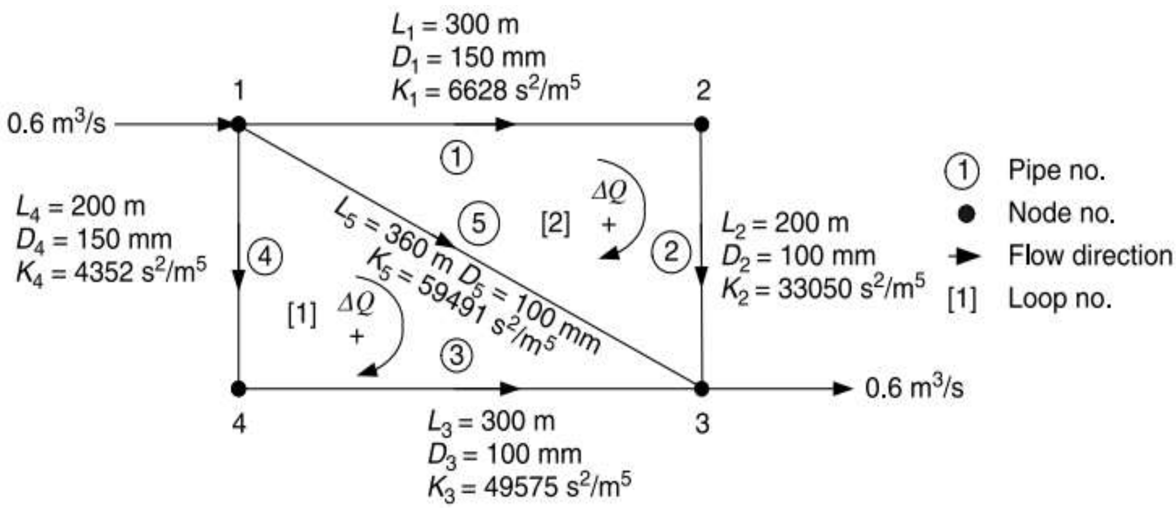
<u>Example 1</u>. The pipe network of two loops as shown in the next Fig has to be analyzed by the Hardy Cross method for pipe flows for given pipe lengths L and pipe diameters D. The nodal inflow at node 1 and nodal outflow at node 3 are shown in the figure. Assume a constant friction factor f = 0.02.

Solution.

The looped network analysis can be conducted as illustrated in this example. The K values for Darcy–Weisbach head loss–discharge relationship are also given in the next Fig. To obtain initial pipe discharges applying nodal continuity equation, the arbitrary pipe discharges equal to the total number of loops are assumed. The total number of loops in a network can be obtained from the following geometric relationship:



To apply continuity equation for initial pipe discharges, the discharges in pipes 1 and 5 equal to 0.1 m3/s are assumed. The obtained discharges are

Q1 = 0.1 m^3/s (flow from node 1 to node 2)

Q2 =0.1 m^3/s (flow from node 2 to node 3)

Q3 = 0.4 m^3/s (flow from node 4 to node 3)

Q4 =0.4 m^3/s (flow from node 1 to node 4)

Q5 =0.1 m^3/s (flow from node 1 to node 3)

Loop 1: Iteration 1

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	−0.40	49,576.12	−7932.18	39,660.89	−0.25
4	−0.40	4352.36	−696.38	3481.89	−0.25
5	0.10	59,491.34	594.91	11,898.27	0.25
Total			−8033.64	55,041.05	
ΔQ			$0.15\ m^3/s$		

Thus the discharge correction ΔQ in loop 1 is $0.15\ m^3/s$. The discharges in loop pipes are corrected as shown in the above table. Applying the same methodology for calculating ΔQ for Loop 2:

Loop 2: Iteration 1

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.10	6528.54	65.29	1305.71	0.19
2	0.10	33,050.74	330.51	6610.15	0.19
5	−0.25	59,491.34	−3598.93	29,264.66	−0.16
Total			−3203.14	37,180.52	
ΔQ			$0.09\ m^3/s$		

