent,

$h_{f1} = h_{f2}$, $Q_1 = Q_2$

$$\therefore \frac{f_1L_1}{D_1^{\,5}}=\frac{f_2L_2}{D_2^{\,5}}$$

$$\Rightarrow L_2=L_1\frac{f_1}{f_2}\left(\frac{D_2}{D_1}\right)^5$$

- **Pipes in parallel**

A combination of two or more pipes connected as shown in fig. so that the flow is divided among the pipes & then is joined again, is a parallel – pipe system.

In parallel pipe – system the head losses are the same in each of the lines & the discharge are cumulative.

Fig.

$h_{f1} = h_{f2} = h_{f3} = \dfrac{P_A}{\gamma}+Z_A-\left(\dfrac{P_B}{\gamma}+Z_B\right)$

$Q = Q_1 + Q_2 + Q_3$

Two types of problems occur:

1) If the head loss b/n A & B is given, Q is determined.
2) If the total flow Q is given, then the head loss & distribution of flow are determined.

Size of pipes, properties, and roughness are assumed to be known. Since this type of problem is more complex, as neither the head loss nor the discharge for any one pipe is known. The procedure is:

1) Assume discharge Q'_1 through pipe 1,
2) Solve for $h'f_1$, using assumed discharge,
3) Using $h'f_1$, find Q'_2 & Q'_3
4) With the three discharges for a common head loss, now assume that the given Q is split up among the pipes in the same proportion as Q'_1, Q'_2 & Q'_3, Thus,

$$Q_1 = \frac{Q_1'}{\sum Q'}Q, \qquad Q_2 = \frac{Q_2'}{\sum Q'}Q, \qquad Q_3 = \frac{Q_3'}{\sum Q'}Q$$

5) Check the correctness of these discharges by computing hf_1, hf_2, & hf_3 for the computed Q_1, Q_2 & Q_3

$\rightarrow Q - Q_1 - Q_2 - Q_3 = 0$

Branching pipes

Let us consider three pipes connected to three reservoirs as in fig. below & connected together or branching at the common junction point J. We shall assume that all the pipes are sufficiently long that minor losses & velocity heads may be neglected. The continuity & energy eqn. require that the flow entering the junction equal the flow leaving it& that the pressure head at J (with open piezometer tube water at elevation P) be common to all pipes.
There being no pumps, the elevation of p must lie b/n the surfaces of reservoirs A& C. If p is level with the surface of reservoir B then water must flow in to B & $Q_1 = Q_2 + Q_3$
If P is below the surface of reservoir B then the flow must be out of B & $Q_1 + Q_2 = Q_3$
So for the situation of the following fig, we have the following governing conditions:

1) $Q_1 = Q_2 + Q_3$
2) Elevation of p is common to all.

a. Length, diameter, &friction factors are required.
b. The flow is steady & minor losses neglected
c. Three basic equations to solve these problems are:-
    i. Continuity equations
    ii. Bernoulli's equation
    iii. Darcy- weisbech equation

- Total rate of in flow at junction = total rate of out flow (continuity equation)

- Pipe 1 — D_1, L_1, V_1, Q_1 h_{f1}; Elevation , Z_1 , Reserv ,A
- Pipe 2 — D_2, L_2, V_2, Q_2, h_{f2}; Z_2, Reserv, B
- Pipe 3 — D_3, L_3, V_3, Q_3, h_{f3}; Z_3, Reserv. C

Junction of elevation

Zj, pressure head p_j/r = total head at junction = $(Zj + p_j/r)$

- Applying Bernoulli's egn b/n the jun citron point & each of reservoirs

$$=> \begin{cases} Z_1 = (pj/r + Zj) + hf_1 \text{ ---------- } (*) \quad (1) \\ Z_2 + h_{f2} = (pj/r + Zj) \text{ ---------- } (**) \quad (2) \\ Z_3 + h_{f3} = (pj/r + Zj) \text{ ---------- } (***) \quad (3) \end{cases}$$

=> If the head of reservoir A is grater than head at junction the flow is in to the junction from A & out of the junction to B&C

=> Q1 = Q2 + Q3 -------------------------- * (4)

$$\frac{\pi}{4}D_1^2 \; V_1 = \frac{\pi}{4}D_2^2 \; V_2 + \frac{\pi}{4} D_3^2 \; V_3 \text{ ------------ } (5)$$

$$=> D_1^2 \; V_1 = D_2^2 \; V2\ V2 + D_3^2 * V3 \text{ ------------- } (6)$$

- There are three types of problem fouling of branching pies :-

Case 1: Given all pipes data (L, D, E, Z_1 & Z_2 Q_1 or Q_2, find Z_3 ?

=> Solution: first hf_1 can be calculated directly (h_{f1} = $f_1 \frac{L_1}{D_1} v_2^1/^2 g$)

Then (p_j/r + Zj) piezometric head at junction can be determine

- ⇨ From eqn (2) h $_{f2}$ & Q_2 can be determined
- ⇨ Q_3 can be detrained from eqn (4) continuity eqn
- ⇨ Then from eqn (3) h_{f3} and finally Z_3 can be determined

Case 2: Given a pipe data, the surface elevation of two reservoirs (A& C) and the flow to or from the second, find Z_3 and Q_1, Q_3?

- ⇨ From eqn (1) & iii) (h_{f1} + h_{f3}) = (Z_1- Z_3) (h_{f1} + h_{f3}) is known & also (Q_1-Q_3) or (Q_3 – Q_1) is known.
- ⇨ Assume trail values of h_{f1} & h_{f3} & from these compute the discharge Q_1+Q_3 & compare with (Q_1-Q_3)
- ⇨ Repeat the procedure until the two values are equal.
- ⇨ From they, piezometric head at junction can be determined
- ⇨ From h +2 & (pj/r Zj) → Z_2 can be determined.

Case:3 Given a pipe lengths , diameters, and the elevation of all the three reservoirs , find Q_1 Q_2, Q_3,

→ In this case the direction of the flow is not known clearly.
→ Assam the elevation of B (Z_2) is equal to the piezometric head (Z_p) & (i.e an flow in pipe 2)
→ From Zp the head losses h_{f1} & h_{f3} determined, and then Q_1 & Q_3 can be obtained
→ If $Q_1 > Q_3$, then Zp must be increased to satisfy continuity eqn at J, causing water to flow into reservoir B, and we will have $Q_1 = Q_2 + Q_3$
→ If Q1<Q3, then Zp must be lowered, causing water to flow out of reservoir B, & we will have $Q_1 + Q_2 = Q_3$