Loop 1: Iteration 2

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	−0.25	49,576.12	−3098.51	24,788.06	−0.21
4	−0.25	4352.36	−272.02	2176.18	−0.21
5	0.16	59,491.34	1522.98	19,037.23	0.20
Total			−1847.55	46,001.47	
ΔQ			$0.04\ m^3/s$		

Loop 2: Iteration 2

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.19	6528.54	226.23	2430.59	0.21
2	0.19	33,050.74	1145.28	12,304.85	0.21
5	−0.20	59,491.34	−2383.53	23,815.92	−0.17
Total			−1012.02	38,551.36	
ΔQ			$0.03\ m^3/s$		

Loop 1: Iteration 3

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	−0.210	49,576.12	−2182.92	20,805.82	−0.197
4	−0.210	4352.36	−191.64	1826.57	−0.197
5	0.174	59,491.34	1799.33	20,692.47	0.187
Total			−575.23	43,324.86	
ΔQ			$0.01\ m^3/s$		

Loop 2: Iteration 3

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.212	6528.54	294.53	2773.35	0.220
2	0.212	33,050.74	1491.07	14,040.10	0.220
5	−0.187	59,491.34	−2084.55	22,272.21	−0.180
Total			−298.95	39,085.67	
ΔQ			$0.008\ m^3/s$		

Loop 1: Iteration 4

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	−0.197	49,576.12	−1915.41	19,489.36	−0.193
4	−0.197	4352.36	−168.16	1711.00	−0.193
5	0.180	59,491.34	1917.68	21,362.18	0.183
Total			−165.89	42,562.54	
ΔQ			$0.004\ m^3/s$		

Loop 2: Iteration 4

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.220	6528.54	316.13	2873.22	0.222
2	0.220	33,050.74	1600.39	14,545.68	0.222
5	−0.183	59,491.34	−2001.85	21,825.91	−0.181
Total			−85.33	39,244.81	
ΔQ			$0.002\ m^3/s$		

Loop 1: Iteration 5

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	−0.193	49,576.12	−1840.21	19,102.92	−0.192
4	−0.193	4352.36	−161.55	1677.07	−0.192
5	0.181	59,491.34	1954.67	21,567.21	0.182
Total			−47.09	42,347.21	
ΔQ			$0.001\ m^3/s$		

Loop 2: Iteration 5

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.222	6528.54	322.40	2901.61	0.223
2	0.222	33,050.74	1632.17	14,689.40	0.223
5	-0.182	59,491.34	−1978.73	21,699.52	−0.182
Total			−24.15	39,290.53	
ΔQ			$0.001\ m^3/s$		

The discharge corrections in the loops are very small after five iterations, thus the final pipe discharges in the looped pipe network in Fig are

Q1 = 0.223 m^3/s

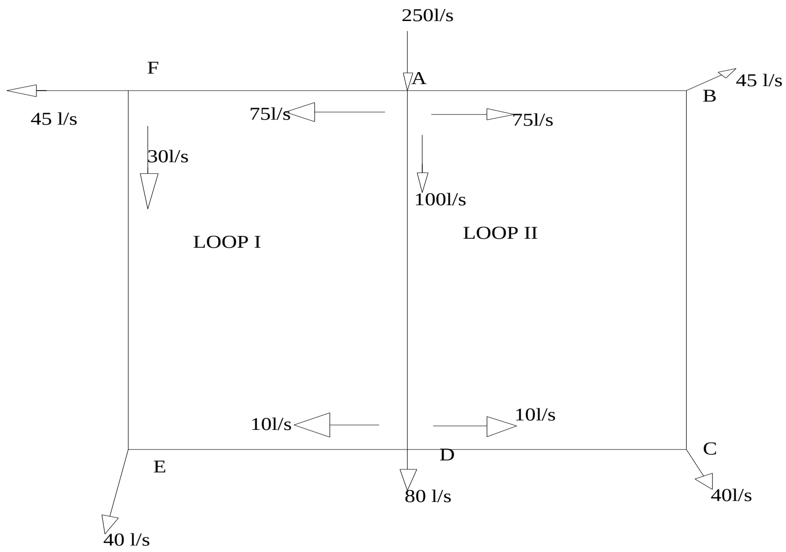
Q2 = 0.223 m^3/s

Q3 = 0.192 m^3/s

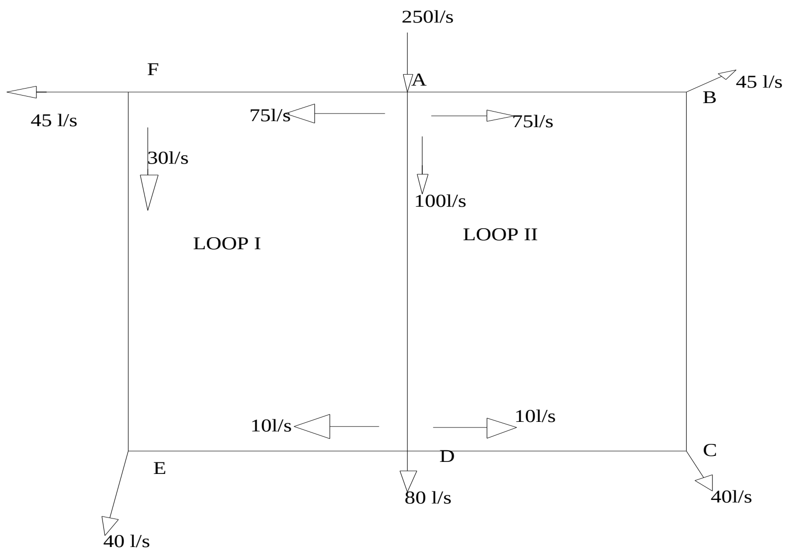
Q4 =0.192 m^3/s

Q5 = 0.182 m^3/s

Example-2 Find the flow distribution in the gravity supply system through the following pipe network shown below. Use Hazen – Williams formula (C= 100). Assume all pipe junctions are at the same elevation.



Solution



Hazen- Williams's formula, $Q = 0.278CD^{2.63}S^{0.54}$

$$S = \left[\frac{Q}{(0.278CD^{2.63})}\right]^{1/.54}$$

Exercise-2

C = 100

1st Correction loop I

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AD	250	1000	100	0.025	25.483	0.255
DE	150	2000	10	0.004	8.628	0.863
EF	200	1000	-30	-0.008	-8.127	0.271
FA	250	2000	-75	-0.015	-29.916	0.399
				∑=	-3.933	1.787

$\Delta Q11 =$ $\sum hL/1.85\sum(hL/Q)$

1.189

1st Correction loop II

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AB	250	2000	75	0.015	29.916	0.399
BC	200	1000	30	0.008	8.127	0.271
CD	100	2000	-10	-0.031	-62.164	6.216
DA	250	1000	-100	-0.025	-25.483	0.255
				∑=	-49.603	7.141

$\Delta Q11 =$ $\sum hL/1.85\sum(hL/Q)$

3.755

2nd Correction loop I

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AD	250	1000	97.435	0.024	24.285	0.249
DE	150	2000	11.189	0.005	10.624	0.949
EF	200	1000	-28.811	-0.008	-7.541	0.262
FA	250	2000	-73.811	-0.015	-29.044	0.393
				∑=	-1.675	1.854

$\Delta Q21 =$ $\sum hL/1.85\sum(hL/Q)$

0.488

2ndCorrection loop II

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AB	250	2000	78.755	0.016	32.749	0.416
BC	200	1000	33.755	0.010	10.111	0.300
CD	100	2000	-6.245	-0.013	-25.997	4.163
DA	250	1000	-97.435	-0.024	-24.285	0.249
				∑=	-7.423	5.127

$\Delta Q22=$ $\sum hL/1.85\sum(hL/Q)$

0.783

3rd Correction loop I

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m	Head loss hL= S*L (m)	hL/Q
AD	250	1000	97.140	0.024	24.150	0.249
DE	150	2000	11.678	0.006	11.499	0.985
EF	200	1000	-28.322	-0.007	-7.306	0.258
FA	250	2000	-73.322	-0.014	-28.689	0.391
				∑=	-0.346	1.882

ΔQ31 = ∑hL/1.85∑(hL/Q)

0.099

3rd Correction loop II

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m	Head loss hL= S*L (m)	hL/Q
AB	250	2000	79.537	0.017	33.354	0.419
BC	200	1000	34.537	0.011	10.549	0.305
CD	100	2000	-5.463	-0.010	-20.289	3.714
DA	250	1000	-97.140	-0.024	-24.150	0.249
				∑=	-0.535	4.687

ΔQ32= ∑hL/1.85∑(hL/Q)

0.062

4th Correction loop I

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m	Head loss hL= S*L (m)	hL/Q
AD	250	1000	**97.240**	0.024	24.196	0.249
DE	150	2000	**11.777**	0.006	11.681	0.992
EF	200	1000	**-28.223**	-0.007	-7.258	0.257
FA	250	2000	**-73.285**	-0.014	-28.662	0.391
				∑=	-0.044	1.889

ΔQ41 = ∑hL/1.85∑(hL/Q)

0.013

4th Correction loop II

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m	Head loss hL= S*L (m)	hL/Q
AB	250	2000	**79.599**	0.017	33.402	0.420
BC	200	1000	**34.599**	0.011	10.584	0.306
CD	100	2000	**-5.401**	-0.010	-19.866	3.678
DA	250	1000	**-97.178**	-0.024	-24.167	0.249
				∑=	-0.047	4.652

ΔQ42= ∑hL/1.85∑(hL/Q)

0.005

5th Correction loop I

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AD	250	1000	**97.247**	0.024	24.199	0.249
DE	150	2000	**11.790**	0.006	11.704	0.993
EF	200	1000	**-28.210**	-0.007	-7.252	0.257
FA	250	2000	**-73.272**	-0.014	-28.653	0.391
				∑=	-0.002	1.890

ΔQ51 = ∑hL/1.85∑(hL/Q)

0.001

5th Correction loop II

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AB	250	2000	**79.605**	0.017	33.406	0.420
BC	200	1000	**34.605**	0.011	10.587	0.306
CD	100	2000	**-5.395**	-0.010	-19.829	3.675
DA	250	1000	**-97.185**	-0.024	-24.170	0.249
				∑=	-0.006	4.649

ΔQ52= ∑hL/1.85∑(hL/Q)

0.001

6th Correction loop I

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AD	250	1000	**97.247**	0.024	24.199	0.249
DE	150	2000	**11.790**	0.006	11.705	0.993
EF	200	1000	**-28.210**	-0.007	-7.252	0.257
FA	250	2000	**-73.271**	-0.014	-28.652	0.391
				∑=	0.000	1.890

ΔQ61 = ∑hL/1.85∑(hL/Q)

0.000

6th Correction loop II

Pipe	Diameter D(mm)	Length L(m)	Flow Q(l/s)	Slope (m/1000m)	Head loss hL= S*L (m)	hL/Q
AB	250	2000	**79.605**	0.017	33.407	0.420
BC	200	1000	**34.605**	0.011	10.587	0.306
CD	100	2000	**-5.395**	-0.010	-19.824	3.675
DA	250	1000	**-97.185**	-0.024	-24.170	0.249
				∑=	0.000	4.649

ΔQ52= ∑hL/1.85∑(hL/Q)

0.000

Therefore, the flow distribution will be

Pipe	D(mm)	L(mm)	Q(l/s)	S	**hL(m)**
AD	250	1000	**97.247**	0.024	**24.20**
DE	150	2000	**11.790**	0.006	**11.70**
EF	200	1000	**-28.210**	-0.007	**-7.25**
FA	250	2000	**-73.271**	-0.014	**-28.65**
AB	250	2000	**79.605**	0.017	**33.41**
BC	200	1000	**34.605**	0.011	**10.59**
CD	100	2000	**-5.395**	-0.010	**-19.82**
DA	250	1000	**-97.185**	-0.024	**-24.17**